Interaction Notes
Note 351
September 1978

Field Excitation of Multiconductor Transmission Lines

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ABSTRACT

This note discusses the excitation of a multiconductor transmission line by an incident electromagnetic field. Specific relations for the terminal (or load) response of a multiconductor line are derived in terms of field-induced voltage and current sources which are distributed along the transmission line. It is shown how these sources are derived for a general multiconductor line and how they may be derived from a knowledge of the incident (or free space) fields by using a field coupling parameter. The components of the exciting fields along the line are then given explicitly for the special case of an incident plane wave with arbitrary angle of incidence.

ACKNOWLEDGEMENT

The authors would like to thank Dr. Carl E. Baum, Air Force Weapons Laboratory, Dr. K.S.H. Lee, Dikewood Industries, Inc., and Dr. S. Frankel, Sidney Frankel and Associates, for their useful discussions on the subject of this report.

SECTION I
INTRODUCTION

Recently there has been a renewed interest in transmission-line theory and its application to the internal interaction problems involving electromagnetic pulse (EMP) excitation of aerospace systems. One new development in this area has been the formulation of an analysis procedure to study large interconnected networks of multi-conductor transmission lines. This analysis, which is described in refs. (1) and (2), and the resulting computer program (ref. 3), will permit not only simple branching of transmission lines within the network, but also complicated looping of lines. Thus, an arbitrarily interconnected set of transmission lines can be analyzed using this approach.

The analysis of the transmission-line networks described in refs. (1) and (2) is based on the network excitation being due to lumped (or discrete) voltage and current sources located at a source position somewhere along each transmission-line section (tube). While this specification of sources may be useful for certain applications, it is not particularly useful for EMP studies, where the transmission-line network is excited by an incident, transient electromagnetic field. In the EMP case, not only is the transmission-line excitation distributed along the line, but the fundamental excitation quantities are the incident electric and magnetic fields ($\overline{E}$ and $\overline{B}$), not the current and voltage sources. Thus, it is necessary to modify the past analysis to permit distributed field excitation of the transmission lines.

Field excitation of simple open two-wire lines has been considered by a number of authors and two separate, but equivalent, approaches used. Taylor, Satterwhite and Harrison (ref. 4) and Smith (ref. 5) derive a coupling model based on the incident tangential electric fields on both wires of the transmission line and on the short wires of the loads at the ends of the ine. This approach is based on the integral form of Maxwell's equations as applied to the closed loop formed by the two parallel wires of the transmission line and the two loads at the ends. In this formulation, there appear distributed voltage sources in both wires of the transmission line, as well as voltage sources at both loads terminating the line.

A different approach has been used by Lee (ref. 6) to determine the distributed field excitation. This is based on the differential forms of Maxwell's equations and yields distributed current and voltage sources along the line, with the voltage source being proportional to the $\overline{H}$ field and the current source being related to the $\overline{E}$ field.

Both of these formulations yield identical results for computing the TEM currents flowing on a two-wire line excited by an incident field. The former approach has been extended to the case of multiconductor transmission lines by Paul (ref. 7) and Frankel (ref. 8), and is similar to that discussed in this report. A slightly different approach has been employed by Kajfez and Wilton in ref. (9), where the concepts of reciprocity have been used to obtain the multiconductor transmission-line response to a small aperture excitation of the line. The method of refs. (4) and (5) has been applied to multiconductor systems by Strawe (ref. 10), but his report is not widely distributed.

The present report discusses in detail the excitation of multiconductor transmission lines by an incident electromagnetic field using the differential formulation. Section II presents the derivation of the equations describing the terminal, or load, current responses of a multiconductor transmission line. These equations have, as sources, both distributed voltage and current generators which are induced by incident magnetic and electric fields. Section III first discusses the derivation of these local sources in terms of the local fields and transmission-line geometry. The concept of an "equivalent separation" between conductors, as commonly used for two-wire lines, is then developed for an arbitrary multiconductor transmission line. Finally, in Section IV, the incident field components which contribute to the distributed sources are given for an incident plane wave striking the line at an arbitrary angle of incidence.

SECTION II

MULTICONDUCTOR TRANSMISSION-LINE RESPONSE TO DISTRIBUTED SOURCES

As discussed in ref. (1), the response of a general transmission-line network may be calculated by decomposing the currents on each tube of the transmisison line into forward and reverse propagating components. At every junction within the network, a scattering matrix can be derived to express all scattered components of current in terms of the incident components. These two sets of relations can be combined to form a large matrix equation for the incident currents. This equation, called the BLT equation, can be inverted numerically and the incident currents determined. Through the scattering matrices, the scattered and, thus, the total currents on the lines, can be determined.

A basic element of the above network analysis is the determination of the propagation properties of the forward and backward waves on the line, as well as their relative excitation by sources along the line. For the purpose of this section, therefore, we will consider only a single section (tube) of multiconductor transmission line.

Consider a lossless section of multiconductor transmission line having no sources, as shown in Figure 1. The length of the line is denoted by ℓ and it contains N wires with the $N+1^{st}$ wire being the reference conductor. The N+1 wires are required to be parallel, but not necessarily coplanar. For such a line, its electrical properties are determined by a capacitive coefficient matrix, $(C'_{n,m})$, and an inductive coefficient matrix, $(L'_{n,m})$, which depend only on line geometry and dielectric properties around the line. For this line, these matrices are nonsingular matrices of order N.

As discussed in ref. (1), the voltages and currents on this line without sources must obey a coupled set of partial differential equations as

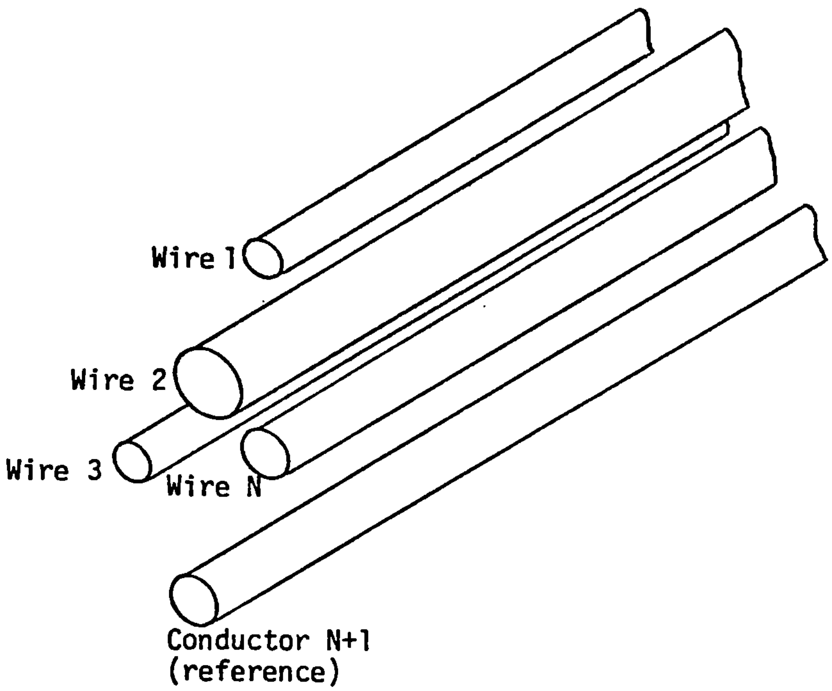

Figure 1. Section of multiconductor transmission line.

$$\frac{\partial}{\partial z}\begin{pmatrix}(\tilde{V}_n(z,s))\\(\tilde{I}_n(z,s))\end{pmatrix} = -s\begin{pmatrix}(0_{n,m}) & (L'_{n,m})\\(C'_{n,m}) & (0_{n,m})\end{pmatrix}\begin{pmatrix}(\tilde{V}_n(z,s))\\(\tilde{I}_n(z,s))\end{pmatrix} \qquad (1)$$

where the notation $(\tilde{V}_n)$ represents an N-vector for the line voltage and a similar notation holds for the current. The parameter s is the complex frequency variable, and the tilde represents a Laplace transformed quantity.

Equation (1) can be manipulated into two separate equations for voltage and current vectors. The current equation becomes

$$\frac{\partial^2(\tilde{I}_n(z,s))}{\partial z^2} - s^2(C'_{n,m})\,(L'_{n,m})\,(\tilde{I}_n(z,s)) = (0_n) \qquad (2)$$

which is a one-dimensional wave equation for the N-vector current.

For a lossless multiconductor section immersed in a uniform, homogeneous dielectric, the matrix product $(C'_{n,m})(L'_{n,m})$ in Equation (2) is diagonal and the individual elements of the current N-vector are themselves a solution to a simple wave equation:

$$\frac{\partial^2 I_n(z,s)}{\partial z} - \frac{s^2}{v^2}\,\tilde{I}_n(z,s) = 0$$

where v is the velocity of wave propagation on the line.

