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Steady-State and Transient Electromagnetic
Coupling Through Slabs

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Abstract

The problem of electromagnetic transmission through a slab, where transmitting and receiving antennas are at finite distances from the slab, is considered. The mathematical formulation of the problem is quite general. A detailed solution is presented for the case of a highly conducting slab exposed to sinusoidal and transient excitations. A discussion is given of the conditions under which measurements with source and receiver at finite distances are equivalent to the same measurements with plane wave excitation.

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1. Introduction

One of the simplest conceivable ways for determining the electromagnetic properties of materials is to measure the electromagnetic field transmitted through a slab of the material under test. The corresponding mathematical model consists of an infinite slab with transmitting and receiving antennas placed on opposite sides of the slab. The model provides a reasonably good approximation to the real situation of a slab of finite extent when the distance between transmitting and receiving points is small compared to the transverse slab dimensions.

Measurements can be made in the sinusoidal or the transient regime. For instance, MIL standards for evaluating the shielding effectiveness of materials [1] require that transmission measurements be made in the steady state at prescribed frequencies and then in a pulsed regime using wire and loop antennas placed at prescribed distances from the slab of shielding material. Although these standards are useful for relative comparisons, a fundamental question remains unanswered: does the measurement depend only on the electromagnetic properties of the slab (and on its thickness), or does it depend also on antenna type and orientation, antenna distance, and (for transient measurements) on transmitted waveform?

A crude but simple method for studying (or, at least, having an estimate of) the field coupled to the inside of an enclosure is to consider the transmission through a slab. The slab may be perforated, or inhomogeneous, or described by stochastic parameters, the last case being relevant to near-millimeter propagation through aerosols used for camouflage tactics. In electromagnetic pulse (EMP) experiments it is customary to

simulate the EMP plane wave signal by using rather sophisticated antennas and guiding devices [2,3]. An attractive alternative to this approach can result from an understanding and exploration of the role played by localized sources at finite distances from the test object.

The objectives of this paper are to reconsider the problem of steady-state and transient coupling through a slab with transmitting and receiving antennas located at finite distances from the slab; to cast the problem in an elegant form; and to show that, at least in the case of a highly conductive slab, simple analytical solutions to the problem can be obtained. An important result of the paper is the determination of antenna positions and (in the transient regime) of incident waveforms that will yield a transmitted field practically the same as that produced by plane wave excitation.

Transmission through highly conductive slabs is certainly not a new problem. For plane wave steady-state excitation, transmission line techniques can easily be applied [4]. For pulsed plane wave excitations, the solution is also available [5]. The situation is much less satisfactory for the case we want to study. It is not the aim of this paper to provide a full bibliography on this subject (for a more complete bibliography, see [6]). We note only that the first attempt to solve this problem was made in 1936 [7] by accommodating the classical results of Maxwell on eddy currents and thin shields to the case of two coaxial loops separated by a plane conducting sheet. Early studies on antenna coupling through plane shields were based on low-frequency [8,9] or quasi-static [10] approximations, were mainly relative to loop excitation [8-10], and required numerical computation [8-12] of integral expressions for the transmitted field. Although the validity of the simple transmission line theory [4] for antennas

at finite distance from the shield, or shields of finite extent has been questioned [13], it appears that all expressions derived in the referenced literature resemble these Shelkunoff formulas [14].

Due to the symmetry of the problem, it can easily be surmised that plane wave expansion techniques provide a powerful tool of analysis for an arbitrary type of excitation of an infinite slab. These techniques have been recently applied [14,15] to the case of electric or magnetic dipole excitation in parallel or coaxial configuration, by computing the transmitted field through the use of fast Fourier numerical programs. In this paper we shall use the same approach. However, we will show that, although the Fourier transformation of the fields is a logical intermediate step of the analysis, it is not needed in the final formulation of the solution. Indeed, the solution can be conveniently expressed in terms of a convolution integral, wherein the presence of the slab is described by an appropriate transfer function. Then, at least for antennas in coaxial configuration, the convolution integral can be analytically evaluated both in steady-state and transient regimes and no numerical work is necessary. Inspection of the solution allows us to answer the original question about the influence of the finite antenna separation on measurements. After all the mathematical machinery has been worked out and simply, physically sound, understandable results are obtained, a discussion of the final results is presented in Section 6.

