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Calculation of the Equivalent Capacitance of a Rib near a Single-Wire Transmission Line

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ABSTRACT

The effect of a rib lying on a ground plane on a single-wire transmission line can be represented by a lumped capacitance. In this paper, the capacitance is evaluated by solving a set of coupled integral equations. Parametric studies are carried out and the results are presented in graphical form.

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SECTION I

INTRODUCTION

As described in a number of recent reports (ref. 1,2,3) the calculation of EMP energy propagation within an electrically complex system, such as an aircraft, often employs simple transmission line theory. Such propagation models usually consist of one or more uniform transmission lines with discrete loads and distributed sources arising from the incident electromagnetic fields. In order to more accurately account for the nonuniform surroundings of actual transmission lines on aircraft, a number of "canonical problems" have been suggested in ref. (4). By solving such problems, it is possible to obtain estimates of the effects of local perturbations of the transmission line fields, and later relate these to

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1. Tesche, F.M., and T.K. Liu, "An Electric Model for a Cable Clamp on a Single Wire Transmission Line," AFWL Interaction Note 307, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, December 1976.

2a. Lam, John, "Equivalent Lumped Parameters for a Bend in a Two Wire Transmission Line: Part I. Inductance," AFWL Interaction Note 303, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, December 1976.

2b. Lam, John, "Equivalent Lumped Parameters for a Bend in a Two Wire Transmission Line: Part II. Capacitance," AFWL Interaction Note 304, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, January 1977.

3. Liu, T.K., "Electromagnetic Coupling between Multiconductor Transmission Lines in a Homogeneous Medium," AFWL Interaction Note 309 , Air Force Weapons Laboratory, Kirtland Air Force Base, NM, December 1976.

4. Tesche, F.M., M.A. Morgan, and B.A. Fishbine, "Evaluation of Present Internal EMP Interaction Technology: Description of Needed Improvements," AFWL EMP Interaction Note 264, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, October 1975.

lumped inductances and capacitances placed appropriately along an otherwise uniform transmission line. This concept is discussed in more detail in ref. (1).

One particular geometry that is often observed in the internal configurations of aircraft cables is shown in Figure 1, where a single wire transmission line of radius a and height b above the ground plane passes over a thin septum of height h. The wire and the septum are mutually perpendicular and not touching. Such a problem will model a cable passing over a rib in an aircraft fuselage or wing root. By considering many periodically spaced septums, pass and stop band characteristics can be determined for the line, as outlined in ref. (2).

In the treatment of this problem, it will be assumed that the septum thickness is very small. This implies that the major effect on the transmission line behavior will be due to a capacitive term in the lumped parameter representation of the obstacle. The equivalent circuit of the septum discontinuity can then be represented, as shown in Figure 2.

This paper describes in detail the calculation of this equivalent capacitance of the septum and presents the results of a parametric study of this canonical problem.

