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CURRENT INDUCED BY A PLANE WAVE ON A THIN INFINITE COATED WIRE ABOVE THE GROUND+

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Abstract

The current induced on a thin infinite coated wire above the ground when an electromagnetic plane wave is incident on it is calculated. The final result is presented in a form which exhibits clearly the various contributions to the total impedance of the wire-ground system. Numerical examples are given, which demonstrate the dependence of the induced current on the conductivity of the ground, the thickness of the coating and the polarization and direction of propagation of the incident wave.

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I. Introduction

The propagation and the excitation of electromagnetic waves on long cables parallel to an interface between two homogeneous half-spaces has been the subject of a number of recent works [1] - [7]. However, in the discussions of an infinite wire in the air only the bare wire case seems to have been treated exactly [4] - [5]. In the present work we calculate the current which is induced on an infinite coated wire above the ground by an incident plane wave. We employ the method developed by Wait [2] and with which he has treated in detail the case of a buried insulated wire [3] , [7].

II. Formulation

We consider an infinitely long thin wire of radius r_w and conductivity σ_w, located at a height h over the ground. The wire is coated with an insulator of relative dielectric constant ε_c, so that the outer radius is r_c. The ground is characterized by its conductivity σ_g and its relative dielectric constant ε_g. We choose the coordinate system so that the wire is parallel to the z axis at x=-h. Let ψ be the angle between the direction of propagation of the incident plane wave and the ground and let θ be the angle between the z axis and the projection of the direction of propagation on the ground plane, x=0 (Fig. 1).

The axial electric field at the location of the wire, due to the incident field (amplitude E_o) and the field reflected from the ground, is given by

$$E_z^a = E_o \sin\theta \left\{1 + R_h e^{-2jk_o h\sin\psi}\right\} e^{-jk_o z \cos\psi \cos\theta} \tag{1}$$