2. Circuit-Like Analysis of Electromagnetic Transmission through a Slab

With reference to Fig. 1, let us consider an infinite slab of thickness s and characterized, in frequency domain, by permittivity $\epsilon = \epsilon_0 \epsilon_r$,

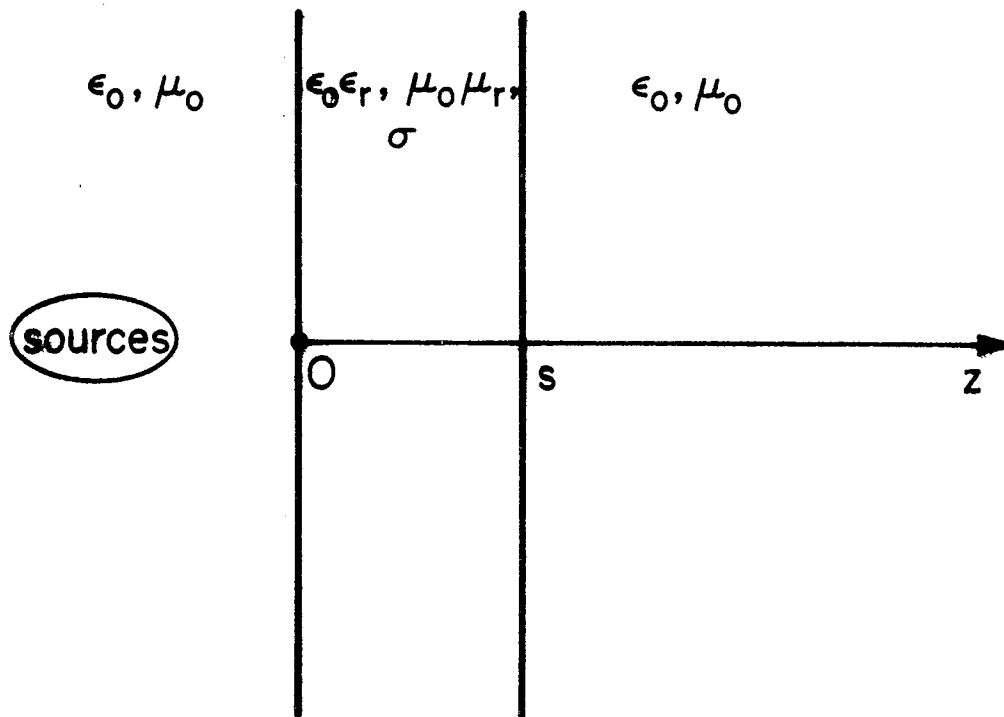


Figure 1. Geometry of the problem.

permeability $\mu = \mu_0 \mu_r$, and conductivity σ . We want to compute the field $\underline{E}^t, \underline{H}^t$ transmitted at any abscissa $z > s$ when the incident field $\underline{E}^i, \underline{H}^i$, i.e., the field produced by the sources when the slab is removed, is known at $z = 0$. For this purpose, it is convenient to expand the incident field in a plane wave set, since the interaction of individual plane wave components with the slab can be conveniently taken into account.

Accordingly, let $H_z^i(x, y, 0), E_z^i(x, y, 0)$ be the z -components of the field incident on the slab surface, with an assumed time dependence $\exp(j\omega t)$. The corresponding spectral components $h_z^i(u, v), e_z^i(u, v)$ are given, at $z = 0$, by

$$h_z^i(u, v) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy H_z^i(x, y, 0) \exp(jux + jvy) \quad (2.1)$$

$$e_z^i(u, v) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy E_z^i(x, y, 0) \exp(jux + jvy) \quad (2.2)$$

At $z = s$, i.e., at the output of the slab, the spectral components $h_z^t(u, v), e_z^t(u, v)$ will be linearly related to the incident components (2.1-2) in the case of a slab made of a linear material. Hence:

$$h_z^t(u, v) = t_H(u, v) h_z^i(u, v) \quad (2.3)$$

$$e_z^t(u, v) = t_E(u, v) e_z^i(u, v) \quad (2.4)$$

The transfer coefficients t_H, t_E can be easily computed for a homogeneous isotropic slab by noting that the transverse spectral components $h_t(u, v), e_t(u, v)$ are related to the longitudinal ones $h_z(u, v), e_z(u, v)$ via the

following relations:

$$\underline{h}_t = \frac{\omega \epsilon \underline{e}_z \underline{\kappa} \times \hat{z} \mp \omega h_z \hat{z} \times \underline{\kappa} \times \hat{z}}{u^2 + v^2} \quad (2.5)$$

$$\underline{e}_t = \frac{-\omega \mu \underline{h}_z \underline{\kappa} \times \hat{z} \mp \omega \epsilon \hat{z} \times \underline{\kappa} \times \hat{z}}{u^2 + v^2} \quad (2.6)$$

wherein $\underline{\kappa} = u\hat{x} + v\hat{y} + w\hat{z}$ and is the propagation vector referred to a Cartesian system of unit vectors $\hat{x}, \hat{y}, \hat{z}$, and upper (lower) signs refer to waves propagating in the positive (negative) direction of the z-axis. Eqs. (2-5,6) represent the total spectral field as a superposition of TE ($e_z = 0$) and TM ($h_z = 0$) parts. And, the medium being identical at both sides of the slab, it is then evident that t_H coincides with the usual slab transmission coefficient for TE plane wave incidence and, similarly, t_E is the same as the slab transmission coefficient for TM plane wave incidence. Letting