A more general line, however, does not have a diagonal result for the $(C'_{n,m})(L'_{n,m})$ matrix, although it is possible to diagonalize it through the use of a nonsingular NxN transformation matrix, denoted by $(T_{n,m})$, which consists of the current eigenmodes, $(\phi_n)_i$, as columns. The ϕ_n's are solutions to the eigenvalue equation

$$s^2(C'_{n,m})(L'_{n,m})(\phi_n)_i = \tilde{\gamma}_i^2\,(\phi_n)_i \qquad (3)$$

where $\tilde{\gamma}_i^2$ is the i^{th} eigenvalue corresponding to the eigenmode $(\phi_n)_i$.

By introducing a change of variables as

$$(\tilde{I}_n(z,s)) = (T_{n,m})(\tilde{i}_n(z,s)) \tag{4}$$

where $(\tilde{i}_n(z,s))$ represents the modal currents, the wave equation for the modal currents becomes

$$\frac{\partial^2(\tilde{i}_n(z,s))}{\partial z^2} = s^2(T_{n,m})^{-1}(C'_{n,m})(L'_{n,m})(T_{n,m})(\tilde{i}_n) = (\tilde{\gamma}_{n,m})^2(\tilde{i}_n) \tag{5}$$

where $(\tilde{\gamma}_{n,m})^2$ is a diagonal matrix containing the $\tilde{\gamma}_i^2$ terms as elements.

Since the matrix $(\tilde{\gamma}_{n,m})^2$ in Equation (5) is diagonalized, the solution for the modal currents can be expressed directly as exponential functions of position, and the total solution for the line currents becomes

$$(\tilde{I}_n(z,s)) = (T_{n,m}) \left(e^{-(\tilde{\gamma}_{n,m})z} (\tilde{\alpha}_n^+) + e^{(\tilde{\gamma}_{n,m})z} (\tilde{\alpha}_n^-) \right) \tag{6}$$

where $(\tilde{\alpha}_n^+)$ and $(\tilde{\alpha}_n^-)$ are N-vectors which define the amplitudes of each of the propagating modes on the line and which depend on the line termination and excitation. The terms $e^{\pm(\tilde{\gamma}_{n,m})z}$ are diagonal matrices having as elements $e^{\pm\tilde{\gamma}_i z}$, where $\tilde{\gamma}_i = +\sqrt{\tilde{\gamma}_i^2}$.

A similar development for the line voltage $(\tilde{V}_n(z,s))$ can be carried out to determine voltage modes and a propagation equation similar to Equation (6). By defining a characteristic impedance matrix as

$$(Z_{C_{n,m}}) = s^{-1}(C'_{n,m})^{-1}(T_{n,m})(\tilde{\gamma}_{n,m})(T_{n,m})^{-1} \tag{7}$$

the line voltage N-vector can be expressed using the same constants $(\tilde{\alpha}_n^+)$ and $(\tilde{\alpha}_n^-)$ as in Equation (6):

$$(\tilde{V}_n(z,s)) = (\tilde{Z}_{C_{n,m}})(T_{n,m})\left(e^{-(\tilde{\gamma}_{n,m})z}(\tilde{\alpha}_n^+) - e^{(\tilde{\gamma}_{n,m})z}(\tilde{\alpha}_n^-)\right) \qquad (8)$$

The unknown constants $(\tilde{\alpha}_n^+)$ and $(\tilde{\alpha}_n^-)$ are determined by taking into account the loads at each end of the line, as well as the excitation. Consider the line shown in Figure 2, which has lumped voltage and current sources at $z = z_s$, as well as load impedances $(\tilde{Z}_{1_{n,m}})$ and $(\tilde{Z}_{2_{n,m}})$ at $z = 0$ and $z = \ell$ respectively. On the section of the line $0 \le z \le z_s$ Equations (6) and (8) are valid, since this section of the line is source free. Similarly, for $z_s \le z \le \ell$ similar equations are valid, but with different constants, $(\tilde{\alpha}_n)$. By relating $(\tilde{V}_n(z,s))$ to $(\tilde{I}_n(z,s))$ at $z = 0$, and $z = \ell$ through the load impedance matrices and by relating the discontinuities of $(\tilde{V}_n(z,s))$ and $(\tilde{I}_n(z,s))$ to the voltage and current sources at $z = z_s$, a set of linear equations can be developed with the $(\tilde{\alpha}_n)$ constants for each section of line as unknowns.

Of special interest are the load currents, i.e., $(\tilde{I}_n(0,s))$ and $(\tilde{I}_n(\ell,s))$. Using the solutions for the $(\tilde{\alpha}_n)$ as well as Equation (6) for $z = 0$ and $z = \ell$, the load currents may be expressed as:

$$\begin{bmatrix}(\tilde{I}_n(0,s))\\(\tilde{I}_n(\ell,s))\end{bmatrix} = \begin{bmatrix}(\delta_{n,m}) + (\tilde{\Gamma}_{1_{n,m}}) & (0_{n,m})\\(0_{n,m}) & (\delta_{n,m}) + (\tilde{\Gamma}_{2_{n,m}})\end{bmatrix} \cdot \begin{bmatrix}-(\tilde{\Gamma}_{1_{n,m}}) & (T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})\ell}\cdot(T_{n,m})^{-1}\\(T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})\ell}\cdot(T_{n,m})^{-1} & -(\tilde{\Gamma}_{2_{n,m}})\end{bmatrix}^{-1} \cdot \begin{bmatrix}(\tilde{\mathcal{I}}_n^-(s))\\(\tilde{\mathcal{I}}_n^+(s))\end{bmatrix} \qquad (9)$$

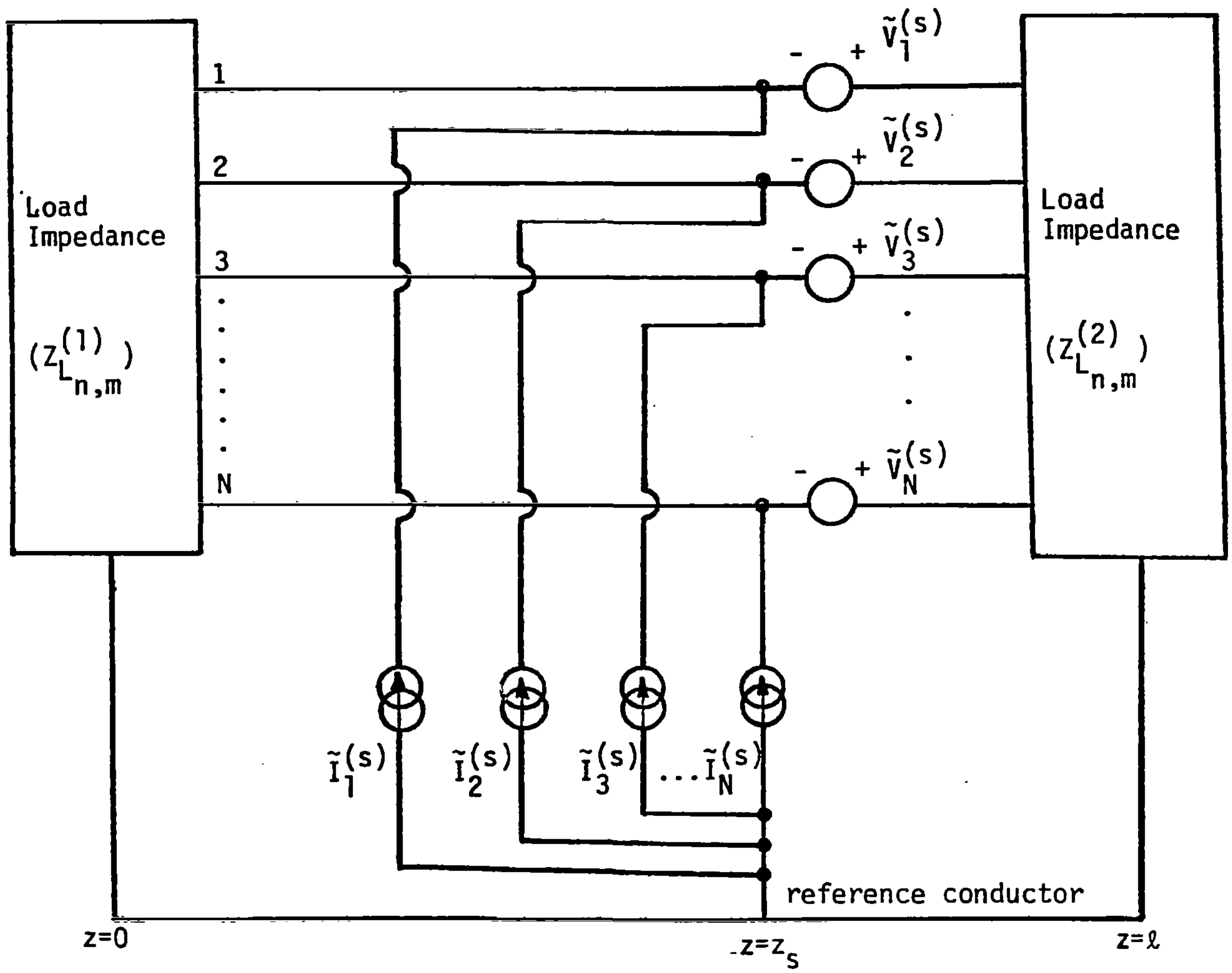

Figure 2. Single length of multiconductor transmission line with loads and lumped sources at $z = z_s$.

