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COUPLING EFFECTS BETWEEN BURIED INSULATED CABLES

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Kuan Min Lee

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1400 San Mateo Blvd., S. E./Suite A
Albuquerque, New Mexico 87108

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The coupling and earth-air interface effects are found for identical, parallel, insulated conductors immersed in a semi-infinite lossy medium. The effects are included as subsequent terms added to the TM mode propagation constant of a single, insulated wire in an infinite, lossy medium. The low-frequency approximation is calculated and a numerical example is given for a frequency of 1 Hz.

COUPLING EFFECTS BETWEEN BURIED INSULATED CABLES

1. INTRODUCTION

In this report, we address the problem of finding the coupling effects of parallel and identical insulated wires immersed in a semi-infinite lossy medium. This problem has been studied by several investigators (refs. 1-3). A review of the problem and the approach for solving it will be described briefly in the next section. Approximations involved in obtaining simple solutions will be discussed. The final expression of the self and mutual impedances are then given. Finally, the formulas are applied to a low frequency problem using typical low frequency data.

2. THE APPROACH TO THE PROBLEM

Consider an insulated wire of circular cross section with a center conductor of radius a, with wave number $k_1 = \omega \sqrt{\mu_0 \tilde{\varepsilon}_1}$, where $\tilde{\varepsilon}_1 = \varepsilon_0 \varepsilon_{r1} - j\, \sigma_1/\omega \cong -j\sigma_1/\omega$ (the skin depth is $\delta_1 = (2/\mu\omega_0\sigma_1)^{\frac{1}{2}}$), and an insulation sheath with radius b and dielectric constant ε_2, with wave number $k_2 = \omega \sqrt{\mu_0 \varepsilon_2}$. Let the insulated wire be immersed in a conducting half space with a dielectric constant $\varepsilon_3 = \varepsilon_0 \varepsilon_{r3}$ and conductivity σ_3, a loss tangent $P_3 = \sigma_3/\omega_0\varepsilon_3$, a skin depth $\delta_3 = (2/\omega\mu_0\sigma_3)^{\frac{1}{2}}$ and a wave number $k_3 = \omega\sqrt{\mu_0 \tilde{\varepsilon}_3}$, where $\tilde{\varepsilon}_3 = \varepsilon_0 \varepsilon_{r3} - j\sigma_3/\omega$. The upper space is assumed to be air with dielectric constant ε_0 and wave number k_0. As shown in figure 1, the wire is buried in the medium at a distance h away from the interface between air and the dissipative medium with the wire axis parallel to the interface. For the purpose of finding the circuit parameters, it is assumed the wire is infinite in length. Consider a second identical wire which is parallel to the first one. To find the common-mode propagation characteristics, let the separation between these two wires be s such that $b \ll s$. The problem is to find the transmission-line parameters of the wires taking the effects due to the interface and the scattering due to the second wire into account.

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1. Sunde, E. D., Earth Conduction Effects in Transmission Systems, Dover Publications, Inc., New York: 1968.

2. Guy, A. W. and G. Hasserjian, "Impedance Properties of Large Subsurface Antenna Arrays," IEEE Trans. Ant. & Prop., May 1963, pp. 232-240.