$$w = \sqrt{\kappa^2 - (u^2 + v^2)} \quad ; \quad w_s = \sqrt{\kappa^2 \left(\epsilon_r + \frac{\sigma}{j\omega\epsilon_0} \right) \mu_r - (u^2 + v^2)} \quad (2.7)$$

$$\gamma_H = \frac{w_s}{\mu_r w} \quad ; \quad \gamma_E = \frac{j\omega\epsilon_0}{\sigma + j\omega\epsilon_0\epsilon_r} \frac{w_s}{w} \quad (2.8)$$

we have:

$$t(u, v) = \frac{4}{(1+\gamma)^2} \frac{\exp(-jw_s s)}{1 - \left(\frac{1-\gamma}{1+\gamma} \right)^2 \exp(-2jw_s s)} \quad (2.9)$$

wherein γ may be taken equal to γ_H or γ_E in order to obtain t_H or t_E , respectively, and $\kappa = \omega\sqrt{\epsilon_0\mu_0}$.

The spectral components h_z, e_z at any $z > s$ are equal to the corresponding values (2.3-4) at $z = s$ times the plane wave transfer function $\exp[-jw(z-s)]$. Accordingly, any z -components $F_z^i(x,y,z)$ of the field transmitted at any arbitrary abscissa $z > s$ will be expressed in terms of the double Fourier integral

$$F_z^t(x,y,z) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv f_z^i(u,v) t(u,v) \exp[-jw(z-s)] \cdot \exp(-jux - jvy) \quad (2.10)$$

wherein f_z^i may be taken equal to h_z^i or e_z^i and, correspondingly, the values of t_H or t_E should be used.

On the other hand, the spectral representation of the z -components of the incident field (the slab is now removed) at any abscissa z is obviously the following:

$$F_z^i(x,y,z) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv f_z^i(u,v) \exp(-jwz) \cdot \exp(-jux - jvy) \quad (2.11)$$

Comparison of (2.10-11) shows that the transmitted field can be computed as the double convolution of the incident field and the double Fourier transform of $t(u,v) \exp(jws)$, hence:

$$F_z^t(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dy' F_z^i(x',y',z) T(x-x',y-y') \quad (2.12)$$

$$T(x,y) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv t(u,v) \exp(jws) \exp(-jux - jvy) \quad (2.13)$$

In the words of system theory, F^t is identified with the output of a linear system described by the unit response function (2.13) and excited by the input F^i .

We further note that relations similar to (2.12) exist between the transmitted and incident transverse components of the field, as easily follows from (2.5-6). It is only necessary to decompose the incident field in its TE and TM parts and then apply superposition.

3. The Azimuthally Symmetric Case

A case of particular interest is obtained when the incident field is not depending on x and y separately but rather upon the transverse coordinate $\rho = \sqrt{x^2 + y^2}$. For instance, if the source is taken equal to an elementary electric or magnetic dipole parallel to the z -axis at $P(0,0,-d)$, then

$$\begin{aligned} F_z^i(x,y,z) &= F_z^i(\rho,z) = -\frac{j\omega}{\kappa} [\kappa^2 \underline{A} + \nabla \nabla \cdot \underline{A}] \cdot \hat{z} = \\ &= -\frac{j\omega}{\kappa^2} \left[\kappa^2 A + \frac{\partial^2 A}{\partial z^2} \right] = \\ &= \frac{j\omega}{\kappa^2} \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial A}{\partial \rho} \end{aligned} \quad (3.1)$$

wherein

$$A(\rho,z) = C \frac{\exp(-j\kappa \sqrt{\rho^2 + (d+z)^2})}{\sqrt{\rho^2 + (d+z)^2}} \quad (3.2)$$

and is an electric or magnetic vector potential, the source intensity being proportional to the constant C .

The integrals (2.12-13) can now be simplified by using the change of coordinates:

$$x = \rho \cos \phi ; y = \rho \sin \phi ; u = \xi \cos \psi ; v = \xi \sin \psi \quad (3.3)$$

Accordingly,

$$\begin{aligned} T(x,y) = T(\rho) &= \int_0^{\infty} \xi d\xi t(\xi) \exp(j\sqrt{\kappa^2 - \xi^2} s) \\ &\cdot \int_0^{2\pi} \exp[-j\rho \xi \cos(\psi - \phi)] d\psi \\ &= 2\pi \int_0^{\infty} \xi d\xi J_0(\rho\xi) t(\xi) \exp(j\sqrt{\kappa^2 - \xi^2} s) \end{aligned} \quad (3.4)$$

and the field transmitted on the axis is given by

$$\begin{aligned} F_z^t(0,0,z) &= \int_0^{\infty} \rho d\rho F_z^i(\rho, d+z) T(\rho) \\ &= \int_0^{\infty} \xi d\xi \exp(j\sqrt{\kappa^2 - \xi^2} s) t(\xi) \int_0^{\infty} \rho d\rho F_z^i(\rho, d+z) J_0(\xi\rho) \end{aligned} \quad (3.5)$$