where the terms $(\tilde{\mathcal{J}}_n^{+}(z_s,s))$ and $(\tilde{\mathcal{J}}_n^{-}(z_s,s))$ represent the source terms for the positive and negative traveling waves on the multi-conductor line. Those are referred to as combined current sources, since they have the dimension of current but arise from both the applied voltage and current sources at $z = z_s$. In this equation, the terms $(\tilde{\Gamma}_{1_{n,m}})$ and $(\tilde{\Gamma}_{2_{n,m}})$ are generalized current reflection coefficient matrices given by

$$(\tilde{\Gamma}_{1_{n,m}}) = \left[(\tilde{Z}_{1_{n,m}}) + (Z_{C_{n,m}})\right]^{-1} \cdot \left[(\tilde{Z}_{1_{n,m}}) - (Z_{C_{n,m}})\right] \tag{10}$$

for the load at $z = 0$, and similarly for $(\tilde{\Gamma}_{2_{n,m}})$ at $z = \ell$ with $(\tilde{Z}_{2_{n,m}})$ as the load impedance. As defined previously, $(Z_{C_{n,m}})$ is the characteristic impedance matrix of the line.

Notice that the matrix equation in Equation (9) has, as its elements, matrices. Thus, it is referred to as a super matrix equation. The double dot operator (:) is used to signify the product between two super matrices by first treating the super matrices as if they were regular matrices and then performing matrix multiplications for each of the individual multiplications of the super matrix product.

The form of the source terms in Equation (9) can be shown to be

$$(\tilde{\mathcal{J}}_n^{-}(s)) = \frac{1}{2}\,(T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})z_s}\cdot(T_{n,m})^{-1}\cdot\Big((Z_{C_{n,m}})\cdot(\tilde{V}_n^{(s)}(z_s,s)) + (\tilde{I}_n^{(s)}(z_s,s))\Big) \tag{11}$$

and

$$(\tilde{\mathcal{J}}_n^{+}(s)) = \frac{1}{2}\,(T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})(\ell - z_s)}\cdot(T_{n,m})^{-1}\cdot\Big((Z_{C_{n,m}})^{-1}\cdot(\tilde{V}_n^{(s)}(z_s,s)) - (\tilde{I}_n^{(s)}(z_s,s))\Big) \tag{12}$$

With these source terms, the terminal response of the transmission line can be determined for lumped voltage and current sources at $z = z_s$. For field excitation of the transmission line, it is necessary to consider distributed excitation, as opposed to the discrete excitation discussed above. This can be regarded as a simple extension of Equations (11) and (12) by integrating over the source terms $(\tilde{V}_n^{(s)})$ and $(\tilde{I}_n^{(s)})$. Doing this, the combined current sources become

$$(\tilde{\mathcal{I}}_n^-(s)) = \frac{1}{2}\int_0^{\ell}\left((T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})\xi}\cdot(T_{n,m})^{-1}\cdot((Z_{c_{n,m}})^{-1}\cdot(\tilde{V}_n'^{(s)}(\xi,s)) + (\tilde{I}_n'^{(s)}(\xi,s))\right)d\xi \quad (13)$$

$$(\tilde{\mathcal{I}}_n^+(s)) = \frac{1}{2}\int_0^{\ell}\left((T_{n,m})\cdot e^{(\tilde{\gamma}_{n,m})(\ell-\xi)}\cdot(T_{n,m})^{-1}\cdot((Z_{c_{n,m}})^{-1}\cdot(\tilde{V}_n'^{(s)}(\xi,s)) - (\tilde{I}_n'^{(s)}(\xi,s))\right)d\xi \quad (14)$$

which follows directly from superposition. Notice that now the voltage and current sources are per-unit-length quantities, and hence denoted by a prime. These quantities must be determined given a knowledge of the incident electromagnetic field on the line, as well as a knowledge of the transmission line cross-sectional geometry. This is discussed in the next section of this report.

SECTION III

DETERMINATION OF DISTRIBUTED VOLTAGE AND CURRENT SOURCES

As indicated in the previous section, the terminal (or load) currents of a multiconductor transmission line can be evaluated using Equations (9), (13) and (14) if the distributed voltage and current sources $(\tilde{V}_n'^{(s)}(z,s))$ and $(\tilde{I}_n'^{(s)}(z,s))$ are known everywhere along the line. In some instances, such as a small aperture or other localized source close to the transmission line, it is possible to approximate the solution using a discrete source position, as in ref. (9). For an arbitrarily incident plane wave, however, this is not possible. Sources distributed over the entire line are necessary.

Consider the case of a single multiconductor cable in free space and with impedance terminations at each end, as shown in Figure 3. Assume that in this bundle there are $n+1$ wires, with the $n+1^{st}$ wire being the reference conductor. The electric and magnetic fields in the vicinity of the line can be divided into two parts. These are the incident components, $\tilde{\tilde{E}}^{inc}$ and $\tilde{\tilde{H}}^{inc}$ and the scattered components $\tilde{\tilde{E}}^{s}$ and $\tilde{\tilde{H}}^{s}$, such that

$$\tilde{\tilde{E}} = \tilde{\tilde{E}}^{inc} + \tilde{\tilde{E}}^{s} \quad (15a)$$

$$\tilde{\tilde{H}} = \tilde{\tilde{H}}^{inc} + \tilde{\tilde{H}}^{s} \quad (15b)$$

The scattered field components are caused entirely by the induced currents and charges on the $n+1$ wires, as well as by the currents on the terminations. The scattered fields from the line can be further subdivided into three different classes. There are TEM, TE and TM transmission line modes, which are produced by "transmission line" currents, having the property that the components of the total current on each of the conductors sum to zero.

In addition to these currents, there are "antenna mode" currents. These are currents which flow on each wire (but with a different magnitude

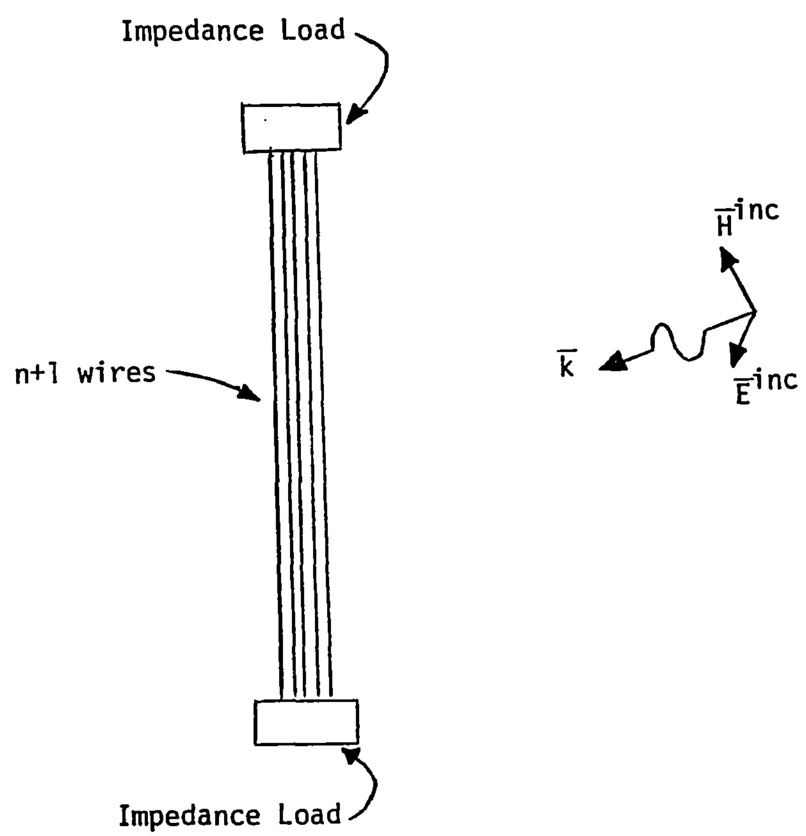

Figure 3. Isolated multiconductor line excited by incident plane wave.

for each wire, in general) and are subject to the constraint that the voltage difference between any two conductors in a transverse plane is zero. Furthermore, these currents go to zero at the ends of the line.