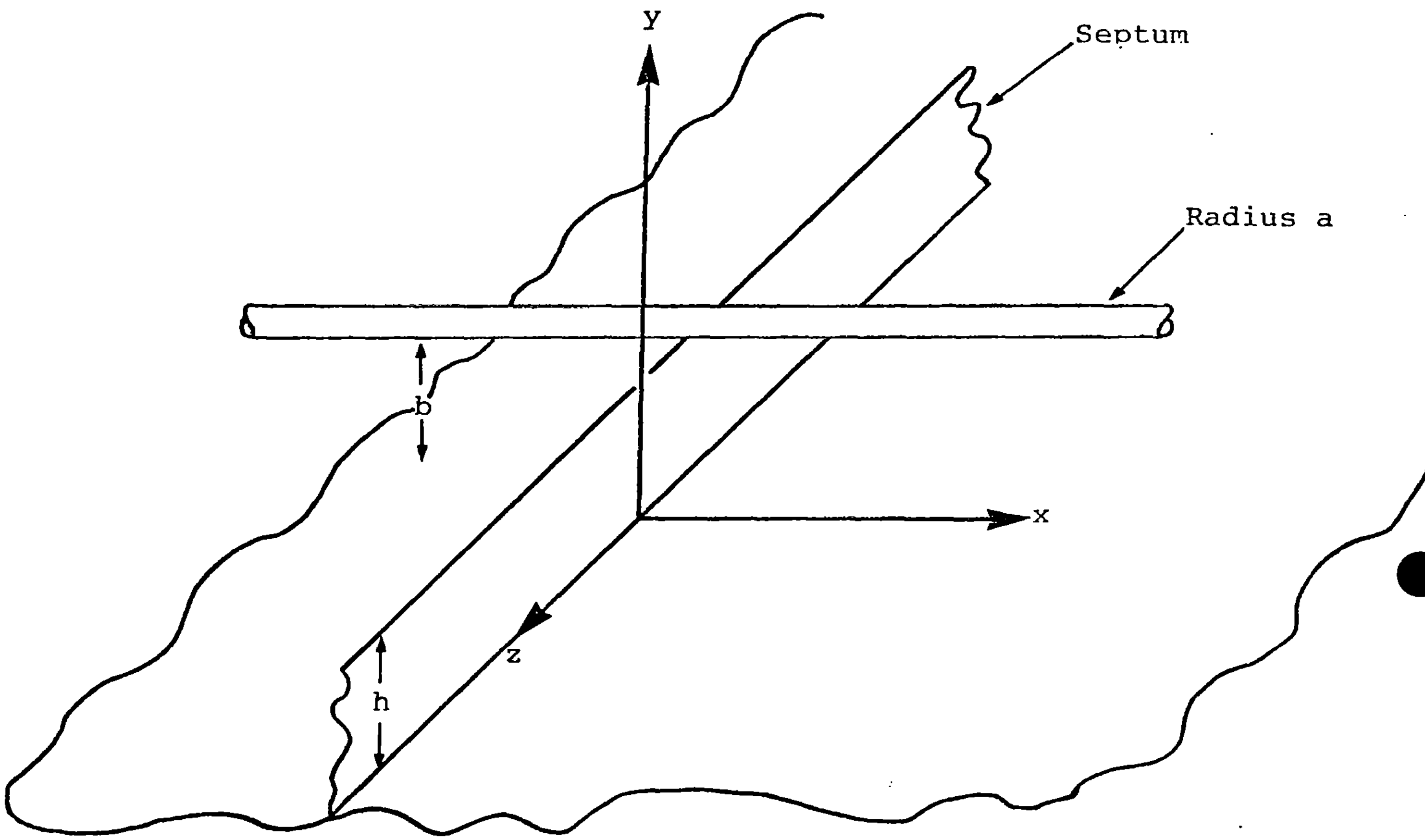

Figure 1. Geometry of the Problem

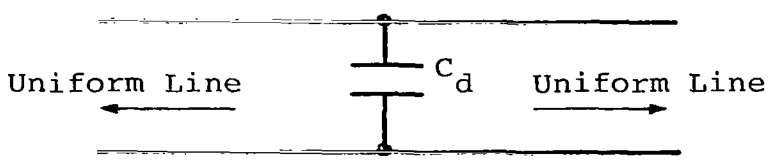

Figure 2. Equivalent Circuit of the
Septum Discontinuity

SECTION II

FORMULATION OF COUPLED INTEGRAL EQUATIONS

The equivalent capacitance of the septum discontinuity shown in Figure 2 may be obtained by solving Laplace's equation, subject to certain boundary conditions. If a Green's function approach is used, the problem may be reduced to the solution of a set of coupled integral equations for the unknown excess charge distribution on the wire and the charge distribtuion on the septum.

The three-dimensional free-space Green's function is given by

$$G_o(\vec{r}/\vec{r}\,') = \left\{4\pi\varepsilon_o \; |\vec{r}-\vec{r}\,'|\right\}^{-1} \tag{1}$$

where ε_o is the permittivity of free-space, $\vec{r}$ is the radius vector to a potential point, and $\vec{r}\,'$ the radius vector to a charge point. Using image theory, the perfectly conducting ground plane may be incorporated in the half-space Green's function G, given by

$$G(\vec{r}/\vec{r}\,') = G_o(\vec{r}/\vec{r}\,') - G_o(\vec{r}/\vec{r}_i\,') \tag{2}$$

where $\vec{r}_i\,'$ is the radius vector to an image charge point.

The superposition principle now shows that

$$\int_{\Gamma_1} \sigma_1(\vec{r}_1')\, G(\vec{r}_1/\vec{r}_1')\, dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2')\, G(\vec{r}_1/\vec{r}_2')\, dS_2' = V, \quad \vec{r}_1 \in \Gamma_1$$

$$\int_{\Gamma_1} \sigma_1(\vec{r}_1')\, G(\vec{r}_2/\vec{r}_1')\, dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2')\, G(\vec{r}_2/\vec{r}_2')\, dS_2' = 0, \quad \vec{r}_2 \in \Gamma_2 \tag{3}$$

where σ_1 is the surface charge density on the wire, σ_2 is the charge distribution on the septum, V is the potential on the wire (with respect to the ground plane), Γ_1 and Γ_2 correspond to the surface of the wire and diaphragm respectively. The coupled integral Equations (3) state that the potential on the wire or the diaphragm is produced by the charge distribtuions σ_1 and σ_2.

The charge distribution σ_1 contains two parts: a uniform charge distribution σ_o in the absence of the septum and an excess charge distribution $\delta\sigma_1$ caused by the septum discontinuity, i.e.,

$$\sigma_1 = \sigma_o + \delta\sigma_1 \tag{4}$$

This allows Equation (3) to be written as

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \; G_{11} \; dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2') \; G_{12} \; dS_2' = V - \int_{\Gamma_1} \sigma_o(\vec{r}_1') G_{11} dS_1', \quad \vec{r}_1 \epsilon \Gamma_1 \tag{5a}$$

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \; G_{21} \; dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2') \; G_{22} \; dS_2' = -\int_{\Gamma_1} \sigma_o(\vec{r}_1') G_{21} dS_1', \quad \vec{r}_2 \epsilon \Gamma_2 \tag{5b}$$

where $G_{mn} = G(\vec{r}_m/\vec{r}_n')$. Note that the integral on the right side of Equation (5a) defines the potential V on the uniform line, and the right-hand side of Equation (5b) is the potential $\psi(\vec{r}_2/\vec{r}_1')$ at $\vec{r}_2$ due to the uniform line at $\vec{r}_1'$. Equations (5a) and (5b) can be reexpressed as:

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \; G_{11} \; dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2') \; G_{12} \; dS_2' = 0, \quad \vec{r}_1 \epsilon \Gamma_1 \tag{6a}$$

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \; G_{21} \; dS_1' + \int_{\Gamma_2} \sigma_2(\vec{r}_2') \; G_{22} \; dS_2' = -\psi(\vec{r}_2/\vec{r}_1'), \quad \vec{r}_2 \epsilon \Gamma_2 \tag{6b}$$

where ψ is known exactly by suitable conformal transformation (cf. ref. 5). Equations (6a) and (6b) are the desired coupled integral equations where $\delta\sigma_1$ and σ_2 are sought. Note that these equations are exact.

5. Silvester, P., Modern Electromagnetic Fields, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1968.

SECTION III

APPROXIMATE SOLUTION OF THE COUPLED INTEGRAL EQUATION

A rigorous solution of Equations (6a) and (6b) involves integrals of the form

$$\sum_{n=0}^{\infty} \int_{-\infty}^{\infty} a_n(\xi)\,d\xi \int_0^{2\pi} \frac{\cos(n\theta')}{\sqrt{(z-\xi)^2 + 4a^2 \sin^2 \frac{\theta-\theta'}{2}}}\, d\theta' \qquad (7)$$

where θ and θ' are standard polar cylindrical coordinates on the wire. Equation (7) is obtained by assuming that

$$\delta\sigma_1(\xi,\theta') = \sum_{n=0}^{\infty} a_n(\xi) \cos(n\theta') \qquad (8)$$

This is equivalent to writing the Fourier series expansion for $\delta\sigma_1$ with coefficients a_n that are functions of ξ. Note that the integral over $[0,2\pi]$ has a logarithmic singularity for $z=\xi$ and $\theta=\theta'$; it cannot be integrated exactly for all values of n. An approximate solution, which is equivalent to truncating the Fourier series expansion at $n=0$, is used. This is often referred to in the literature as the "thin-wire approximation", and is an adequate approximation provided that the wire radius is small compared to other dimensions of the problem. A second approximation is introduced by assuming that the charge distribution on the septum is uniform in the y_o direction.

Note that the charge distribution on the septum has a square root singularity at the edge of the septum; the last approximation may be thus regarded as a first order approximation. The variational properties of the capacitance, however, lead to a second order approximation in the capacitance results. Here, (x_o, y_o, z_o) denotes the actual dimension in cartesian coordinates.