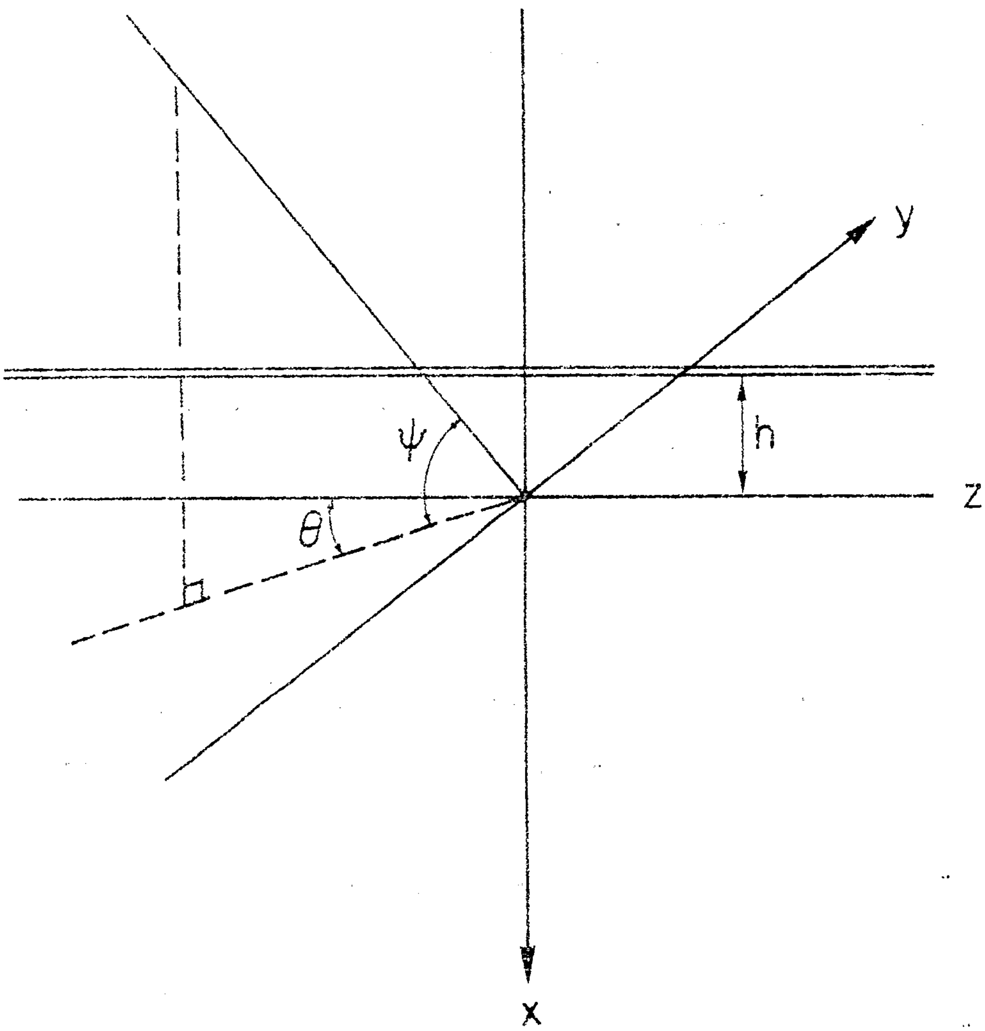


Fig. 1. Geometry of wire, at $x=-h$, above the ground $(x=0)$. The angles θ and ψ define the direction of propagation of the incident wave.

for the E (or horizontal) polarization and by

$$E_z^a = E_o \sin\psi \cos\theta \left\{1-R_v e^{-2jk_o h \sin\psi}\right\} e^{-jk_o z\cos\psi \cos\theta} \qquad (2)$$

for the H (or vertical) polarization. Here R_h and R_v are the Fresnel reflection coefficients

$$R_h = \frac{\sin\psi - [\varepsilon_g(1 + \sigma_g/j\omega\varepsilon_g\varepsilon_o) - \cos^2\psi]^{\frac{1}{2}}}{\sin\psi + [\varepsilon_g(1 + \sigma_g/j\omega\varepsilon_g\varepsilon_o) - \cos^2\psi]^{\frac{1}{2}}} \qquad (3)$$

$$R_v = \frac{\varepsilon_g(1 +\sigma_g/j\omega\varepsilon_g\varepsilon_o)\sin\psi - [\varepsilon_g(1 + \sigma_g/j\omega\varepsilon_g\varepsilon_o) - \cos^2\psi]^{\frac{1}{2}}}{\varepsilon_g(1 +\sigma_g/j\omega\varepsilon_g\varepsilon_o)\sin\psi + [\varepsilon_g(1 + \sigma_g/j\omega\varepsilon_g\varepsilon_o) - \cos^2\psi]^{\frac{1}{2}}} \qquad (4)$$

and $k_o = \omega(\varepsilon_o\mu_o)^{\frac{1}{2}}$.

The fields due to the induced current in the wire are expressed in terms of the z components π and π^* of electric and magnetic Hertz vectors respectively [2]. Using a cylindrical coordinate system (r,ϕ,z), which is coaxial with the wire, we can express the field components H_ϕ and E_z in terms of π and π^* in the form

$$H_\phi = \frac{1}{r}\frac{\partial^2\pi^*}{\partial z\partial\phi} - j\omega\varepsilon\frac{\partial\pi}{\partial r} \qquad (5)$$

$$E_z = \left\{k^2 + \frac{\partial^2}{\partial z^2}\right\}\pi \qquad (6)$$

All fields will depend on z through the factor $e^{-\Gamma z}$ only, as dictated by the external field, where $\Gamma = jk_o\cos\psi\cos\theta$. The field components inside the thin wire and its coating will have the form

$$E_z = e^{-\Gamma z}(k^2 + \Gamma^2)\tilde{\pi} \tag{7}$$

$$H_\phi = - j\omega\varepsilon e^{-\Gamma z} \frac{\partial\tilde{\pi}}{\partial r} \tag{8}$$

where $\tilde{\pi}$ satisfies the equation

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + v^2\right)\tilde{\pi} = 0 \tag{9}$$

and $v^2 = k^2 + \Gamma^2$. The appropriate solutions inside the coating, $r_w < r < r_c$, are

$$E_z^c = e^{-\Gamma z}\ v_c^2\ [A_c J_o(v_c r) + B_c Y_o(v_c r)]$$

$$H_\phi^c = e^{-\Gamma z}\ j\omega\varepsilon_c\ v_c[A_c J_1(v_c r) + B_c Y_1(v_c r)] \tag{11}$$

where $v_c^2 = k_c^2 + \Gamma^2$, $k_c^2 = \omega^2\varepsilon_c\varepsilon_o\mu_o$. Inside the conductor, $r < r_w$, the field components are

$$E_z^w = e^{-\Gamma z}\ v_w^2 A_w I_o(jv_w r) \tag{12}$$

$$H_\phi^w = -e^{-\Gamma z}\ j\sigma_w v_w A_w I_1(jv_w r) \tag{13}$$

where $v_w^2 = k_w^2 + \Gamma^2$, $k_w^2 = -j\omega\mu_o\sigma_w$.