Finally, there can exist quasi-static current and charge distributions which contribute to the scattered field but have a net current or charge of zero on each conductor. Although these latter currents and charges do not play a role in computing the transmission-line response directly, they are important in determining the coupling of electromagnetic fields to the transmission line.

A complete and rigorous solution for the field induced currents on the multiconductor line in Figure 3 can be obtained by formulating and solving a set of coupled integral equations for the wire and load currents, given a particular incident field. In many cases, however, such a complete solution for the current is not needed. For lines which are long compared with the wire separation, the currents due to TE and TM fields attenuate rapidly from the loads or other line terminations, giving rise, therefore, to a current distribution which corresponds primarily to the TEM currents plus the other scattering currents mentioned above. Moreover, in many cases, only the transmission-line current response is desired since the antenna mode currents do not contribute to the load response in the general case, and if the transmission line is next to a reference ground plane, the antenna mode currents are not excited at all. Under the assumption that the TE and TM currents are negligible and neglecting the effects of load currents, the total $\bar{E}$ and $\bar{H}$ fields in the vicinity of the transmission line can be written as

$$\tilde{\bar{E}} = \tilde{\bar{E}}^{inc} + \tilde{\bar{E}}^{ant} + \tilde{\bar{E}}^{TEM} + \tilde{\bar{E}}^{st} \tag{16a}$$

and

$$\tilde{\bar{H}} = \tilde{\bar{H}}^{inc} + \tilde{\bar{H}}^{ant} + \tilde{\bar{H}}^{TEM} + \tilde{\bar{H}}^{st} \tag{16b}$$

where the subscript (inc) refers to the incident (or free space) fields, (ant) denotes the fields produced by the antenna mode currents, (TEM) stands for the fields due to the transmission-line currents, and (st) is for the portion of the fields caused by the static distribution of current and charge on the wires, determined with the condition that the total current and charge be zero on each wire.

Following the approach used in ref. (11) for single-wire lines and in ref. (7) for multiconductor lines, Maxwell's equations can be used to derive a v-i relation for the transmission line currents. Consider a uniform section of multiconductor line shown in Figure 4. For a time dependence of e^{st}, Maxwell's equation may be written as

$$\nabla \times \tilde{\bar{E}} = -s\, \tilde{\bar{B}} \tag{17}$$

and on a path C_1, from the reference conductor to wire 1 (where $d\bar{\ell}_1$ represents an element of the path, and $\hat{n}_1$ is the normal to the path), we can integrate Equation (17) to yield the following:

$$-\frac{d}{dz} \int_{a\,C_1}^{b} \tilde{\bar{E}} \cdot d\ell = s \int_{a\,C_1}^{b} \tilde{\bar{B}} \cdot \hat{n}\, d\ell \tag{18}$$

This result is standard, and its derivation will not be repeated here.

Noting that the line integral of the electric field in Equation (18) is the negative of the voltage between the two conductors, this equation may be written as

$$\frac{d\tilde{V}_1}{dz} = j\omega \int_{a\,C_1}^{b} \tilde{\bar{B}}^{TEM} \cdot \hat{n}\, d\ell + j\omega \int_{a\,C_1}^{b} \tilde{\bar{B}}^{ant} \cdot \hat{n}\, d\ell$$

$$- j\omega \int_{a\,C_1}^{b} (\tilde{\bar{B}}^{inc} + \tilde{\bar{B}}^{st}) \cdot \hat{n}\, d\ell \tag{19}$$

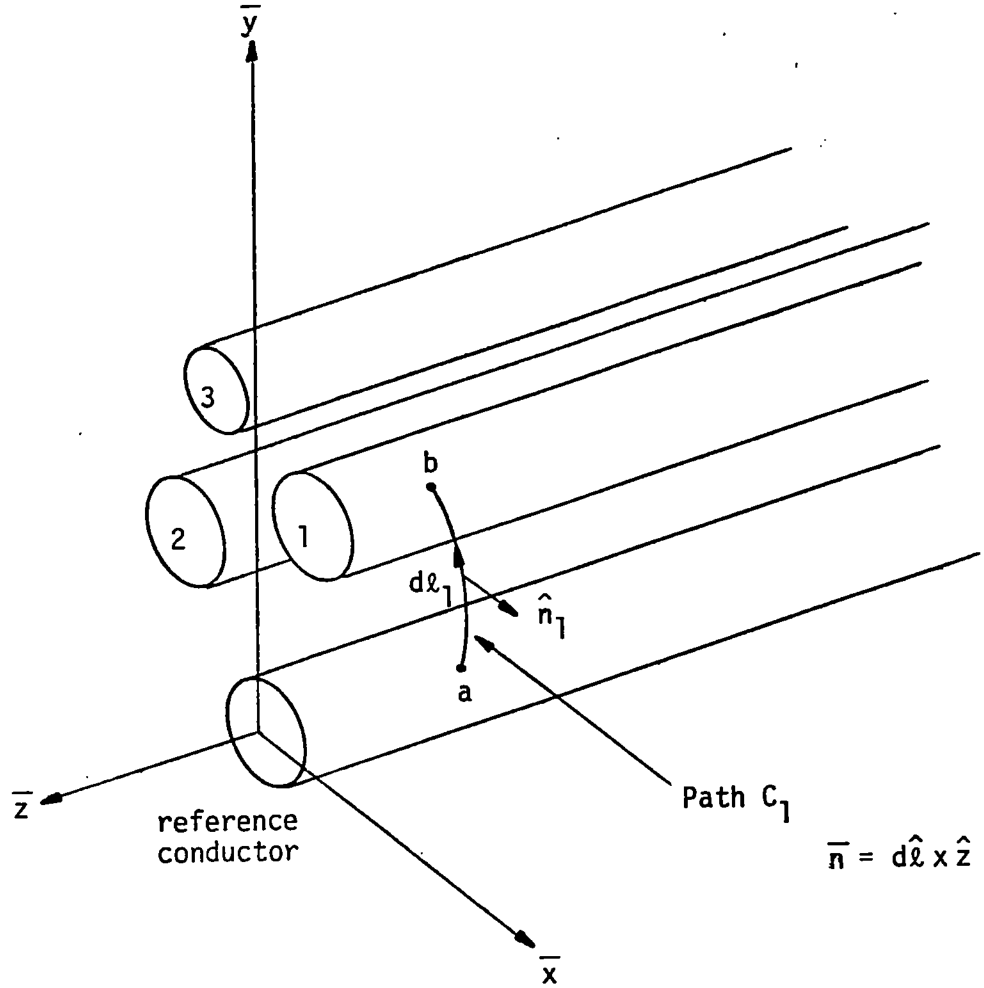

Figure 4. Cross section of multiconductor line showing integration path C_1 from point a to b .

As discussed by Paul in ref. (7), the term involving the $\tilde{\bar{B}}^{TEM}$ field, which arises from all TEM currents on the multiconductor line and is a magnetic flux per unit length, can be computed in terms of the inductance coefficient matrix elements as

$$- \oint_{1\,a}^{b} \tilde{\bar{B}}^{TEM} \cdot \hat{n}\, d\ell \equiv \phi_1 = L_{11}\, \tilde{I}_1 + L_{12}\, \tilde{I}_2 + \ldots L_{1n}\, \tilde{I}_n \qquad (20)$$

where $\tilde{I}_1 \ldots \tilde{I}_n$ represent the currents on the n (non-reference) conductors.