Applying the thin-wire approximation to Equation (6) we have

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \, G_{11} \, dS_1' \cong \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \delta\tau_1(x_o') \, K_{11}(x_o/x_o') \, dx_o' \tag{9a}$$

$$\int_{\Gamma_1} \delta\sigma_1(\vec{r}_1') \, G_{21} \, dS_1' \cong \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \delta\tau_1(x_o') \, K_{21}(y_o, z_o/x_o') \, dx_o' \tag{9b}$$

where $\delta\tau_1(x_o')$ is a linear charge density related to $\delta\sigma_1(\vec{r}_1')$ via the relation

$$\delta\tau_1(x_o') = 2\pi a \; \delta\sigma_1(\vec{r}_1') \tag{10}$$

and the kernels K_{11} and K_{21} are given by

$$K_{11}(x_o/x_o') = \left\{(x_o-x_o')^2 + a^2\right\}^{-\frac{1}{2}} - \left\{(x_o-x_o')^2 + 4b^2\right\}^{-\frac{1}{2}} \tag{11}$$

$$K_{21}(y_o, z_o/x_o') = \left\{(x_o')^2 + z_o^2 + (y_o-b)^2\right\}^{-\frac{1}{2}}$$

$$-\left\{(x_o')^2 + z_o^2 + (y_o+b)^2\right\}^{-\frac{1}{2}} \qquad (12)$$

Now assuming that

$$\sigma_2(\vec{r}_2') = f(y_o') \ \tau_2(z_o') \qquad (13)$$

where $\tau_2(z_o')$ is a linear charge density and $f(y_o')$ is uniform along the y_o' direction and has dimensions of Coul/m; Equation (6) now shows that

$$\int_{\Gamma_2} \sigma_2(\vec{r}_2') \ G_{12} \ dS_2' \cong \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \tau_2^*(z_o') dz_o' \int_0^h K_{12}(x_o/y_o', z_o') dy_o' \qquad (14)$$

$$\int_{\Gamma_2} \sigma_2(\vec{r}_2') \ G_{22} \ dS_2' \cong \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \tau_2^*(z_o') dz_o' \int_0^h K_{22}(y_o, z_o/y_o', z_o') dy_o' \qquad (15)$$

where the kernels K_{12} and K_{22} are given by

$$K_{12}(x_o/y_o', z_o') = \left\{x_o^2 + (b-y_o')^2 + (z_o')^2\right\}^{-\frac{1}{2}}$$

$$-\left\{x_o^2 + (b+y_o')^2 + (z_o')^2\right\}^{-\frac{1}{2}} \qquad (16)$$

$$K_{22}(y_o, z_o/y_o', z_o') = \left\{(y_o - y_o')^2 + (z_o - z_o')^2\right\}^{-\frac{1}{2}}$$

$$- \left\{(y_o + y_o')^2 + (z - z_o')^2\right\}^{-\frac{1}{2}} \qquad (17)$$

Note that $f(y_o')$ has been assumed constant and has been incorporated in τ_2^*. τ_2^* has dimensions of $\mathrm{Coul/m^2}$.

Equations (9) through (17) allow the coupled integral equations (6) to be written as

$$\int_{-\infty}^{\infty} \delta\tau_1(x_o')\ K_{11}(x_o/x_o')\ dx_o' + \int_{-\infty}^{\infty} \tau_2^*(z_o')\ dz_o' \int_0^h K_{12}(x_o/y_o', z_o')dy_o' = 0,$$

$$-\infty < x_o < \infty \qquad (18a)$$

$$\int_{-\infty}^{\infty} \delta\tau_1(x_o')\ K_{21}(y_o, z_o/x_o')\ dx_o' + \int_{-\infty}^{\infty} \tau_2^*\ (z_o')dz_o' \int_0^h K_{22}(y_o, z_o/y_o', z_o')dy_o'$$

$$= -4\pi\varepsilon_o\ \psi(y_o,\ z_o)$$

$$0 \leq y_o \leq h,\ -\infty < z_o < \infty\ . \qquad (18b)$$

For computational purposes it is convenient to introduce dimensional variables and functions given by

$$
\left.
\begin{aligned}
&(x_o, y_o, z_o / x_o', y_o', z_o') = b(x, y, z/x', y', z') \\
&\delta\tau_1(x_o') = V\,\varepsilon_o\,\xi(x') \\
&\tau_2^*(z_o') = Vb^{-1}\,\varepsilon_o\,\eta(z') \\
&\psi(y_o, z_o) = V\,\phi(y, z) \\
&K_{ij}(r_o/r_o') = b^{-1}\,L_{ij}(r/r'), \quad i,j = 1,2
\end{aligned}
\right\} \qquad (19)
$$

With this choice of variables and functions, the coupled integral equations (18) take the form

$$
\int_{-\infty}^{\infty} \xi(x')\, L_{11}(x/x')\, dx' + \int_{-\infty}^{\infty} \eta(z')\, dz' \int_{0}^{h/b} L_{12}(x/y', z')\, dy' = 0
$$

$$
-\infty < x < \infty \qquad (20a)
$$

$$
\int_{-\infty}^{\infty} \xi(x')\, L_{21}(y, z/x')\, dx' + \int_{-\infty}^{\infty} \eta(z')\, dz' \int_{0}^{h/b} L_{22}(y, z/y', z')\, dy'
$$

$$
= -4\pi\, \phi(y, z)
$$

$$
0 \le y \le h/b, \; -\infty < z < \infty \qquad (20b)
$$

Note that the integration of L_{12} and L_{22} over $(0, h/b)$ may be performed exactly (cf. ref. 6); Equation (20) is thus reduced to

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6. Gradshteyn, I.S. and Ryzhik, I.M., Table of Integrals Series and Products, Academic Press, New York, 1965.

$$\int_{-\infty}^{\infty} \xi(x')L_{11}(x/x')dx' + \int_{-\infty}^{\infty} \eta(z')L_{12}^{*}(x/z')dz' = 0$$

$$-\infty < x < \infty \qquad (21a)$$

$$\int_{-\infty}^{\infty} \xi(x')L_{21}(y,z/x')dx' + \int_{-\infty}^{\infty} \eta(z')L_{22}^{*}(y,z/z')dz'$$

$$= -4\pi \ \phi(y,z)$$

$$0 \le y \le h/b, \quad -\infty < z < \infty \qquad (21b)$$

where L_{12}^{*} and L_{22}^{*} are given by

$$L_{12}^{*}(x/z') = \ln \frac{\sqrt{x^2 + (1-h/b)^2 + (z')^2} - (1 - h/b)}{\sqrt{x^2 + 1 + (z')^2} - 1}$$

$$- \ln \frac{\sqrt{x^2 + (1+h/b)^2 + (z')^2} + (1 + h/b)}{\sqrt{x^2 + (z')^2 + 1} + 1} \qquad (22)$$

$$L_{22}^{*}(y,z/z') = \ln \frac{\sqrt{(h/b-y)^2 + (z-z')^2} + (h/b - y)}{\sqrt{y^2 + (z-z')^2} - y}$$

$$- \ln \frac{\sqrt{(h/b+y)^2 + (z-z')^2} + (h/b + y)}{\sqrt{y^2 + (z-z')^2} + y} \qquad (23)$$