wherein the order of integration has been reversed. Upon substitution of (3.1) in (3.5) the inner integral can be evaluated by repeated integration by parts as follows:

$$\begin{aligned} \int_0^{\infty} \rho d\rho F_z^i(\rho, d+z) J_0(\rho\xi) &= \\ &= -\frac{j\omega C}{\kappa^2} \xi^2 \int_0^{\infty} \frac{\rho \exp(-jk\sqrt{\rho^2 + (d+z)^2})}{\sqrt{\rho^2 + (d+z)^2}} J_0(\rho\xi) d\rho \\ &= -\frac{\omega C}{\kappa^2} \xi^2 \frac{\exp[-j\sqrt{\kappa^2 - \xi^2} (d+z)]}{\sqrt{\kappa^2 - \xi^2}} \end{aligned} \quad (3.6)$$

the last expression stemming out from a known Fourier-Bessel transform [16]. Note that $\sqrt{\kappa^2 - \xi^2} = -j\sqrt{\xi^2 - \kappa^2}$ for $\xi^2 > \kappa^2$ and that we have implicitly assumed in this section $\kappa \neq 0$.

The formal expression for the z-component of the field transmitted through the slab is now the following:

$$F_z^t(0,0,z) = \omega C \kappa \int_{\Gamma} (1-u^2) t(u) \exp(-j\kappa \ell u) du \quad (3.7)$$

wherein $\ell = d - s + z$, the integration path Γ is depicted in Fig. 2, and the substitution $\kappa^2 - \xi^2 = \kappa^2 u^2$ has been used.

4. The Case of an Electric Plane Shield. Steady-State Excitation.

A case particularly interesting for applications is obtained when $\mu_r = 1$, $\sigma \gg \omega \epsilon_0 \epsilon_r$, i.e., when a highly conducting nonmagnetic slab is used as a shielding screen. As already noted in Section 1, this is an important configuration in shielding theory and practice. The solution to this problem is available in numerical form [14,15] for prescribed sinusoidal time variation and arbitrary spatial dependence for the fields; and in analytical form [5] for prescribed plane wave excitation and arbitrary time variation.

The case of a magnetic dipole excitation is considered first. The expression for $t(u)$ pertinent to this case is the following:

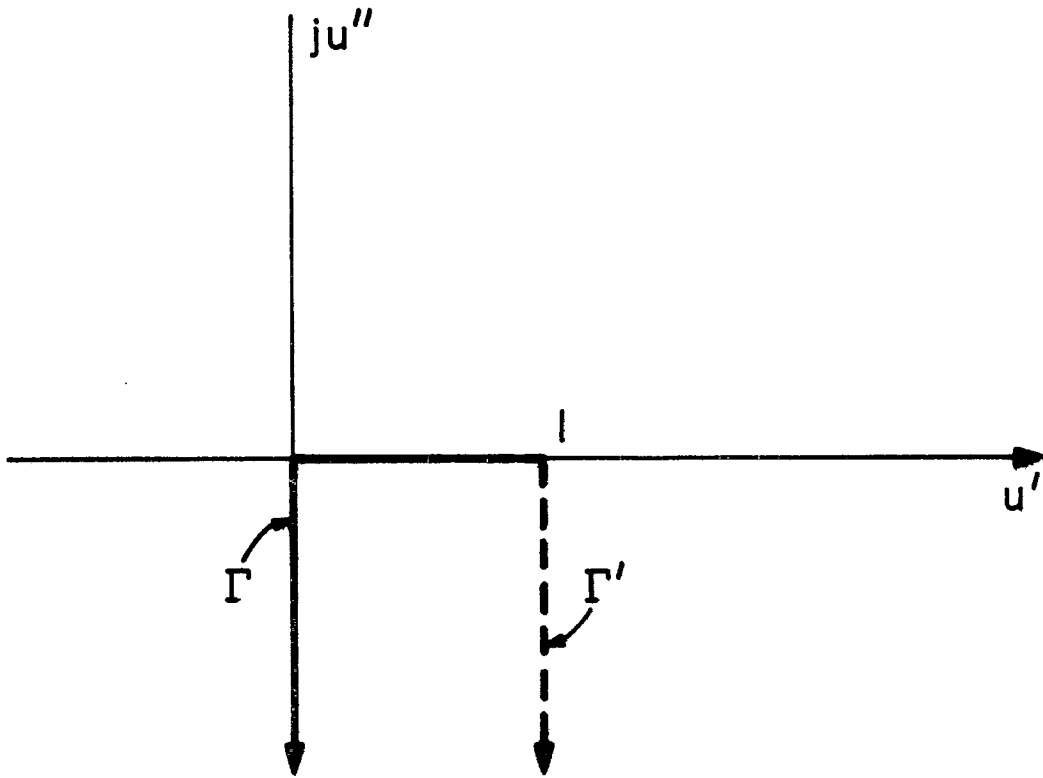


Figure 2. Integration path in the complex u -plane.