3. Stratton, J. A., Electromagnetic Theory, McGraw-Hill. New York: 1941, pp. 545-554.

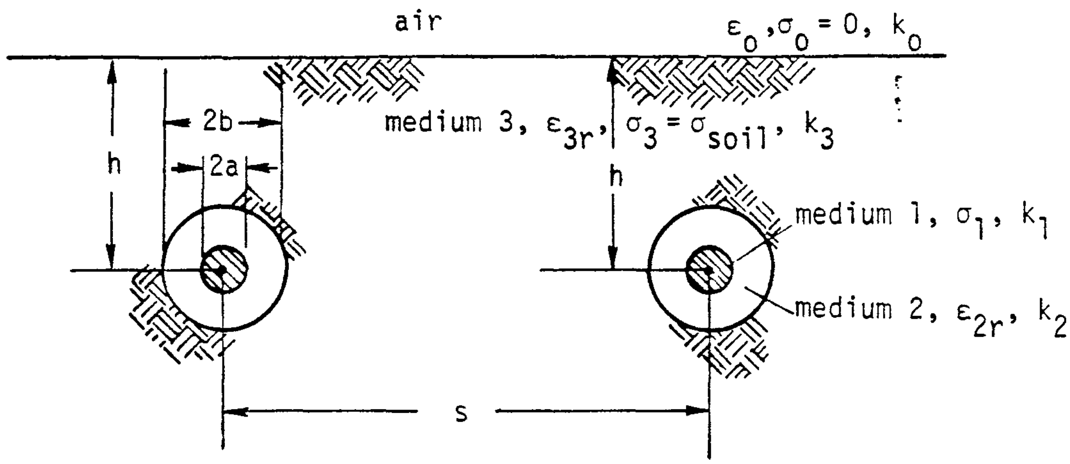

Figure 1. Coupled Insulated Wires in a Dissipative Half Space

The following assumptions are used to find the solutions:

(a) $(a/\lambda_o) < (b/\lambda_o) \ll 1$, where λ_o is the free space wavelength;

(b) $|k_3^2| \gg |k_2^2|$, where k_2 and k_3 as defined in the above paragraph can be complex as well as real;

(c) $h \gg b$, such that the TM wave assumption is true in the dielectric region;

(d) $s \gg b$, such that the TM wave assumption is again true in the dielectric region.

The approach to the problem can be divided into three steps:

Step 1: Referring to figure 2, we first consider a single insulated wire when the depth $h = \infty$. Assuming the lowest TM wave is the dominated mode when the wire is excited by a δ-function generator, the problem can be solved easily by standard technique as shown by Stratton (ref. 3).

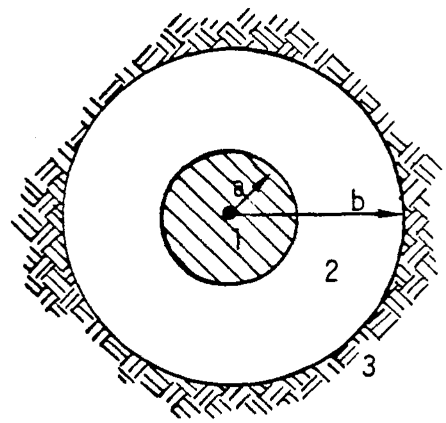

Figure 2. Single Insulated Wire in a Dissipative Medium

Step 2: Referring to figure 3, the effect due to the interface on the single wire is then introduced by assuming that the TM mode is still dominant in the dielectric region while the axial electric field in the dissipative medium is modified by the reflected wave due to the interface. Once the modification due to the reflection is obtained, the problem can again be solved by the standard technique. The procedure is given in reference 2.

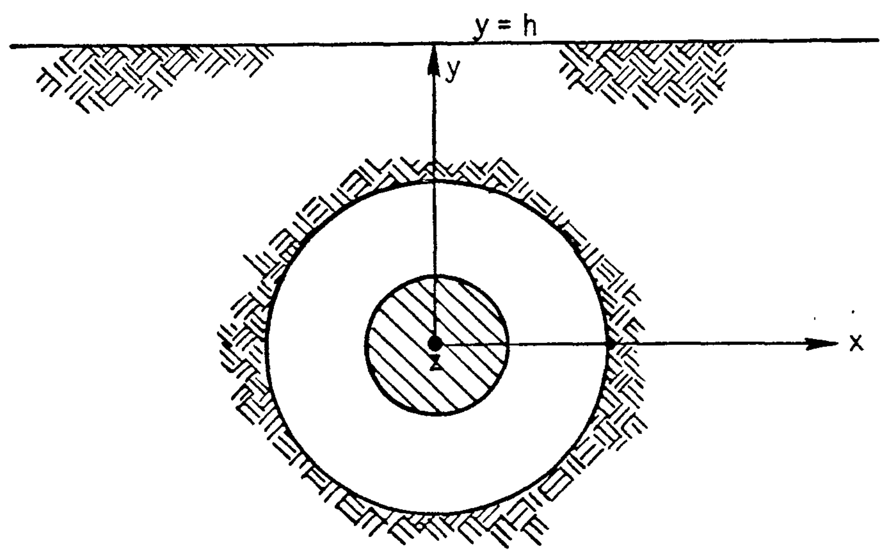

Figure 3. Single Insulated Wire in a Dissipative Half Space.