The coupled integral equation (21) may be solved by using the following Galerkin's approach. First integrating Equation (21b) with respect to y gives

$$\int_{-\infty}^{\infty} \xi(x')L_{11}(x/x')dx' + \int_{-\infty}^{\infty} \eta(z')L_{12}^{*}(x/z')dz' = 0 \qquad -\infty < x < \infty$$

$$\int_{-\infty}^{\infty} \xi(x')L_{21}^{*}(z/x')dx' + \int_{-\infty}^{\infty} \eta(z')L_{22}^{**}(z/z')dz' = \eta(z) \qquad -\infty < z < \infty \tag{24}$$

where the kernels L_{21}^{*} and L_{22}^{**} are obtained by integrating L_{21} and L_{22}^{*} with respect to y over $(0,h/b)$. Similarly, η is obtained by integrating $-4\pi\phi$ over $(0,h/b)$. These are given by

$$L_{21}^{*}(z/x') = L_{12}^{*}(x/z')$$

$$L_{22}^{**}(z/z') = \frac{h}{b}\ \ell n \frac{\sqrt{(z-z')^2 + (h/b)^2} + (h/b)}{\sqrt{(z-z')^2 + (h/b)^2} - (h/b)}$$

$$+ \frac{2h}{b}\ \ell n \frac{\sqrt{(z-z')^2 + (h/b)} + (h/b)}{\sqrt{(z-z')^2 + (2h/b)^2} + (2h/b)}$$

$$+ 3|z-z'| + \sqrt{(z-z')^2 + (2h/b)^2} - 4\sqrt{(z-z')^2 + (h/b)^2} \tag{26}$$

$$\eta(z) = -\frac{2\pi}{\ell n\ (a/2b)} \left\{ \ell n \frac{(1+z^2)^2}{[(1-h/b)^2 + z^2][(1+h/b)^2 + z^2]} \right.$$

$$+ \frac{h}{b}\ \ell n \frac{(1-h/b)^2 + z^2}{(1+h/b)^2 + z^2} - 2z \left[\tan^{-1}\left(\frac{1-h/b}{z}\right) + \tan^{-1}\left(\frac{1+h/b}{z}\right)\right.$$

$$\left.\left. - 2\tan^{-1}(1/z) \right] \right\} \qquad (27)$$

where, again, use was made of table of integrals (ref. 6). Note that $L_{22}^{**}(z/z')$ has a logarithmic singularity $-\ell n\ |z-z'|$ and $L_{11}(x/x')$ has a sharp peak at $x=x'$. For computational purposes, these may be treated as follows: First write

$$L_{22}^{**}(z/z') = -\frac{2h}{b}\ \ell n\ |z-z'| + F(z/z') \qquad (28)$$

where $F(z/z')$ is continuous throughout the interval $(-\infty,\infty)$. Next integrate by parts the integral in equation (24) which contains $L_{11}(x/x')$. Equation (24) then becomes

$$\left.\begin{aligned}
&\int_{-\infty}^{\infty} \frac{\partial\xi(x')}{\partial x'} L_{11}^{*}(x/x')\,dx' + \int_{-\infty}^{\infty} \eta(z')\,L_{12}^{*}(x/z')\,dz' = 0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad -\infty < x < \infty \\
&\int_{-\infty}^{\infty} \xi(x')\,L_{21}^{*}(z/x')\,dx' + \int_{-\infty}^{\infty} \eta(z')\,F(z/z')\,dz' \\
&\qquad - \frac{2h}{b}\int_{-\infty}^{\infty} \ell n\ |z-z'|\ \eta(z')\,dz' = \eta(z) \\
&\qquad\qquad\qquad\qquad\qquad\qquad -x < z < \infty
\end{aligned}\right\} \qquad (29)$$

where

$$L_{11}^{*}(x/x') = \ell n \frac{\sqrt{(x'-x)^2 + (a/b)^2} + (x'-x)}{\sqrt{(x'-x)^2 + 4} + (x'-x)}$$

Equation (29) is noted to have a smoother integrand than Equation (24), which is more amenable to numerical computations; the integrand containing the logarithm will be evaluated in closed form in Section V. The second stage in the use of Galerkin's approach for obtaining the solution to Equation (29) requires knowledge of the properties of Hermite polynomials and the Gauss-Hermite formula. These will be discussed in the following section.

SECTION IV

HERMITE POLYNOMIALS

The Hermite polynomials $H_j(\xi)$ are defined over the infinite interval $(-\infty, \infty)$ and satisfy the orthogonality conditions

$$\int_{-\infty}^{\infty} e^{-\xi^2} H_i(\xi)\, H_j(\xi)\, d\xi = \sqrt{\pi}\; 2^i\; i!\; \delta_{ij} \tag{30}$$

where δ_{ij} is the Kronecker delta.

The first few Hermite polynomials are:

$$H_o(\xi) = 1, \quad H_1(\xi) = 2\xi, \quad H_2(\xi) = 4\xi^2 - 2 \tag{31}$$

and the recurrence relation is

$$H_{n+1}(\xi) - 2\xi\, H_n(\xi) + 2n\, H_{n-1}(\xi) = 0 \tag{32}$$

These arise in integration over $(-\infty, \infty)$, and the Gauss-Hermite formula for approximating the integral is given by

$$\int_{-\infty}^{\infty} e^{-\xi^2} f(\xi)\, d\xi = \sum_{k=1}^{N} w_k\, f(\xi_k) \tag{33}$$

where ξ_k is the k^{th} zero of $H_n(\xi)$ and the weights w_k is given by

$$w_k = \frac{2^{n-1}\, n!\, \sqrt{\pi}}{n^2\, [H_{n-1}(\xi_k)]^2} \tag{34}$$

The weights w_k and abscissas ξ_k are given, for example, by Abramowitz and Stegun (ref. 7).