$$t_H(u) = \frac{4u \sqrt{\alpha^2 + u^2}}{(u + \sqrt{\alpha^2 + u^2})^2} \frac{\exp(-j \sqrt{\alpha^2 + u^2} \kappa s)}{1 - \left[\frac{u - \sqrt{\alpha^2 + u^2}}{u + \sqrt{\alpha^2 + u^2}} \right]^2 \exp(-2j \sqrt{\alpha^2 + u^2} \kappa s)} \quad (4.1)$$

$$\alpha^2 = \frac{\sigma + j\omega\epsilon_0(\epsilon_r - 1)}{j\omega\epsilon_0} \approx \frac{\sigma}{j\omega\epsilon_0} \quad (4.2)$$

It is noted that $t_H(u)$ exhibits no singularity in the lower right quadrant of the complex u -plane, so that the integration path Γ can be freely deformed therein, e.g., in the new path Γ' (see Fig. 2). When expression (4.1) is substituted in (3.7), it is noted that we can neglect u^2 with respect to α^2 provided the integrand is negligible when $u > |\alpha|$. Accordingly, when $\kappa\ell|\alpha| \gg 1$ the integral (3.7), specified to the case at hand, becomes

$$H_Z^t(0,0,\ell) = -j\omega C\kappa \frac{4}{\alpha} \frac{\exp(-j\alpha\kappa s)}{1 - \exp(-2j\alpha\kappa s)} \exp(-j\kappa\ell) \cdot \int_0^\infty v(-jv^2 + 3v + 2j) \exp(-\kappa\ell v) dv \quad (4.3)$$

and the origin of coordinates is now $\ell = 0$. The integral is now straightforward to evaluate and can be conveniently normalized to the value of the incident field $H_Z^i(0,0,\ell)$. We have

$$\frac{H_Z^t(0,0,\ell)}{H_Z^i(0,0,\ell)} = \left[\frac{4}{\alpha} \frac{\exp(-j\alpha\kappa s)}{1 - \exp(-2j\alpha\kappa s)} \right] \left[\frac{1 + \frac{3}{j\kappa\ell} + \frac{3}{(j\kappa\ell)^2}}{1 + \frac{1}{j\kappa\ell}} \right] = t_0(\alpha, \kappa s) \Omega_H(\kappa\ell) \quad (4.4)$$

It is noted that the first bracketed term $t_0(\alpha, \kappa s)$ is just the plane wave transmission coefficient under normal incidence and appropriate to a highly conducting screen. The second term $\Omega(\kappa\ell)$ depends on the mutual

distance l between transmitting and receiving points and approaches 1 when $\kappa l \gg 1$. Accordingly, it follows that simple plane wave transmission coefficient can be used for evaluating shielding effectiveness provided that transmitting and receiving antennas are a few wavelengths apart.

On the other hand, when κl is small $\Omega_H(\kappa l) \approx 3/j\kappa l$, and

$$\frac{H_Z^t(0,0,l)}{H_Z^i(0,0,l)} \approx \frac{3}{j\kappa l} t_0(\alpha, \kappa s) = \frac{\exp(-j\alpha \kappa s - j\pi/4) 3\delta}{[1 - \exp(-2j\alpha \kappa s)]\sqrt{2} l} \quad (4.5)$$

wherein δ is the skin depth of the screen. Note that Eq. (4.5) is valid provided that $\delta/l \ll 1$, otherwise the assumption $\kappa|\alpha|l \gg 1$ is no longer met.

The case of an electric dipole excitation can be treated similarly.

We have

$$t_E(u) = \frac{4\alpha^2 u \sqrt{\alpha^2 + u^2}}{(\alpha^2 u + u + \sqrt{\alpha^2 + u^2})^2} \frac{\exp(-j \sqrt{\alpha^2 + u^2} \kappa s)}{1 - \left(\frac{\alpha^2 u + u - \sqrt{\alpha^2 + u^2}}{\alpha^2 u + u + \sqrt{\alpha^2 + u^2}} \right)^2 \exp(-2j \sqrt{\alpha^2 + u^2} \kappa s)} \quad (4.6)$$

We can now neglect u^2 with respect to α^2 without serious limitation in the validity of the results. The integral corresponding to (4.3) is the following:

$$E_Z^t(0,0,l) = \omega C \kappa t_0(\alpha, \kappa s) \cdot \left\{ \int_1^\infty \frac{\exp(-j\kappa l u)}{u} du + \right. \\ \left. + j \int_0^\infty (1 - ju) \exp(-\kappa l v) dv \right\} \quad (4.7)$$

which can be easily evaluated to yield

$$\frac{E_z^t(0,0,l)}{E_z^i(0,0,l)} = t_0(\alpha, \kappa s) \Omega_E(\kappa l) \quad (4.8)$$

$$\Omega_E(\kappa l) = \frac{1}{2} \frac{1 + j\kappa l + (j\kappa l)^2 \exp(j\kappa l) [Ci(\kappa l) - jsi(\kappa l)]}{1 + \frac{1}{j\kappa l}}$$

wherein the cosinus integral $Ci(x)$ and sinus integral $si(x)$ functions [17] do appear.