Step 3: Referring back to figure 1, the effects due to the next wire is then introduced by making a similar assumption as in Step 2, that TM mode will be dominant in the dielectric region while the axial electric field in the dissipative medium is modified by the interface reflection and that due to the scattered field of the second wire.

3. THE SOLUTION TO THE PROBLEM

If the wire carries an axial current on the center conductor with $I = I_o e^{j\omega t - jhz} = I_o e^{j\omega t - \Gamma z}$, (where $\Gamma = jh$ is the propagation constant), then using the procedure outlined in Section 2, the following characteristic equation can be derived for solving the propagation constant $\Gamma = jh = j(\beta - j\alpha)$:

$$\frac{Y_o(\lambda_2 a) - \dfrac{\mu_1\lambda_1 k_2^2 J_o(\lambda_1 a)}{\mu_2\lambda_2 k_1^2 J_1(\lambda_1 a)} Y_1(\lambda_2 a)}{J_o(\lambda_2 a) - \dfrac{\mu_1\lambda_1 k_2^2 J_o(\lambda_1 a)}{\mu_2\lambda_2 k_1^2 J_1(\lambda_1 a)} J_1(\lambda_2 a)} = \frac{Y_o(\lambda_2 b) - \dfrac{\mu_3\lambda_3 k_2^2 G(\lambda_3 b)}{\mu_2\lambda_2 k_3^2 H_1^{(2)}(\lambda_3 b)} Y_1(\ _2 b)}{J_o(\lambda_2) - \dfrac{\mu_3\lambda_3 k_2^2 G(\lambda_3 b)}{\mu_2\lambda_2 k_3^2 H_1^{(2)}(\lambda_3 b)} J_1(\lambda_2 b)} \tag{1}$$

where $J_n(x)$, $Y_n(x)$ and $H_n^{(2)}(x)$ are Bessel's functions of the first kind and second kind, and the Hankel's function of the second kind respectively, and,

$$\lambda_1 = \sqrt{k_1^2 - h^2}$$

$$\lambda_2 = \sqrt{k_2^2 - h^2}$$

$$\lambda_3 = \sqrt{k_3^2 - h^2}$$

$\mu_1 = \mu_2 = \mu_3 = \mu_o$, the permeability of free space

$$h = \beta - j\alpha \text{ with } \beta = \text{phase constant} \tag{2}$$
$$\alpha = \text{attenuation constant}$$

$h = \alpha + i\beta$ in Stratton's notation,

$jh = \Gamma$ in Guy and Hasserjian's papers.

In case of $I = I_o e^{-i\omega t + ihz}$, $h = \beta + i\alpha$ (and replace j in all quantities in the following by -i).

The function $G(\lambda_3 b)$ is given by,

$$\text{in Step 1,} \quad G(\lambda_3 b) \equiv H_o^{(2)}(\lambda_3 b) \tag{3}$$

$$\text{in Step 2,} \quad G(\lambda_3 b) \equiv H_o^{(2)}(\lambda_3 b) + I_o(o,o,h) \tag{4}$$

$$\text{in Step 3,} \quad G(\lambda_3 b) \equiv H_o^{(2)}(\lambda_3 b) + I_o(o,o,h) + H_o^{(2)}(\lambda_3 s) + I_o(s,o,h) \tag{5}$$

with

$$I_o(x,y,h) \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sqrt{k_3^2 - \zeta^2} - \sqrt{k_o^2 - \zeta^2}}{\sqrt{k_3^2 - \zeta^2} + \sqrt{k_o^2 - \zeta^2}} \; \frac{\exp\left[j\zeta x + j(y-2h)\sqrt{k_3^2 - \zeta^2}\right]}{\sqrt{k_3^2 - \zeta^2}} \, d\zeta \quad . \tag{6}$$