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7. Abramowitz, M., and J.A. Stegun, Handbook of Mathematical Functions, New York: Dover Publications, 1964.

SECTION V

NUMERICAL SOLUTIONS OF THE COUPLED INTEGRAL EQUATIONS

The coupled integral equation (29) is reduced to a set of algebraic equations by using a Galerkin's approach with Hermite polynomials as base functions. Assuming that

$$\xi(x') = e^{-(x')^2} \sum_{n=0}^{N} a_n H_n(x')$$
$$\tag{35}$$
$$\eta(z') = e^{-(z')^2} \sum_{m=0}^{N} b_m H_m(z')$$

where a_n, b_m are coefficients to be determined, and $H_\ell(\cdot)$ is Hermite polynomial of order ℓ. Further, note that

$$-\frac{2h}{b} \int_{-\infty}^{\infty} e^{-z'^2} H_n(z') \ln |z-z'| dz' = \int_{-\infty}^{\infty} e^{-z'^2} H_{n+1}(z') S(z/z') dz' \quad (36)$$

where $S(z/z') = \frac{2h}{b} (z'-z) [\ln |z'-z| - 1]$. This is obtained by integration by parts and the recurrence relation for Hermite polynomials. The integral equation (29) now takes the form

$$\sum_{n=0}^{N} a_n \int_{-\infty}^{\infty} e^{-(x')^2} H_{n+1}(x') \; L_{11}^{*}(x/x')dx'$$

$$+ \sum_{m=0}^{N} b_m \int_{-\infty}^{\infty} e^{-(z')^2} H_m(x') \; L_{12}^{*}(x/z')dz = 0 \qquad -\infty < x < \infty$$

$$\sum_{n=0}^{N} a_n \int_{-\infty}^{\infty} e^{-(x')^2} H_n(x') \; L_{21}^{*}(z/x')dx'$$

$$+ \sum_{m=0}^{N} b_m \int_{-\infty}^{\infty} e^{-(z')^2} \left\{ S(z/z')H_{m+1}(z') + F(z/z')H_m(z') \right\} dz'$$

$$= \eta(z) \qquad -\infty < z < \infty \tag{37}$$

The integrals may be evaluated approximately by using the Gauss-Hermite quadrature (33); the result is

$$\sum_{n=0}^{N} a_n \, g_n^{(1)}(x) + \sum_{m=0}^{N} b_m \, g_m^{(2)}(x) = 0 \qquad -\infty < x < \infty$$

(38)

$$\sum_{n=0}^{N} a_n \, g_n^{(3)}(z) + \sum_{m=0}^{N} b_m \, g_m^{(4)}(z) = f\,(z) \qquad -\infty < z < \infty$$

where

$$g_n^{(1)}(x) \cong \sum_{k=0}^{M_1} W_k \; L_{11}(x/x_k') \; H_n(x_k') \tag{39a}$$

$$g_m^{(2)}(x) \cong \sum_{k=0}^{M_1} W_k \; L_{12}(x/z_k') \; H_m(z_k') \tag{39b}$$

$$g_n^{(3)}(z) \cong \sum_{k=0}^{M_1} W_k \; L_{21}(z/x_k') \; H_n(x_k') \tag{39c}$$

$$g_m^{(4)}(z) \cong \sum_{k=0}^{M_1} W_k [S(z/z_k') \; H_{m+1}(z_k') + F(z/z_k') \; H_m/Z_k')] \tag{39d}$$

and M_1 is the order of this quadrature.

Applying Gauss-Hermite formula (33) once more to Equations (38) results

$$\begin{aligned} &\sum_{n=0}^{N} a_n \; T_{jn}^{(1)} + \sum_{m=0}^{N} b_m \; T_{jm}^{(2)} = 0, \quad j = 0,1,2,\ldots N \\ &\sum_{n=0}^{N} a_n \; T_{\ell n}^{(3)} + \sum_{m=0}^{N} b_m \; T_{\ell n}^{(4)} = d_\ell, \quad \ell = 0,1,2,\ldots N \end{aligned} \tag{40}$$

Equation (40) is a set of linear equations for the determinations of the unknown coefficients a_n, b_n, $n = 0,1,2,...N$, which may be solved on a digital computer by means of standard matrix inversion routine.

Once the coefficients a_n, b_m are determined, the capacitance of the discontinuity C_d may be obtained by integrating the excess charge density on the wire. The orthogonality conditions of Hermite polynomials may be used to perform this integration exactly; the result is

$$C_d = \sqrt{\pi}\ a_o \tag{41}$$

Notice, therefore, that the capacitance C_d depends only on the first term of the Hermite polynomials expansion for the excess charge distribution. In fact, it can be shown that the capacitance of the discontinuity, as given by Equation (41), is stationary with respect to arbitrary small variations in the functional form of the excess charge distribution and is lower bound (c.f., Section IV of ref. 8).

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8. S. Coen and G.M.L. Gladwell, "A Legendre approximation method for circular microstrip disk problem," IEEE Transactions on Microwave Theory and Techniques, Vol MTT-25, No. 1, January 1977.

SECTION VI

NUMERICAL RESULTS

No exact or approximate results to the present problem are known to the authors. It is intended, therefore, that the results for the equivalent capacitance of the septum, obtained from the present analysis, be compared with experimental results in the future (ref. 9).

Figures 3 through 5 present dimensionless excess charge density on the wire, obtained from the present analysis, each for a particular h/b and for $a/b = 0.00., 0.01, 0.1$. Note that the excess charge density is an even function of x, thus only the positive range of x is shown. These results have been obtained with $N = 10$ in Equation (35), and the order of the Gauss-Hermite quadrature is $M_1 = 22$ in equation (39). Note that the curves displace kinks around $x = 1.6$. This phenomenon is more evident for larger values of h/b and larger values of a/b. It is attributable to the fact that only a finite order of the Gauss-Hermite quadrature is taken. One would obtain smoother curves if higher order quadratures were used.

Figure 6 presents the equivalent capacitance of the septum C_d, normalized to the capacitance of the uniform line

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9. Dr. Larry Scott, Mission Research Corporation, Albuquerque, New Mexico, private communication, December 1976.

(in the absence of the septum discontinuity) per unit length times the height of the thin wire from the ground plane, as a function of h/b for a range of a/b. The same plot is presented in Figure 7 with h/b as a parameter and a/b as variables in logarithmic scale.