It is again noted that $\Omega_E(\kappa l) \rightarrow 1$ when $\kappa l \gg 1$, as easily follows upon use of the asymptotic series expansions [17] of the functions $Ci(x)$ and $si(x)$, so that expression (4.8) reduces again to the plane wave transmission coefficient $t_0(\alpha, \kappa s)$ provided that transmitting and receiving antennas are a few wavelengths apart. On the contrary, when κl is small, a proper series expansion [17] shows that $\Omega_E(\kappa l) \approx j\kappa l/2$ and

$$\begin{aligned} \frac{E_z^t(0,0,l)}{E_z^i(0,0,l)} &\approx \frac{j\kappa l}{2} t_0(\alpha, \kappa s) = \\ &= j \frac{\exp(-j\alpha \kappa s + j\pi/4)}{1 - \exp(-2j\alpha \kappa s)} (\kappa l)^2 \frac{\delta}{\sqrt{2}l} \end{aligned} \quad (4.9)$$

5. The Case of an Electric Plane Shield. Transient Excitation.

We have shown under Section 4 that the steady-state z-components of the field transmitted through a highly conductive plane shield are given by

$$F_z^t(0,0,l) = F_z^i(0,0,l) t_0(\alpha, \kappa s) \Omega(\kappa l) \quad (5.1)$$

It is then evident that the z-components of the transient transmitted field can be obtained by time-convolving the transient z-components of

the incident field with the inverse Fourier transforms of $t_0(\omega)$ and $\Omega(\omega)$, say $T_0(t)$ and $\Omega(t)$. Use of Laplace inversion tables [18] shows that

$$\begin{aligned} T_0(t) &= 2 \sqrt{\frac{\tau}{\pi}} \sum_{n=1}^{\infty} \frac{\exp(-n^2 \eta/t)}{t^{5/2}} (2n^2 \eta - t) \\ &= 4 \sqrt{\frac{\tau}{\pi}} \sum_{n=1}^{\infty} \frac{d}{dt} \frac{\exp(-n^2 \eta/t)}{t^{1/2}} = 4 \sqrt{\frac{\tau}{\pi}} \sum_{n=1}^{\infty} \frac{d}{dt} S_0(t) \end{aligned} \quad (5.2)$$

where $\tau = \sqrt{\epsilon_0/\sigma}$ and is the relaxation time of the material of the shield, $\eta = s^2/c^2 \tau$ and is the diffusion time through the shield thickness.

A qualitative behavior of the first term $n=1$ of $T_0(t)$ is given in Fig. 3, wherein $S_{\max} = 4 \sqrt{\tau/2\pi\eta} = S_0(2\eta)$ and is the maximum value of the function $S_0(t)$ (see also Fig. 4. In S_{\max} "e" is the Neper's constant.)

The behavior of successive terms of the series (5.2) is similar to that depicted in Fig. 3. The maxima occur at later times and their absolute values are smaller by the factor $\exp[-2.6(n^2-1)]/n^3$. Accordingly, they can be safely neglected and we can take the only first term of the series (5.2).

After some algebra, Laplace inversion [19] of the two functions $\Omega(\omega)$ leads to

$$\Omega_H(t) = \delta(t) + \frac{3 - \exp(-t/T)}{T} U(t) \quad (5.3)$$

$$\begin{aligned} \Omega_E(t) &= \delta(t) - \frac{2}{T} \exp(-t/T) U(t) + \\ &+ \frac{3}{T} \int_0^{t/T} \frac{\exp(-u)}{(1 + \frac{t}{T} - u)^4} du \end{aligned} \quad (5.4)$$