Equation (1) can be solved for $h = \beta - j\alpha$ provided that the following approximations are used:

$$\left|k_2^2\right| << \left|k_3^2\right|,$$

$$\left|k_2^2\right| << \left|k_1^2\right|,$$

$$\lambda_3 = \sqrt{k_3^2 - h^2} \doteq k_3,$$

h is of the same numerical order of k_2, i.e., $h \cong O(k_2)$

$$|\lambda_2 a| < |\lambda_2 b| << 1$$

$$J_o(\lambda_2 a) \doteq 1, \quad J_1(\lambda_2 a) \doteq 0, \quad Y_o(\lambda_2 a) \doteq \frac{2}{\pi}\ln\lambda_2 a, \quad Y_1(\lambda_2 a) \doteq \frac{2}{\pi\lambda_2 a}$$

$$J_o(\lambda_2 b) \doteq 1, \quad J_1(\lambda_2 b) \doteq 0, \quad Y_o(\lambda_2 a) \doteq \frac{2}{\pi}\ln\lambda_2 b, \quad Y_1(\lambda_2 b) \doteq \frac{2}{\pi\lambda_2 b}$$

$$H_o^{(2)}(\lambda_3 r) \doteq H_o^{(2)}(k_3 r)$$

$$H_1^{(2)}(\lambda_3 r) \doteq H_1^{(2)}(k_3 r) \quad . \qquad (7)$$

It is found that h is given in Step 1 by:

$$h = k_2\left[1 + \frac{H_o^{(2)}(k_3 b)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} - \frac{J_o(k_1 a)}{(k_1 a)\ln(b/a)J_1(k_1 a)}\right]^{1/2} ; \qquad (8)$$

in Step 2 by

$$h = k_2\left[1 + \frac{H_o^{(2)}(k_3 b)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} + \frac{j}{\pi}\frac{I(h/\delta)}{k_3 b\ln(b/a)H_1^{(2)}(k_3 b)} - \frac{J_o(k_1 a)}{(k_1 a)\ln(b/a)J_1(ka)}\right]^{1/2} ; \qquad (9)$$

in Step 3 by

$$h = k_2\left[1 + \frac{H_o^{(2)}(k_3 b)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} + \frac{j}{\pi}\frac{I(h/\delta)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} + \frac{H_o^{(2)}(k_3 s)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} + \frac{I_o(s,0,h)}{(k_3 b)\ln(b/a)H_1^{(2)}(k_3 b)} - \frac{J_o(k_1 a)}{(k_1 a)\ln(b/a)J_1(k_1 a)}\right]^{1/2} , \qquad (10)$$

where $I_n(s,0,h)$ is defined in eq. (6) and $I(h/\delta) \equiv I_o(o,o,h)(\pi/j)$ with $I_o(o,o,h)$ given in eq. (6). In each case, one can express the characteristic impedance of the insulated wire as (refs. 2,3,4)

$$Z_o = \frac{60\,h}{\varepsilon_{r2}\,k_o}\,\ell n(b/a) \quad . \tag{11}$$

With h and Z_o given by eqs. (10) and (11), it is easy to find the series impedance per unit length, Z_L, and shunt admittance per unit length, y_L, from the following relations:

$$Z_o = (Z_L/y_L)^{\frac{1}{2}} \tag{12a}$$

$$h = (-Z_L \cdot y_L)^{\frac{1}{2}} \quad . \tag{12b}$$

Using eqs. (10) and (11), it is found that

$$y_L = \frac{j\omega\cdot 2\pi\varepsilon_2}{\ell n(b/a)} = \text{shunt admittance per unit length} \tag{13a}$$

$$Z_L = Z_1^i + Z_3^i + Z^e \tag{13b}$$

$$Z_1^i = \frac{-j\omega\mu_o}{2\pi}\;\frac{J_o(k_1a)}{(k_1a)J_1(k_1a)} = \text{internal impedance per unit length of the center conductor} \tag{13c}$$

$$Z^e = \frac{j\omega\mu_o}{2\pi}\,\ell n(b/a) = \text{external impedance per unit length between the center conductor and the surrounding medium} \tag{13d}$$

$$Z_3^i = Z_{3a}^i + Z_{3b}^i + Z_{3c}^i + Z_{3d}^i = \text{internal impedance per unit length of the surrounding medium} \tag{13e}$$

$$Z_{3a}^i = \frac{j\omega\mu_o}{2\pi}\;\frac{H_o^{(2)}(k_3b)}{(k_3b)H_1^{(2)}(k_3b)} = \text{earth impedance due to the first wire} \tag{13f}$$