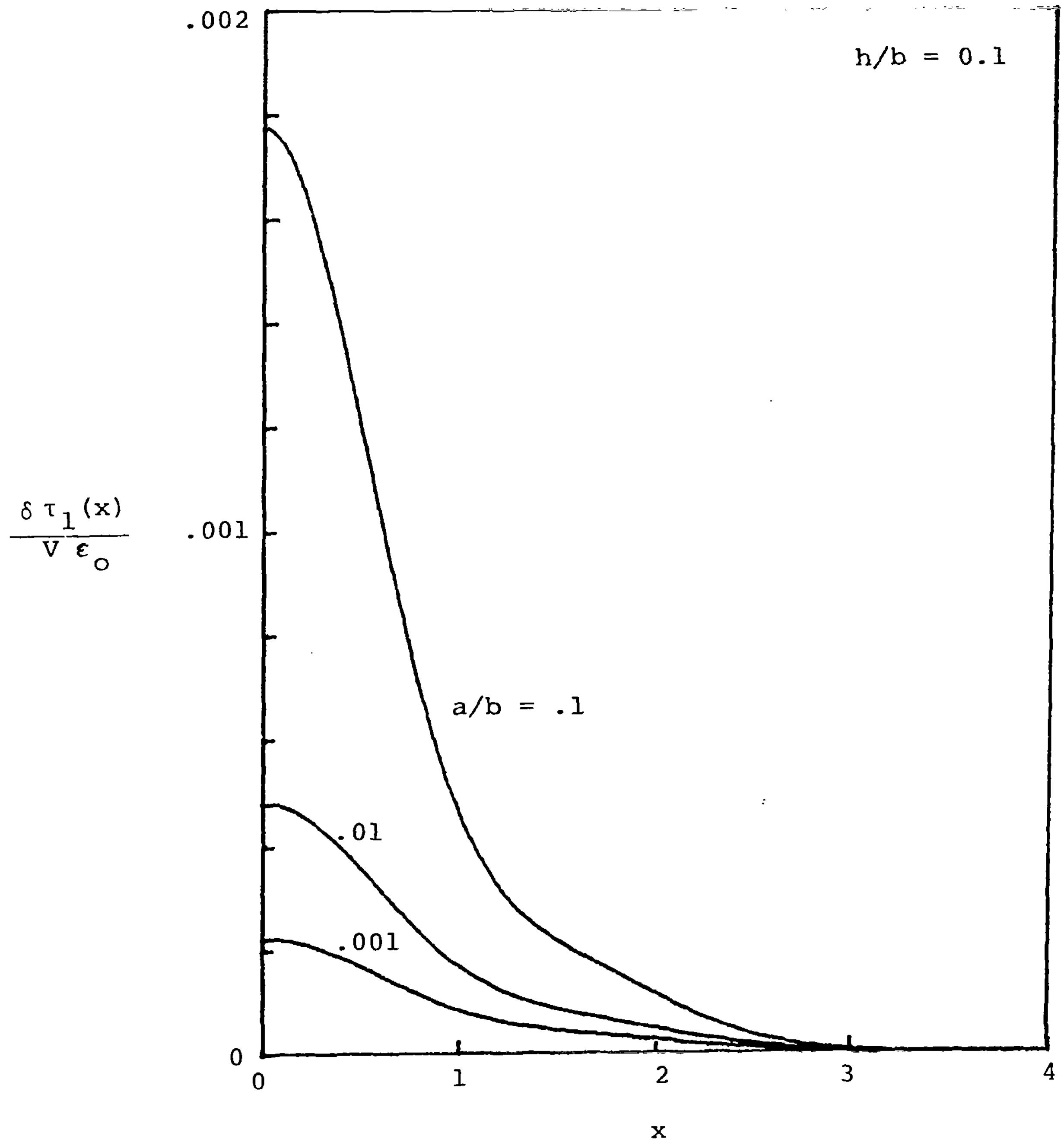

Figure 3. Distribution of normalized excess charge for $h/b = 0.1$. Note that $\delta\tau_1$ is an even function of x.