From our definition of the "antenna mode" currents, the voltage between wire 1 and the reference is zero for these currents, which implies that the antenna current flux term is also zero. See ref. (12). Thus, we have the relation

$$\oint_{1\,a}^{b} \tilde{\bar{B}}^{ant} \cdot \hat{n}\, d\ell \equiv 0 \qquad (21)$$

With these substitutions, Equation (19) can be written as

$$\frac{d\tilde{V}_1}{dz} = -j\omega\, (L'_{11}\tilde{I}_1 + L'_{12}\tilde{I}_2 + \ldots L'_{1n}\tilde{I}_n) + s \oint_{1\,a}^{b} (\tilde{\bar{B}}^{inc} + \tilde{\bar{B}}^{st}) \cdot \hat{n}\, d\ell \qquad (22)$$

This procedure may be repeated for each of the n wires in the bundle, and the resulting equations expressed in matrix form are

$$\frac{d(\tilde{V}_n)}{dz} = -s\ (L'_{n,m})\cdot(\tilde{I}_n) + s\left(\oint_{a\ n}^{b_n} (\tilde{\bar{B}}^{inc} + \tilde{\bar{B}}^{st})\cdot\hat{n}_n\ d\ell_n\right) \tag{23}$$

The last term in this equation has dimensions of (volts/unit length) and is essentially a distributed voltage source for the transmission line. Denoting this by $(\tilde{V}_n'^{(s)})$, we then have

$$(\tilde{V}_n'^{(s)}) = s\mu_o\left(\oint_{a\ n}^{b_n} (\tilde{\bar{H}}^{inc} + \tilde{\bar{H}}^{st})\cdot\hat{n}_n\ d\ell_n\right) \tag{24}$$

where the relation $\tilde{\bar{B}} = \mu_o\tilde{\bar{H}}$ has been used. The differential equation for voltage and current in Equation (23) then becomes

$$\frac{d(\tilde{V}_n)}{dz} + s(L'_{n,m})\cdot(\tilde{I}_n) = (\tilde{V}_n'^{(s)}) \tag{25}$$

A similar manipulation can be performed using the other Maxwell equation

$$\nabla x\tilde{\bar{H}} = s\varepsilon\ \tilde{\bar{E}} \tag{26}$$

to obtain the second telegrapher's equation containing sources. Applying this to the contour C_1 in exactly the same manner as in ref. (11), the following relation may be derived.

$$-\frac{d}{dz}\oint_{a\ 1}^{b} \bar{H}\cdot\hat{n}\ d\ell = s\ \varepsilon\oint_{a\ 1}^{b} \bar{E}\cdot d\bar{\ell} \tag{27}$$

By inserting Equation (16a) into this last equation and noting that the antenna mode contributions vanish, since by definition of the antenna currents, $\oint \overline{E}^{ant} \cdot d\overline{\ell} = 0$ and $\oint \overline{H}^{ant} \cdot \hat{n}\, d\ell = 0$, this equation can be written as

$$- \frac{d}{dz} \oint_{1\,a}^{b} (\tilde{\overline{H}}^{inc} + \tilde{\overline{H}}^{st} + \tilde{\overline{H}}^{TEM}) \cdot \hat{n}\, d\ell = s\, \varepsilon \oint_{1\,a}^{b} \tilde{\overline{E}} \cdot d\overline{\ell} \qquad (28)$$

or, as done by Lee (ref. 11), expressed as

$$- \frac{d}{dz} \oint_{1\,a}^{b} \tilde{\overline{H}}^{TEM} \cdot \hat{n}\, d\ell = s\, \varepsilon \oint_{a}^{b} \tilde{\overline{E}} \cdot d\ell - s\, \varepsilon \oint_{1\,a}^{b} (\tilde{\overline{E}}^{inc} + \tilde{\overline{E}}^{s}) \cdot d\overline{\ell} \qquad (29)$$

Using Equation (20) and recognizing that $\oint_{1\,a}^{b} \tilde{\overline{E}} \cdot d\ell$ is the voltage $-V_1$, Eqaution (29) becomes

$$\frac{1}{\mu} \frac{d}{dz} (L'_{11} \tilde{I}_1 + \ldots L'_{1n} \tilde{I}_n) = -s\, \varepsilon\, \tilde{V}_1 - s\, \varepsilon \oint_{1\,a}^{b} (\tilde{\overline{E}}^{inc} + \tilde{\overline{E}}^{s}) \cdot d\overline{\ell} \qquad (30)$$

for the first wire. This process can be repeated for each wire, and the following matrix equation can be developed for the transmission line currents $(\tilde{I}_n)$ and voltages $(\tilde{V}_n)$:

$$\frac{1}{\mu} (L'_{n,m}) \cdot \frac{d(\tilde{I}_n)}{dz} = -s\, \varepsilon (\tilde{V}_n) - s\, \varepsilon \left(\oint_{n\,a}^{b_n} (\tilde{\overline{E}}^{inc} + \tilde{\overline{E}}^{s}) \cdot d\overline{\ell}_n \right) \qquad (31)$$

Rearranging terms slightly yields the second telegrapher's equation

$$\frac{d(\tilde{I}_n)}{dz} + s(C'_{n,m})\cdot(\tilde{V}'_n) = (\tilde{I}'^{(s)}_n) \tag{32}$$

where the source term $(\tilde{I}'^{(s)}_n)$ is given by

$$(\tilde{I}'^{(s)}_n) = -s(C'_{n,m})\cdot\left(\oint_{a_n}^{b_n} (\tilde{\overline{E}}^{inc} + \tilde{\overline{E}}^{s})\cdot d\overline{\ell}_n\right) \tag{33}$$

Note that in deriving this relation, the assumption that

$$(L'_{n,m})\cdot(C'_{n,m}) = \mu_o\,\varepsilon \tag{34}$$

has been employed, a result which implies that the lines are within a uniform, homogeneous dielectric medium.

In an inhomogeneous dielectric region, say for the case of each conductor having a separate dielectric jacket, it is known that true TEM modes cannot exist. However, an approximate analysis can be carried out by assuming that Equations (25) and (32) are applicable. The validity of this "quasi-TEM" assumption lies in the reasonable comparison of theoretical and experimental results for the multi-conductor system (ref. 13).

It is to be noted that the basic telegrapher's equations derived here for the transmission line currents and voltages are different in form than those developed by Paul (ref. 7). This is due to the fact that Paul has integrated from the center of one conductor to the other center, not from one surface to another of the thin, widely spaced conductors which he considers. For the more general case of fat, closely spaced wires, the total static electric and magnetic field in any transverse plane must be used to compute the equivalent line sources.

Aside from a difference in the definition of the unit normal vector $\hat{n}$, the major difference between the formulation of Lee in ref. (11) and the present analysis is the existence of an additional antenna mode source term in Lee's two-wire analysis. This two-wire analysis could be extended to a multiwire case, and thus would imply the existence of similar source terms in the present multiconductor analysis. As discussed by Frankel (ref. 12), the apparent discrepancy arises out of different choices for the "antenna current" by Lee, which thus has an effect on the remaining transmission-line current.

As stated earlier, our choice of the "antenna current" is that current flowing in each wire which produces a voltage difference of zero between any conductor and another at any transverse plane in the line. This choice is also used by Uchida (ref. 14), and thus leads to a decoupling of the transmission line currents from the antenna mode currents.

Although explicit expressions for the voltage and current sources have been developed in Equations (24) and (33), it still remains necessary to evaluate the scattered static fields $\tilde{\tilde{E}}^s$ and $\tilde{\tilde{H}}^s$, before the source terms can be used in Equations (13) and (14) to determine the load response of the multiconductor line. To determine these source terms, it is necessary to solve two static boundary value problems. To determine the current source in Equation (33), it is necessary to solve the two-dimensional static problem illustrated in Figure 5. An incident (free space) electric field strikes a collection of conductors, on which the net charges are zero. A static scattered field is produced by the local charges induced on each wire, and the