4. King, R. W. P., "The Insulated Conductor as a Scattering Antenna in a Relatively Dense Medium," IEEE Trans. Ant. & Prop., May 1976, pp. 327-330.

$$Z_{3b}^{i} = \frac{j\omega\mu_o}{2\pi} \frac{j}{\pi} \frac{I(h/\delta)}{(k_3b)H_1^{(2)}(k_3b)} = \text{impedance due to interface reflection of first wire field} \quad (13g)$$

$$Z_{3c}^{i} = \frac{j\omega\mu_o}{2\pi} \frac{H_o^{(2)}(k_3s)}{(k_3b)H_1^{(2)}(k_3b)} = \text{impedance due to the scattered field of the second wire} \quad (13h)$$

$$Z_{3d}^{i} = \frac{j\omega\mu_o}{2\pi} \frac{I_o(s,0,h)}{(k_3b)H_1^{(2)}(k_3b)} = \text{impedance due to the field generated from the interface reflection of the second wire field} \quad . \quad (13i)$$

The self and mutual impedance per unit length of the insulated wire are defined as

$$Z_{11} = Z_1^i + Z^e + Z_{3a}^i + Z_{3b}^i = \text{self impedance} \quad (14a)$$

$$Z_{12} = Z_{3c}^i + Z_{3d}^i = \text{mutual impedance} \quad . \quad (14b)$$

The desired solution is now complete except the evaluation of the integral $I_o(x,y,h)$. This will be discussed in the following section.

An equivalent circuit representation can be shown as in figure 4, where each circuit element is defined in the above paragraphs.

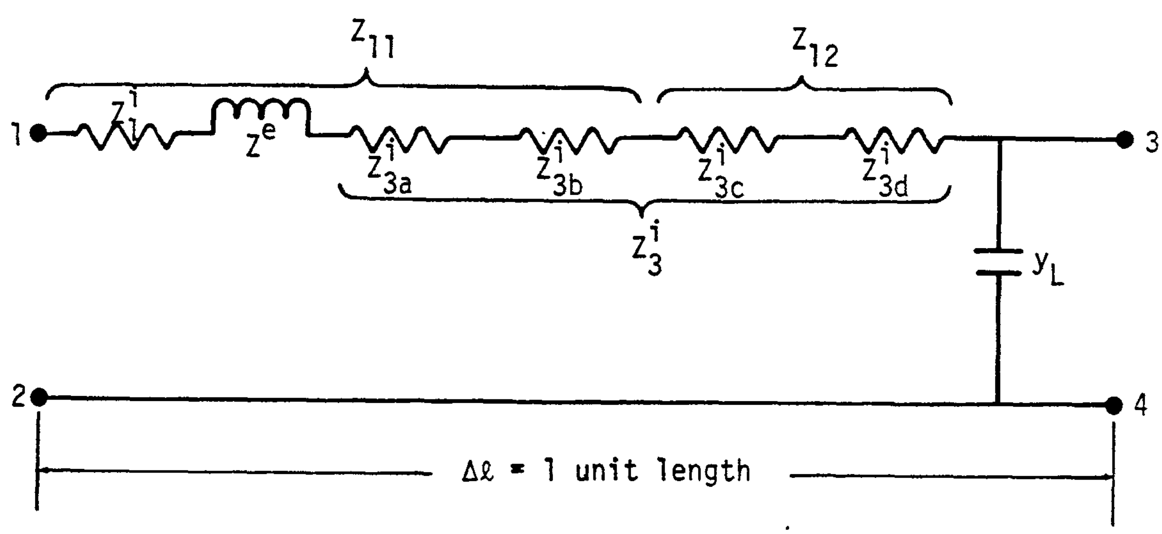

Figure 4. Equivalent Circuit of One Unit Length of the Insulated Cable

Note that, in Step 1, $Z^i_{3b} = Z^i_{3c} = Z^i_{3d} = 0$ since there is no interface reflection and no scattered fields due to the second wire. In Step 2, $Z^i_{3c} = Z^i_{3d} = 0$ since there is no scattered fields due to the second wire.

4. THE EVALUATION OF THE INTEGRALS

The evaluation of integrals $I(h/\delta)$ and $I_o(s,0,h)$ involved in the impedances is discussed in reference 2. We will follow that approach in this section. Some corrections are found and included here.