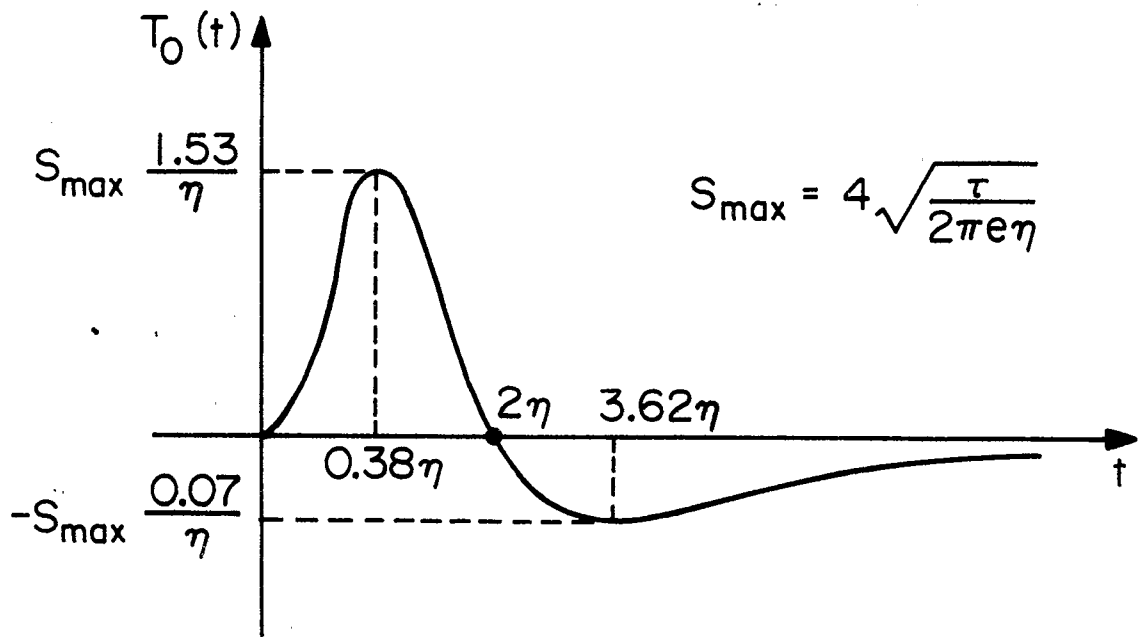


Figure 3. Qualitative behavior of the first series term of the function $T_0(t)$.

where $\delta(t)$ and $U(t)$ are the Dirac and the unit step function, respectively, $T = \ell/c$ and is the free-space transit time from the transmitting to the receiving antenna.

Convolution of (5.2) with the $\delta(t)$ terms of (5.3,4) just reproduce the function $T_0(t)$. Convolution with the other terms may become significant only after a time of order T . Accordingly, if the incident field has a time duration small compared with T , i.e., its spatial length is small compared with the in-between antennas distance ℓ , then the time dependence of the transmitted field is simply given by the time convolution of the incident signal and the function $T_0(t)$. This transmitted field is the same that would be obtained for the case of plane wave excitation. Accordingly, the result is obtained that the finite distance between antennas plays no significant role if the incident waveform is sufficiently short in time. For instance, if the incident signal is a pulse of unit amplitude and time duration T' , then

$$F_Z^t(0,0,\ell,t^*) = 4 \sqrt{\frac{T}{\pi}} \frac{\exp(-\frac{\eta}{t^*})}{\sqrt{t^*}}, \quad t^* \leq T' \quad (5.5)$$

$$F_Z^t(0,0,\ell,t^*) = 4 \sqrt{\frac{T}{\pi}} \left\{ \frac{\exp(-\frac{\eta}{t^*})}{\sqrt{t^*}} - \frac{\exp(-\frac{\eta}{t^*-T'})}{\sqrt{t^*-T'}} \right\}, \quad t^* > T' \quad (5.6)$$

where $t^* = t - (\ell/c)$ and is the retarded time. A qualitative sketch of (5.5,6) is given in Fig. 4 for $T' > 2\eta$. When $T' \ll 2\eta$, then the transmitted field is just given by (5.2) times T' .