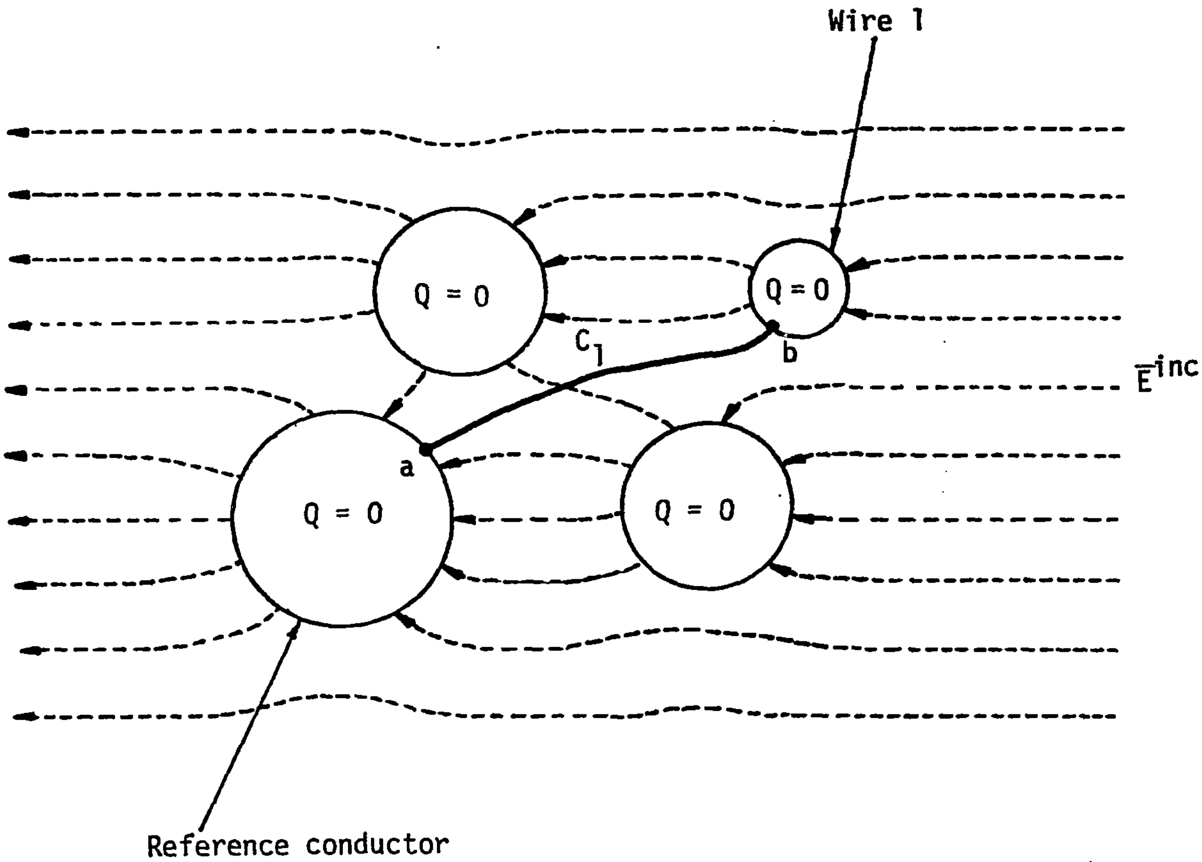

Figure 5. Cross section of multiconductor cable in incident $\bar{E}$ field showing typical field distribution and integration path from a to b . Each conductor has zero net charge.

integrals in Equation (33) are then evaluated along any contour from point a to b , using the total scattered field, $\bar{E}^{inc} + \bar{E}^{s}$.

The solution to this problem for the multiconductor case is similar to the two-wire problem discussed by Lee, but extended to more wires. It is solved by looking for the solution to

$$\nabla^2 \phi = 0 \tag{34}$$

exterior to the wires, with the condition that ϕ = constant on each of the conductors subject to the constraint that

$$\oint \frac{\partial \phi_i}{\partial n_i} dS_i = 0 \tag{35}$$

on each conductor, i , and that at infinity, the potential is

$$\phi \Rightarrow \phi^{inc} = -\bar{r} \cdot \bar{E}^{inc} \tag{36}$$

Here ϕ^{inc} represents the incident or free-space potential field in the absence of the transmission line. Once this equation is solved (usually by numerical means) the potentials of each wire, ϕ_i , can be determined, and the integrals of Equation (33) can be determined directly

$$\int_{a}^{b_i} (\tilde{\bar{E}}^{inc} + \tilde{\bar{E}}^{s}) \cdot d\bar{\ell}_i = -(\phi_i - \phi_{n+1}) \tag{37}$$

It is possible, however, to express the integral in Equation (37) in a simpler form, using only the incident field, $\bar{E}^{inc}$, and a vector equivalent distance, $\bar{h}_i$, in a manner similar to that of ref. (11). Consider an auxilliary problem which has a potential field given by ϕ^* and is defined by the relations

$$\nabla^2 \phi^* = 0 \tag{38}$$

with ϕ_j^* = constant (but unknown) on each of the i conductors of the multiconductor bundle, and with

$$\oint \frac{\partial \phi_j}{\partial n_j} dS_j = 0 \tag{39}$$

for all conductors except for the i^{th} conductor and the reference conductor, where we have the constraint

$$\oint_{\text{wire } i} \frac{\partial \phi_i}{\partial n} dS_i = - \frac{Q_i^*}{\varepsilon} \tag{40}$$

and

$$\oint_{\text{reference}} \frac{\partial \phi_{n+1}}{\partial n_{n+1}} dS_{n+1} = \frac{Q_i^*}{\varepsilon} \tag{41}$$

The solution to this auxiliary problem can be used to find the field excitation of the transmission line by using Green's identity,

$$\phi^* \nabla^2 \phi - \phi \nabla^2 \phi^* = 0 \tag{42}$$

and applying Gauss' theorem to give the expression

$$\oint_{\text{all conductors}} \left(\phi^* \frac{\partial \phi}{\partial n} - \phi \frac{\partial \phi^*}{\partial n} \right) dS + \oint_{S_\infty} \left(\phi^* \frac{\partial \phi}{\partial n} - \phi \frac{\partial \phi^*}{\partial n} \right) dS = 0 \tag{43}$$

where S_∞ is a closed surface at infinity. Using the facts that ϕ and ϕ^* are constant on the conductors, that

$$\oint \frac{\partial \phi_j^*}{\partial n_j} dS = 0 \tag{44}$$

for all conductors except the i^{th} and the reference conductor, and that

$$\oint_{S_\infty} \left(\phi^* \frac{\partial \phi}{\partial n} - \phi \frac{\partial \phi^*}{\partial n} \right) dS' = \int_{\text{wires}} \left(\phi^{inc} \frac{\partial \phi^*}{\partial n} \right) dS = - \frac{1}{\varepsilon} \int_{\text{wires}} \phi^{inc} \sigma^* \, dS \tag{45}$$

where

$$\sigma_j^* = -\varepsilon \frac{\partial \phi_j^*}{\partial n} \tag{46}$$

is the charge density on each conductor for the auxiliary problem and is a known quantity. Equation (43) can then be expressed as

$$\frac{1}{\varepsilon}(\phi_i - \phi_{n+1}) \int_{S_i} \sigma_i \, dS_i = -\frac{1}{\varepsilon}\int_{S_1} \phi^{inc} \sigma_1^* \, dS_1 - \frac{1}{\varepsilon}\int_{S_2} \phi^{inc} \sigma_2 \, dS_2 - \ldots \tag{47}$$

Using Equation (40) and the relation $\phi^{inc} = -\overline{E}\cdot\overline{r}$, this last equation takes the form

$$(\phi_i - \phi_{n+1}) = -\overline{E}^{inc}\cdot\overline{h}_i \tag{48}$$

where the vector $\overline{h}_i$ is defined as

$$\overline{h}_i = \frac{\oint_{S_1} r\sigma_1^* \, dS_1 + \oint_{S_2} r\, \sigma_2^* \, dS_2 + \ldots \oint_{S_{n+1}} r\, \sigma_{n+1}^* \, dS_{n+1}}{\oint_{S_i} \sigma_i^* \, dS_i} \tag{49}$$

With this expression, Equation (37) can be conveniently expressed as

$$\int_{a_i}^{b_i} (\overline{E}^{inc} + \overline{E}^{S})\cdot d\overline{\ell}_i = \overline{E}^{inc}\cdot\overline{h}_i \tag{50}$$

and the N vector equivalent current source becomes

$$(\tilde{I}_n'(s)) = -s(C_{n,m}')\cdot\left((\overline{E}^{inc}\cdot\overline{h}_n)_m\right) \tag{51}$$

The vectors $\overline{h}_i$ are referred to as the "field coupling vectors" for the line, and also as the "effective height" of the conductors.

Physically, they correspond to the vector distance between the charge centroids on the multiconductor system, given a total charge Q on the i^{th} conductor, -Q on the reference conductor and zero net charge on all others. Figure 6 illustrates these relationships.

For the case of thin, widely separated wires, the vectors $\bar{h}_i$ are simply the distances from the center of the reference conductor to each of the wires' centers. For more closely spaced wires, the field coupling parameters must be calculated, using the integral equation approach outlined by Giri in ref. (15).

A similar procedure can be carried out for determining the distributed voltage source in Equation (24) by solving a magnetostatic problem. The details of this are identical to that described by Lee (ref. 11), modified by the presence of more than just two conductors. The results are that the same field coupling parameters, $\bar{h}_i$, that are used for the electric field calculations may be used for the magnetic fields. This results in the following equation for the distributed voltage source.

$$(\tilde{V}_n'^{(s)}) = s\mu_o \left((\bar{h}_n \times \hat{1}_z)\cdot\bar{H}^{inc}\right) \tag{52}$$

The preceding discussion has been for the field excitation of an isolated multiconductor line, in which one of the conductors in the bundle serves as the reference. An often encountered situation, however, is not this configuration, but one with an n-wire bundle next to a flat, conducting ground plane. For this case, the ground plane serves as the reference conductor, and the antenna mode currents are not excited.