The integral to be evaluated is

$$I_o(x,y,h) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sqrt{k_3^2 - \zeta^2} - \sqrt{k_o^2 - \zeta^2}}{\sqrt{k_3^2 - \zeta^2} - \sqrt{k_o^2 - \zeta^2}} \quad \frac{\exp\left[j\zeta x + j(y-2h)\sqrt{k_3^2 - \zeta^2}\right]}{\sqrt{k_3^2 - \zeta^2}} \, d\zeta \qquad (15)$$

when $|k_3| >> k_o$, $x = y = 0$, with $k_3^2 \cong -j\, 2/\delta_3^2$ where δ_3 is the skin depth of the earth,

$$I_o(0,0,h) = \frac{j}{\pi} \int_{-\infty}^{\infty} \frac{\left(\sqrt{j + \zeta^2} - \sqrt{\zeta^2}\right)}{\left(\sqrt{j + \zeta^2} + \sqrt{\zeta^2}\right)} \cdot \frac{\exp\left[-2^{3/2}(h/\delta)(j + \zeta^2)^{1/2}\right]}{\sqrt{j + \zeta^2}} \, d\zeta$$

$$= \frac{j}{\pi} \; I(h/\delta) \qquad . \qquad (16)$$

This integral is evaluated numerically by Guy and Hasserjian (ref. 2) and is quoted in figure 5 for reference.

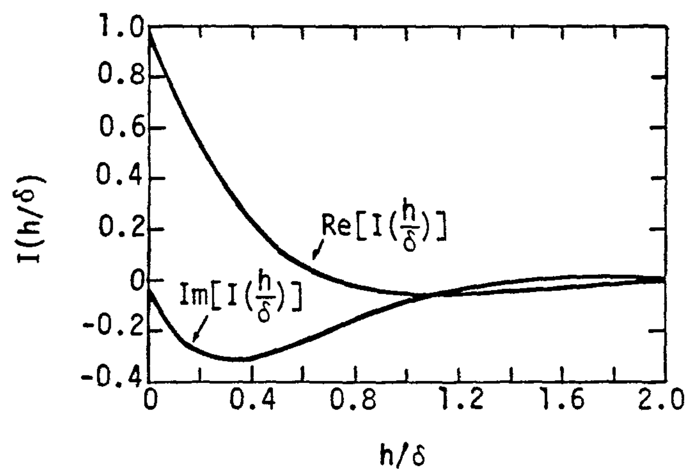

Figure 5. Plot of the Function $I(h/\delta)$
(From reference 2)

The next integral to be evaluated is $I_0(x,0,h)$, which can be rewritten as, when $|k_3| >> k_o$ and $x >> 2h$,

$$I_o(x,o,h) = (I_2 - 2I_1)\frac{1}{k_3^2 - k_o^2} . \qquad (17)$$

where I_2 and I_1 can be shown to be:

$$I_2 = (k_3^2 + k_o^2)H_o^{(2)}\left(k_3\sqrt{x^2+4h^2}\right) + 2\frac{\partial^2}{\partial x^2}H_o^{(2)}\left(k_3\sqrt{x^2+4h^2}\right) \qquad (18)$$

$$I_1 = -j\frac{\partial}{\partial x}\frac{\partial}{\partial y}H_o^{(2)}\left(k_4\sqrt{x^2+4h^2}\right) - \exp(-j2k_3h)\frac{\partial^2}{\partial y^2}H_o^{(2)}\left(k\sqrt{x^2+y^2}\right)\Bigg|_{y=0.} \qquad (19)$$

When the differentiation is carried as indicated, the final formula is

$$I_o(x,0,h) \cong \frac{1}{\left(k_3^2 - k_o^2\right)}\left\{k_3^2 + k_o^2 - \frac{2k_3^2x^2}{R'^2} - \frac{4j\ k_3^2xh}{R'^2}\right\}H_o^{(2)}(k_3R')$$
$$+\frac{2}{\left(k_3^2 - k_o^2\right)}\frac{k_3}{R'}\left\{1 + j\frac{4xh}{R'^2}\right\}H_1^{(2)}(k_3R')$$
$$-\exp[-j2\ k_3h]\frac{2k_o}{\left(k_3^2 - k_o^2\right)x}\ H_1^{(2)}(k_ox) \qquad (20)$$

where $R' = \sqrt{x^2 + 4h^2}$.