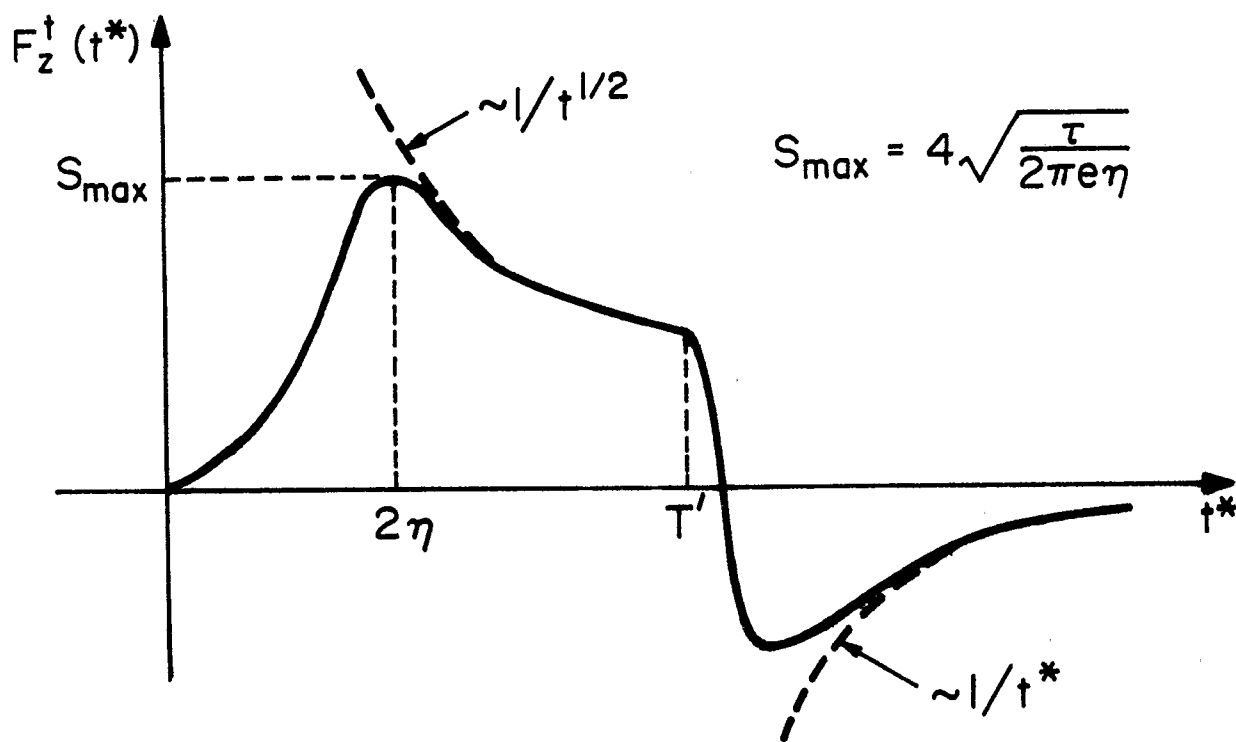


Figure 4. Qualitative behavior of a pulsed field of time duration T' after transmission through a highly conductive slab.

6. Conclusions. Practical Considerations.

We have considered the problem of transmission of steady-state and transient electromagnetic waves through a slab. An analytical solution has been obtained for the case of a linear, homogeneous, isotropic, highly conducting infinite slab excited by collinear electric or magnetic dipoles. The transmitted z-components of the field are expressed as the product (steady-state case) or the convolution (transient case) of the corresponding incident field components and a two-term factor. In the frequency domain the first term of this factor, see (5.1), is exactly the transmission coefficient of a plane wave normally incident on the slab. The second term takes into account the finite distance between the transmitting and receiving antennas and becomes significant only when this distance is of the order of, or smaller than, the free-space wavelength (steady-state case) or the spatial length of the incident pulse (transient case). It is therefore possible to obtain plane wave excitation results even when sources (and receivers) are located at finite distances. For this, all that is needed is the proper choice of distance between antennas.

It is certainly true that these results have been obtained under the conditions that the transmitting antenna is a dipole oriented normal to the slab; that the transmitted field is computed along the axial direction of the dipole; and that only the z-components of the field are used in the comparison. However, we believe that our analysis has a more general validity. For instance, results of the collinear configuration can easily be extended to transmitted field points off the axis. We should only substitute

$$J_0(\xi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \phi'}) \quad (6.1)$$

for $J_0(\xi\rho)$ in Eq. (2.12). Then expansion [20] of the Bessel function (6.1) and integration in ϕ' gives

$$F_Z^t(\rho, \ell) = \omega C \kappa \int_{\Gamma} (1-u^2) t(u) J_0(\kappa\rho\sqrt{1-u^2}) \exp(-j\kappa\ell u) du \quad (6.2)$$

which is the generalization of (3.7) to the case $\rho \neq 0$. Then $\partial F_Z^t(\rho, \ell)/\partial \rho = 0$ for $\rho = 0$, which implies that results of our analysis are certainly valid also in the neighbors of the axis. Furthermore, use of Maxwell's equations, with (6.2) as longitudinal fields, shows that the same is true for transverse fields.

Should further study show that the above considerations can be extended to more complicated geometries, all simulation studies for shielding purposes might be worth reconsidering.

Some few practical notes are now in order. Reference is made to a copper slab ($\sigma = 5.8 \times 10^7$ siemens/m) of thickness $s = 1$ mm, so that $\tau = 1.52 \times 10^{-19}$ sec and $\eta = 70$ μ sec. Only the plane wave transmission coefficient will be considered. For incident pulses of unit amplitude and time duration $T' \ll \eta$, the peak of transmitted field is equal to $6.9 \times 10^{-8} T'/\eta$, therefore linearly decreasing with the bandwidth $\sim 1/T'$ of the signal. In the sinusoidal excitation case, the attenuation due to the mismatch, $4|\alpha|$, equals that due to the damping inside the slab material, $\exp(-|\alpha|\kappa s/\sqrt{2})$, at the frequency $f = 0.72$ MHz. At this frequency, the transmitted field is equal to 11×10^{-12} times the incident one. At higher frequencies, the signal is decreasing exponentially with the square-root of the frequency.

For moderate antenna spacings, it is noted that the transmitted field can be computed using the plane-wave transmission coefficient only when the attenuation is very high. However, this may not be the case if even small apertures exist in the screen. Accordingly, we believe it is worthwhile to extend the analysis presented in this paper to other canonical problems, which are amenable to the same analytical approach. Among those, we list the problem of an infinite conductive screen with a single hole; and that of a conductive screen with a regular lattice of equal small apertures. The former problem can take advantage of the solution of a plane wave diffraction by apertures in conducting screens [21-23] and, eventually, of symmetrization procedures [24]. The latter could make use of artificial dielectric theory [25] properly accommodated to this single sheet problem.

7. References

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