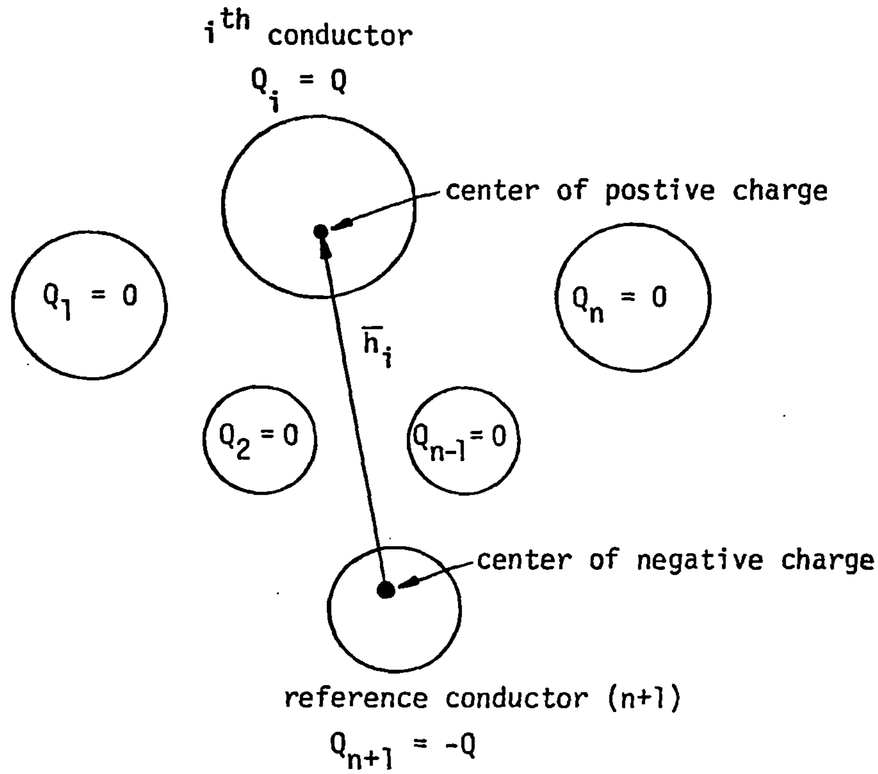

Figure 6. Cross section of isolated n+1 wire multiconductor line, showing field coupling vector for the i^{th} conductor.

For this case, the field coupling parameters are still calculated as above. For example, as shown in Figure 7, the coupling parameter $\overline{h}_i$ is calculated by placing a charge Q on wire i and no net charge on the other wires. By image theory, there is an image charge of -Q on the image of wire i and the resulting charge centroids may be computed. The coupling parameter vector is directed away from the ground plane and has a magnitude equal to the shortest distance from the ground plane to the i^{th} wire's charge center.

In this case, note that the incident fields $\overline{E}^{inc}$ and $\overline{H}^{inc}$ which are used in Equations (51) and (52) must include the reflection effects of the ground plane. Thus, if $\overline{E}^{inc}$ and $\overline{H}^{inc}$ represent the free space fields in the absence of the ground plane, the exciting fields of the line to be used in the above equations are

$$E_n = 2(\overline{E}^{inc} \cdot \hat{n}) \tag{53}$$

and

$$H_t = 2(\hat{k} \times \hat{n}) \cdot \overline{H}^{inc} \tag{54}$$

where $\hat{n}$ is a unit normal to the plane, $\hat{k}$ is the direction of propagation of the incident wave and the subscripts n and t represent field components normal to and parallel to the ground plane, respectively.

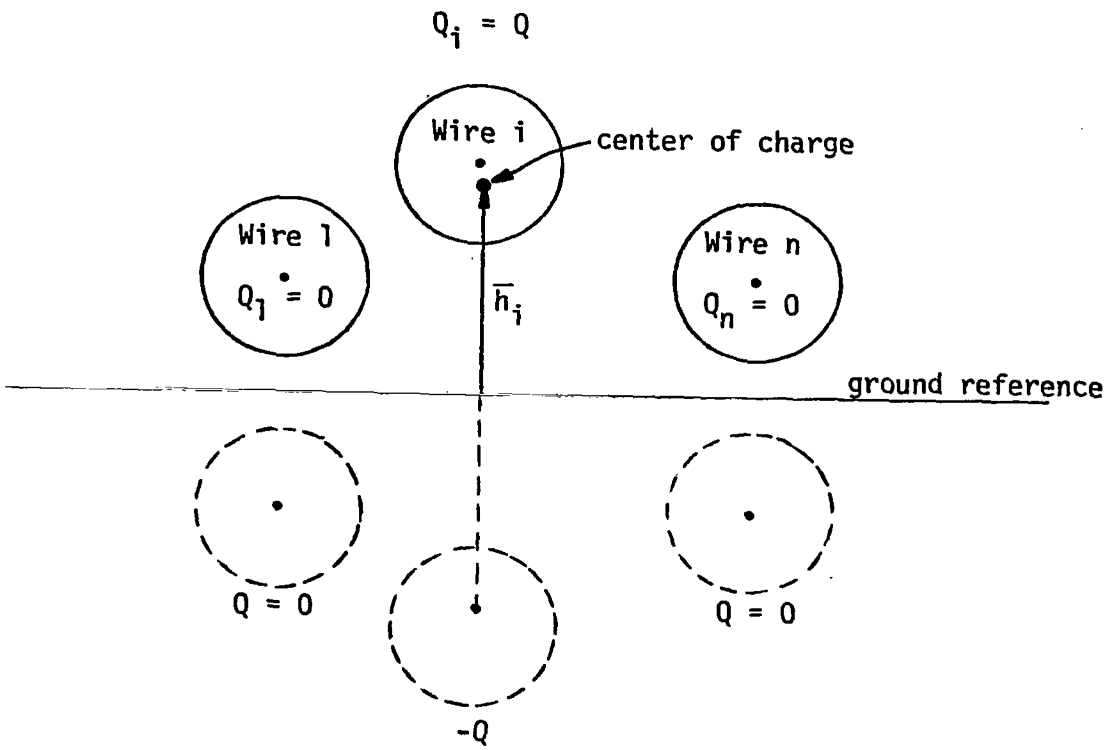

Figure 7. Field coupling vector for Wire i of multiconductor line over a ground plane.

SECTION IV
EXCITATION FIELDS DUE TO INCIDENT PLANE WAVE

The expressions for the distributed current and voltage sources in Equations (51) and (52) in the previous section are quite general and depend only on the local incident electric and magnetic fields on the line. One type of incident field which is useful to consider is a plane wave of arbitrary angle of incidence.

Consider a single transmission-line tube being illuminated by a plane electromagnetic field. As shown in Figure 8a, the tube is in the $\hat{z}$ direction and the $\bar{k}$ vector of the incident field arrives with angles ψ_o with respect to the $\hat{z}$ axis and θ_o, which is the inclination angle of the incident field. Two different polarizations of the incident field are possible, and are denoted as TE and TM, respectively. The TE case occurs when the incident $\bar{E}$ field is perpendicular to the plane of incidence, which is defined as the plane formed by the $\bar{k}$ vector and its two-dimensional projection in the x-y plane. The TM case, conversely, occurs when the $\bar{E}$ field lies within the plane of incidence. Figure 8b illustrates these different polarizations.

For both of these polarizations, the field components at the multiconductor tube can be expressed as follows:

<u>TE Fields</u>

$$H_z^{inc} = -H^{inc} \sin \psi_o \qquad E_z^{inc} = -E^{inc} \sin \theta_o \cos \psi_o$$

$$H_x^{inc} = H^{inc} \cos \psi_o \qquad E_x^{inc} = -E^{inc} \sin \theta_o \sin \psi_o$$