If one compares eq. (20) with eq. (34) in the Appendix of reference 2, it is found that there is a minus sign missed before the fourth term in the first bracket of that Appendix, and an extra factor x^2 appears before the second bracket. When a numerical example is worked out, it is found that this x^2 factor gives a very large error in the calculation and should be discarded. In eq. (30) of that appendix, a factor 2 is missed in the third term, and in eq. (31), the notation γ^2 should be k^2 (or k_o^2 in our notation). The final result, eq. (34), should be corrected for the minus sign and the x^2 factor as mentioned above.

5. SUMMARY OF THE RESULTS

For $|k_3| >> k_o$ and $s^2 >> 4h^2$,

Self impedance per unit length:

$$Z_{11} = \frac{j\omega\mu_o \ell n(b/a)}{2\pi} + \frac{j\omega\mu_o}{2\pi} \frac{H_o^{(2)}(k_3 b)}{k_3 b\ H_1^{(2)}(k_3 b)} + \frac{j\omega\mu_o}{2\pi} \frac{jI(h/\delta)}{\pi k_3 b H_1^{(2)}(k_3 b)}$$

$$- \frac{j\omega\mu_o}{2\pi} \frac{J_o(k_1 a)}{(k_1 a) J_1(k_1 a)} \tag{21}$$

Mutual impedance per unit length:

$$Z_{12} = \frac{j\omega\mu_o}{2\pi} \frac{H_o^{(2)}(k_3 s)}{(k_3 b) H_1^{(2)}(k_3 b)} + \frac{j\omega\mu_o}{2\pi} \frac{I_o(s,0,h)}{(k_3 b) H_1^{(2)}(k_3 b)} \tag{22}$$

$$I(h/\delta) = \int_{-\infty}^{\infty} \frac{(j+\zeta^2)^{\frac{1}{2}} - |\zeta|}{(j+\zeta^2)^{\frac{1}{2}} + |\zeta|} \frac{\exp[-2\ \ (h/\delta)(j+\zeta^2)^{\frac{1}{2}}]}{(j+\zeta^2)^{\frac{1}{2}}}\ d\zeta \tag{23}$$

$$I_o(s,0,h) = \frac{1}{(k_3^2 - k_o^2)} \left[k_3^2 + k_o^2 - \frac{2k_3^2 s^2}{(s^2+4h^2)} - \frac{j4k_3^2 hs}{(s^2+4h^2)} \right] H_o^{(2)}\left(k_3 \sqrt{s^2+4h^2}\right)$$

$$+ \frac{2\ k_3}{(k_3^2 - k_o^2)\sqrt{s^2+4h^2}} \left(1 + j\ \frac{4h}{s}\right) H_1^{(2)}\left(k_3 \sqrt{s^2+4h^2}\right)$$

$$- \exp[-2j\ k_3 h] \frac{2\ k_o}{(k_3^2 - k_o^2)s}\ H_1^{(2)}(k_o s) \quad . \tag{24}$$

For low frequency problems, the following approximations are appropriate:

$$H_0^{(2)}(k_3b) \cong 1 - \frac{2j}{\pi}\left(0.57722 + \ln \frac{k_3b}{2}\right)$$

$$H_1^{(2)}(k_3b) = \frac{+2j}{\pi k_3 b}$$

$$J_0(k_1a) \cong 1$$

$$J_1(k_1a) \cong \frac{k_1a}{2}$$

$$H_0^{(2)}(k_3s) \cong 1 - \frac{2j}{\pi}\left(0.57722 + \ln \frac{k_3s}{2}\right)$$

$$H_1^{(2)}(k_0s) \cong \frac{2j}{\pi k_0 s} \qquad . \qquad (25)$$

We shall work out an example using these approximations.

6. A NUMERICAL EXAMPLE

Let

Frequency = 1 Hz

$\omega = 2\pi$ rad.