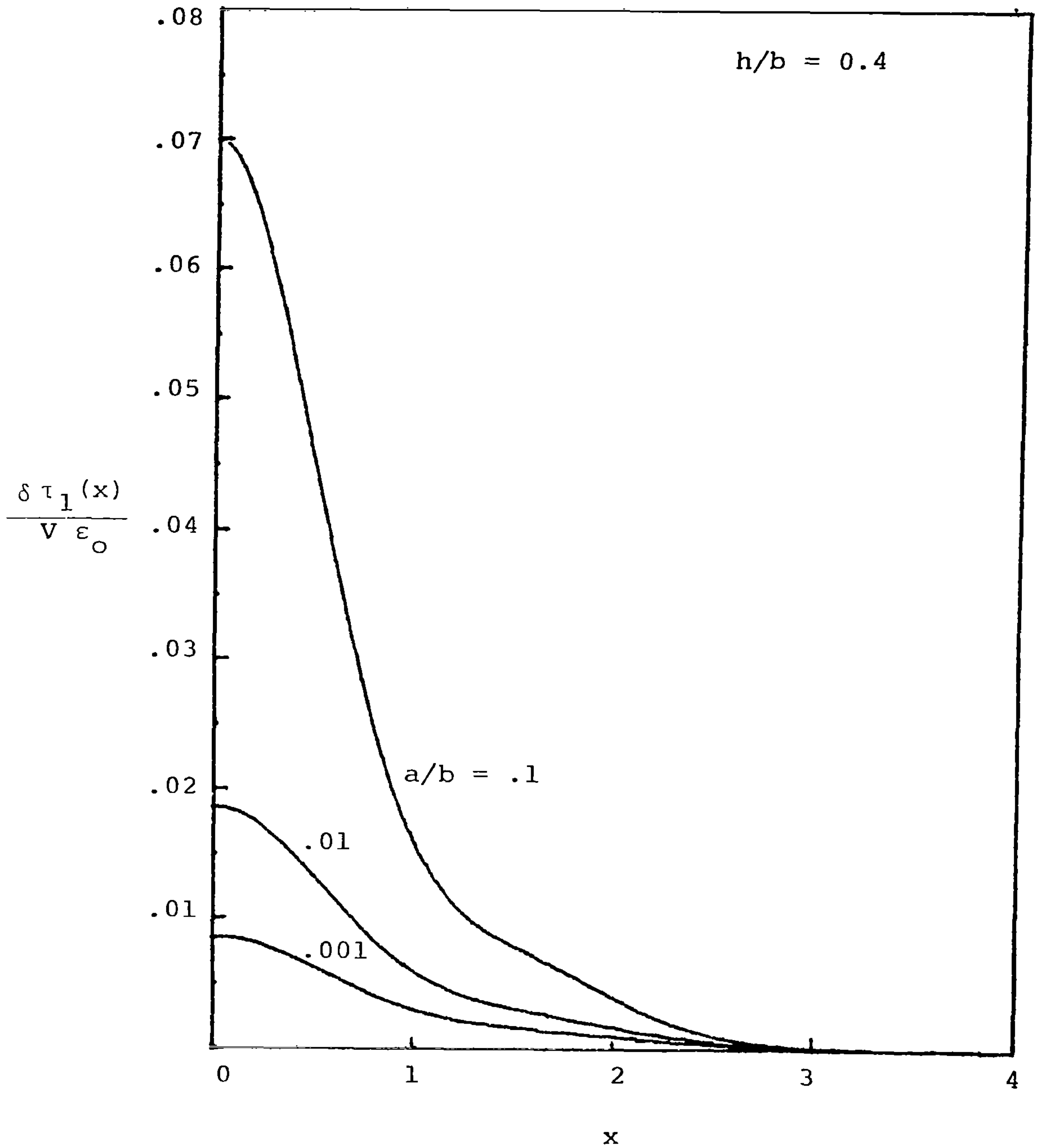

Figure 4. Distribution of normalized excess charge for h/b = 0.4.

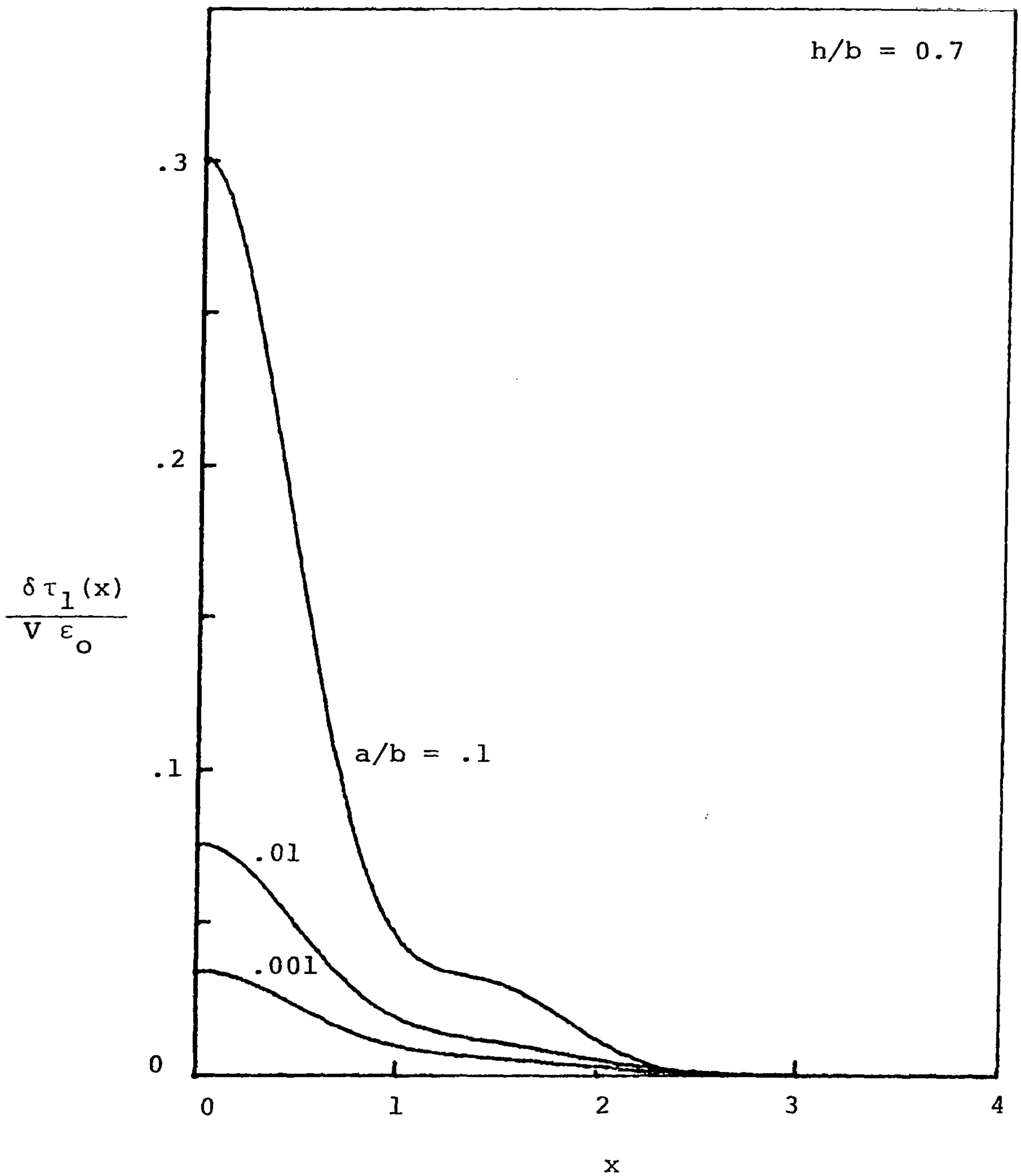

Figure 5. Distribution of normalized excess charge for h/b = 0.7.