$$H_y^{inc} = 0 \qquad E_y^{inc} = E^{inc} \cos \theta_o$$

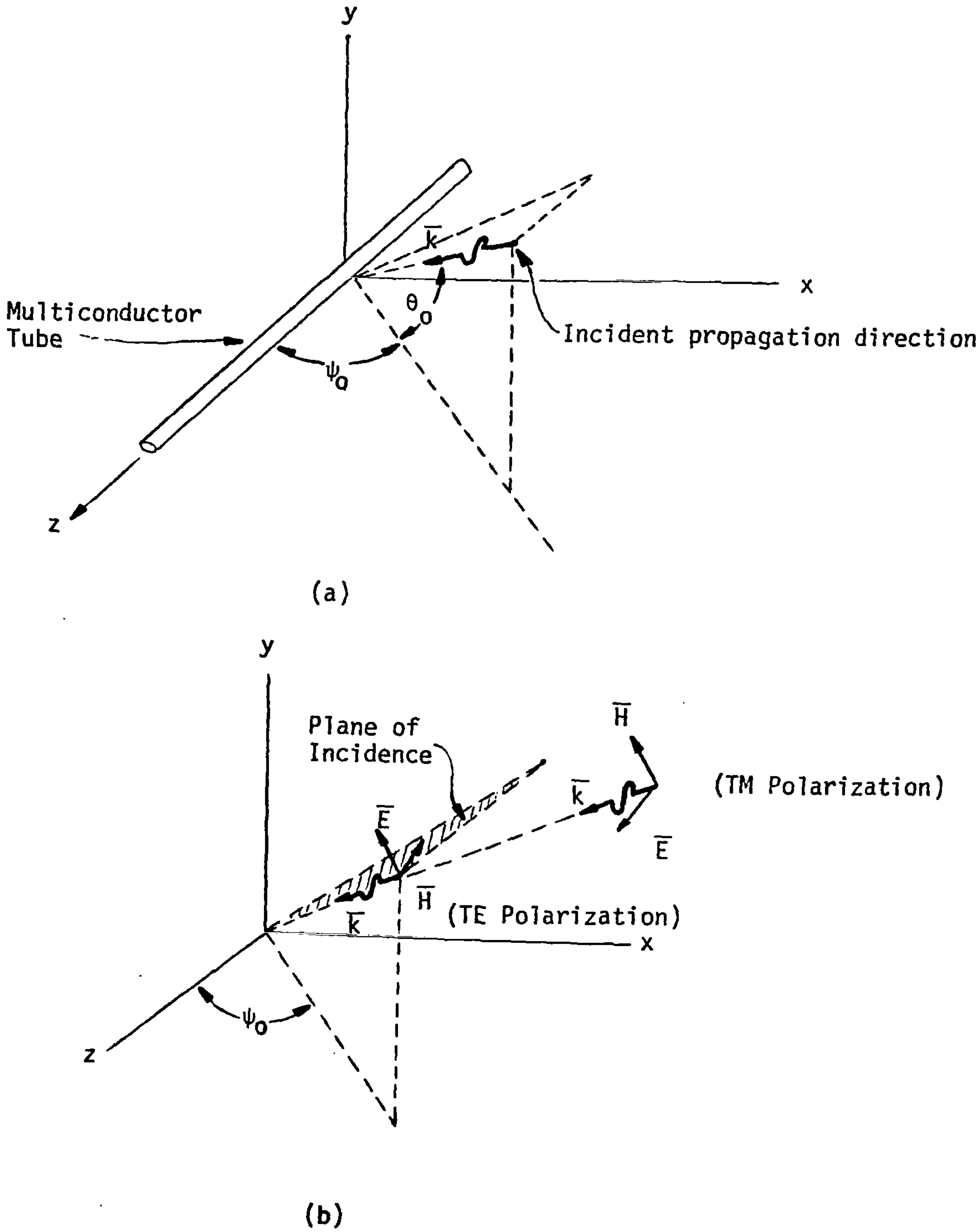

Figure 8. Geometry and polarization of the incident plane wave.

TE Fields

$$H_z^{inc} = -H^{inc} \sin \theta_o \cos \psi_o \qquad E_z^{inc} = E_o \sin \psi_o$$

$$H_x^{inc} = -H^{inc} \sin \theta_o \sin \psi_o \qquad E_x^{inc} = -E_o \cos \psi_o$$

$$H_y^{inc} = H^{inc} \cos \theta_o \qquad E_y^{inc} = 0$$

As seen from Equations (51) and (52), the important quantities for determining the distributed sources are the electric field component parallel to the vectors $\bar{h}_i$, and the magnetic field component perpendicular to $\bar{h}_i$. Consider the geometry shown in Figure 9. The i^{th} conductor is shown with its coupling vector having an angle θ_i with respect to the chosen x axis, and a magnitude h_i . For this case, the components of the electric field in the direction parallel to $\bar{h}_i$ are given by the following expressions for the i^{th} conductor:

$$E_{\|_i} = E_y^{inc} \sin \theta_i + E_x \cos \theta_i$$

$$= E^{inc} (\sin \theta_i \cos \theta_o - \cos \theta_i \sin \theta_o \sin \psi_o) \quad \text{(TE polarization)} \tag{53a}$$

$$= -E^{inc} \cos \theta_i \cos \psi_i \quad \text{(TM polarization)} \tag{53b}$$

and

$$H_{\perp_i} = -H_y \cos \theta_i + H_x \sin \theta_i$$

$$= H^{inc} \sin \theta_i \cos \psi_o \quad \text{(TE polarization)} \tag{54a}$$

$$= -H^{inc} (\cos \theta_i \cos \theta_o + \sin \theta_i \sin \theta_o \sin \psi_o) \quad \text{(TM polarization)} \tag{54b}$$

With these field components, the distributed vector current and voltage sources take the form

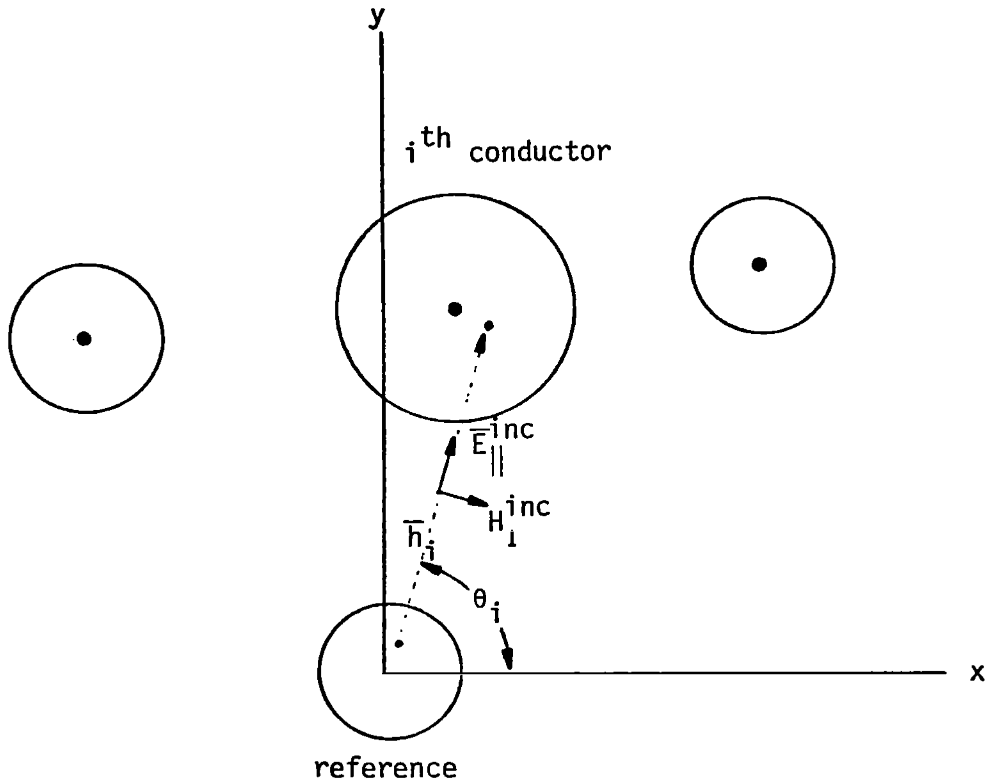

Figure 9. Cross section of multiconductor line showing field coupling parameter and pertinent field components.

$$(\tilde{I}_n'^{(s)}) = -s\ (\bar{C}_{n,m}')\ (h_n\ E_{\|_n})\tag{55}$$

and

$$(\tilde{V}_n'^{(s)}) = s\mu_o\ (h_n\ H_{\perp_n})\tag{56}$$

and should be used in Equations (13) and (14) to evaluate the line response for an incident plane wave.

SECTION V
CONCLUSIONS

This report has presented a discussion of the field excitation of multiconductor transmission lines. First, a general expression for the current response at the terminations of an N-wire multiconductor cable has been developed in terms of distributed voltage and current sources. In Section III relationships between these sources and the total static electric and magnetic fields in the vicinity of the transmission line are then derived. These are then related to the free space (or incident) fields through a vector field coupling parameter or equivalent separation of the lines. Finally, Section IV expresses the distributed source terms for the multiconductor line in terms of the angles of incidence and polarization of an incident plane wave.

This work expands upon the past studies of field excitation of two-wire transmission lines. The field coupling parameters for a multiconductor line are seen to be determined from a series of calculations involving specifying a zero net charge on all conductors except the reference and the conductor for which the coupling parameter is being determined. It is noted, furthermore, that the excitation of the line depends strongly on the line's orientation with respect to the incident fields.

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