$a = 1.27E-2m$

$b = 2.54E-2m$

$\sigma_1 = 5.8E-7$ mho/m

$\sigma_3 = \sigma_{soil} = 10^{-2}$ mho/m

$\varepsilon_{r3} = \varepsilon_{3rsoil} = 10$

$s = 10^3 m$

$h = 1m$

$\mu_0 = 4\pi E-7$ henry/m

$\varepsilon_0 = 8.85E-12$ farad/m

$\varepsilon_{2r} = 1.0$

Then

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} = 2.09E-8$$

$$k_3 = \omega\sqrt{\mu_0(\varepsilon_0\varepsilon_{r3} - j\sigma_3/\omega)} \doteq \sqrt{-j}\ \ 2.81E-4$$

$$k_2 = k_0 = 2.09E-8$$

$$k_1 = \omega\sqrt{\mu_0(\varepsilon_0\varepsilon_{r1} - j\ \sigma_1/\omega)} \doteq \sqrt{-j}\ \ 21.40$$

$$|k_1 a| = |\sqrt{-j}\ \ 0.27| << 1$$

$$|k_3 b| = |\sqrt{-j}\ \ 7.14E-6| << 1$$

$$|k_3 s| = 0.281 << 1$$

$$\delta_{soil} = \sqrt{\frac{2}{\omega\mu_o\sigma_{soil}}} = 5.03E\ 3\ m >> 1$$

$$\delta_{cond} = 6.61E-2\ m$$

$$\frac{h}{\delta_{soil}} = 1.99E-4 << 1$$

$$\frac{|k_3|}{|k_2|} = \frac{2.81E-4}{2.09E-8} = 1.34\ E\ 4 >> 1$$

$$s = 10^3\ m >> b = 0.0254\ m$$

$$h = 1\ m >> b = 0.0254\ m.$$

Hence, the required conditions for the approximations are satisfied. From figure 1 of reference 2 (figure 5 in this report),

$$I(h/\delta) = I(0.000199) \cong I(0) = 1. + j\ 0.$$

It is found that

$$Z_{11} = 3.55E-5 + j\ 1.65E-5$$

with the numerical values in (24), (25), and (22),

$$I_0(1000,\ 0,\ 1) \cong -0.4968 - j\ 0.9487$$

$$Z_{12} \cong 6.9087E-9 - j\ 1.3126E-7$$

so

$$\frac{|Z_{12}|}{|Z_{11}|} \cong \frac{1.3144E-7}{3.9147E-5} = 3.358E-3 = 0.34\%.$$

The mutual impedance per unit length is less than 1% of the self impedance per unit length in this example, and hence is almost negligible.

7. DISCUSSION

In the derivation of the wave number h and the impedances, we have assumed that the differential currents running in wire 1 and wire 2 are negligible. This is no longer true when s is not much larger than b. However, it is not too difficult to derive the parameters for the differential mode using a similar approach.

The method outlined in sections 2 and 3 is applicable to more than two wires as long as s >> b is satisfied. An example is given in reference 2 where an infinite number of arrays is solved. Again, when the differential currents are large, a new formulation is required.

Note that the condition $|k_3^2| >> |k_2^2|$ and $|a/\lambda_0| < |b/\lambda_0| << 1$ must be emphasized for the approximate solutions to be applied. However, this covers a large range of parameters in the practical use. The case $|k_3^2|$ near $|k_2^2|$ has recently been studied for case 1 of section 2 (ref. 5). The generalization to cases 2 and 3 seems to be possible if one follows the same procedure.

5. Lee, K. M., T. T. Wu and R. W. P. King, "Theory of the Insulated Linear Antenna in a Dissipative Medium," Radio Science, April 1977.

REFERENCES

1. Sunde, E. D., Earth Conduction Effects in Transmission Systems, Dover Publications, Inc. New York: 1968.

2. Guy, A. W. and G. Hasserjian, "Impedance Properties of Large Subsurface Antenna Arrays," IEEE Trans. Ant. & Prop., May 1963, pp. 232-240.

3. Stratton, J. A., Electromagnetic Theory, McGraw-Hill. New York: 1941, pp. 545-554.

4. King, R. W. P., "The Insulated Conductor as a Scattering Antenna in a Relatively Dense Medium," IEEE Trans. Ant. & Prop., May 1976, pp. 327-330.

5. Lee, K. M., T. T. Wu and R. W. P. King, "Theory of the Insulated Linear Antenna in a Dissipative Medium," Radio Science, April 1977.