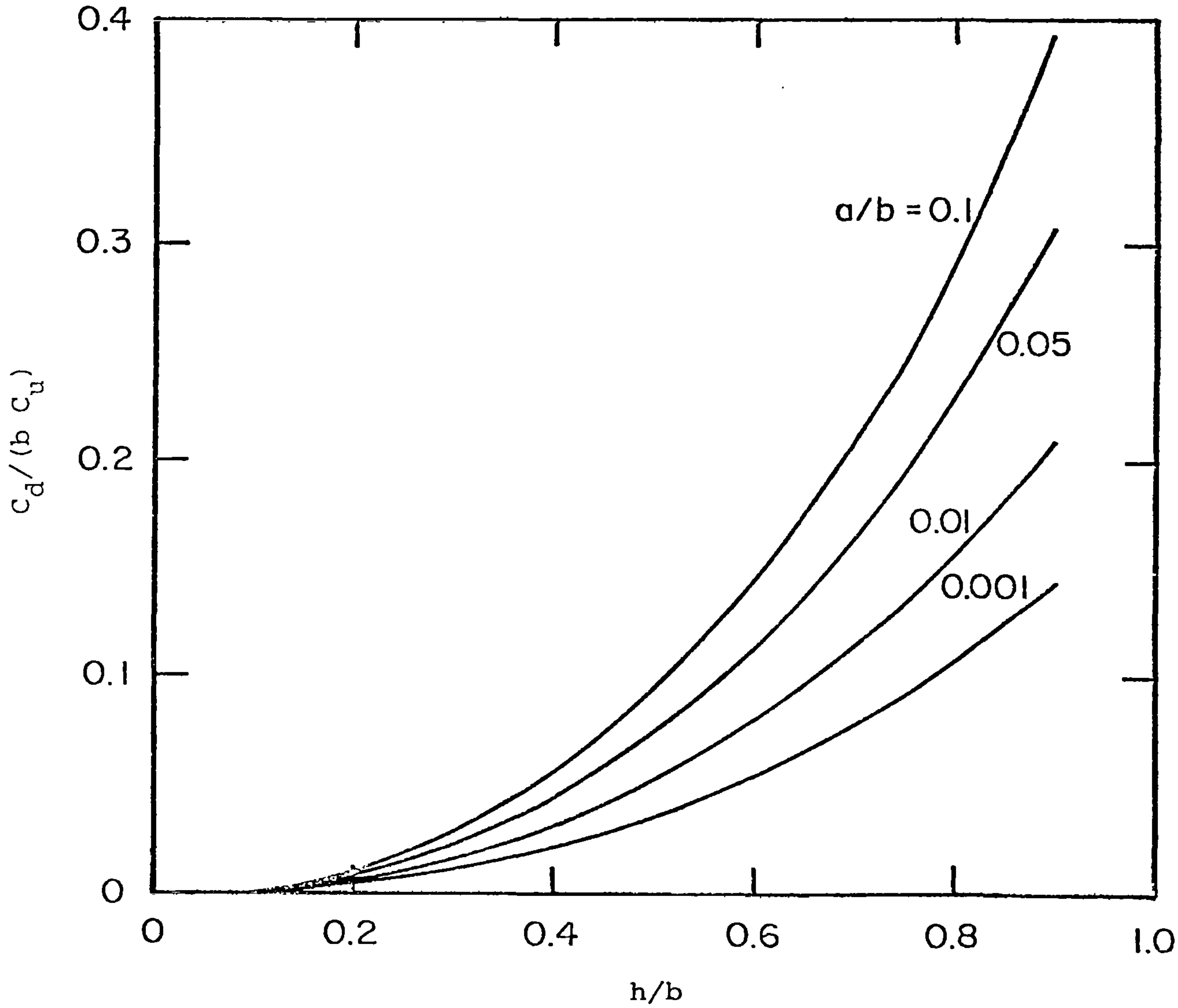

Figure 6. Capacitance due to septum as function of septum height. C_u is capacitance per unit length of unperturbed wire.

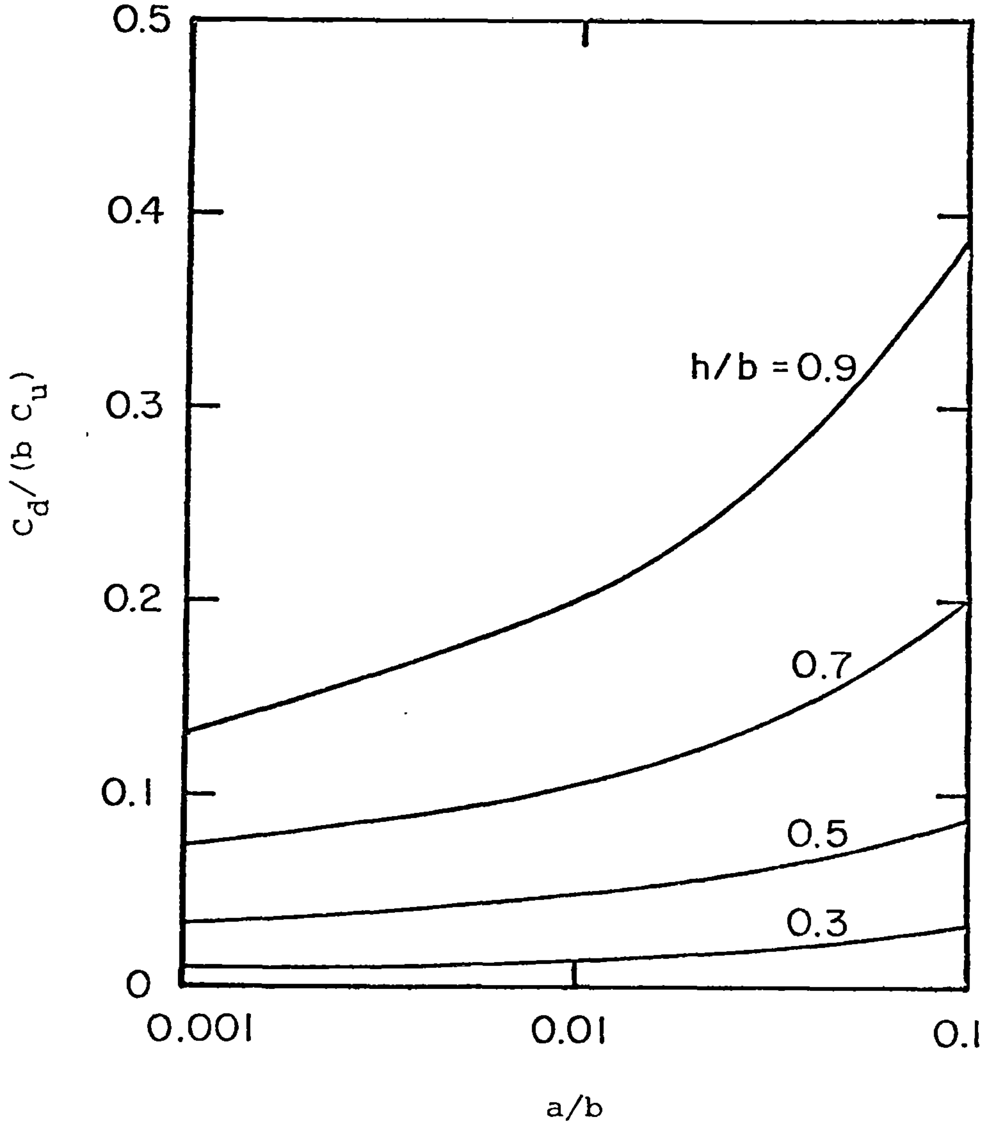

Figure 7. Capacitance due to septum as function of septum height.

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