Applying the continuity requirements on E_z and H_ϕ at $r=r_w$ we can eliminate two of the three constants A_c, B_c, A_w. We obtain for the fields inside the coating

$$E_z^c = e^{-\Gamma z}\ v_c^2 A_c [J_o(v_c r) - GY_o(v_c r)] \tag{14}$$

$$H_\phi^c = e^{-\Gamma z}\, j\omega\varepsilon_c v_c A_c [J_1(v_c r) - G Y_1(v_c r)] \tag{15}$$

where

$$G = \frac{J_o(v_c r_w) - jJ_1(v_c r_w)[k_c^2 v_w I_o(jv_w r_w)/k_w^2 v_c I_1(jv_w r_w)]}{Y_o(v_c r_w) - jY_1(v_c r_w)[k_c^2 v_w I_o(jv_w r_w)/k_w^2 v_c I_1(jv_w r_w)]} \tag{16}$$

In the absence of the ground the potential in the air due to the current in the wire would be given by

$$\pi_o = e^{-\Gamma z} K_o(jv_o r) \tag{17}$$

where $v_o^2 = k_o^2 + \Gamma^2$. The modified Bessel function can be expanded in the form [8]

$$K_o(jv_o r) = \frac{1}{2}\int_{-\infty}^{\infty} dt\, \frac{e^{-jty - u_o(h+x)}}{u_o} \tag{18}$$

where $u_o = (t^2 - v_o^2)^{\frac{1}{2}}$. To account for the presence of the ground we have to add to π_o a reflection term and a magnetic potential term, which we write as integrals analogous to (18) [2]. The potentials in the region $-h < x < o$ will be

$$\pi_o = e^{-\Gamma z}\int_{-\infty}^{\infty} dt\, \frac{e^{-jty}}{u_o}\, [e^{-u_o(h+x)} + Re^{u_o(x-h)}] \tag{19}$$

$$\pi_o^* = e^{-\Gamma z}\int_{-\infty}^{\infty} dt\, \frac{e^{-jty}}{u_o}\, Me^{u_o x} \tag{20}$$

and those in the ground, $x > o$, will be

$$\pi_g = e^{-\Gamma z}\int_{-\infty}^{\infty} dt\, \frac{e^{-jty}}{u_g}\, Te^{-u_g x} \tag{21}$$

$$\pi_g^* = e^{-\Gamma z} \int_{-\infty}^{\infty} dt \frac{e^{-jty}}{u_g} Ne^{-u_g x} \tag{22}$$

where $u_g = (t^2 - v_g^2)^{\frac{1}{2}}$, $v_g^2 = k_g^2 + \Gamma^2$, $k_g^2 = \omega\mu_o(\omega\varepsilon_g\varepsilon_o - i\sigma_g)$.

The four coefficients R, M, T, N can be determined from the four continuity requirements on E_y, E_z, H_y, H_z at x=0. In the following calculation only R will be needed, for which we find

$$R = -1 + \frac{2k_o^2 u_o}{v_o^2} \cdot \frac{t^2 - u_g u_o}{k_o^2 u_g + k_g^2 u_o} \tag{23}$$

The fields in the air, at the location of the wire, x=-h, will have the form

$$E_z^o = A_o e^{-\Gamma z} v_o^2 \left[K_o(jv_o r) + \int_o^{\infty} dt \frac{e^{-2hu_o}}{u_o} R\right] \tag{24}$$

$$H_\phi^o = -A_o e^{-\Gamma z} \omega\varepsilon_o v_o K_1(jv_o r) \tag{25}$$

Requiring E_z and H_ϕ to be continuous at $r = r_c$, using eqs. (14) (15) (24) (25) for the fields, and noting that the current I in the wire is equal to $2\pi r_c H_\phi^o$ we obtain

$$I = E_z^a / Z \tag{26}$$

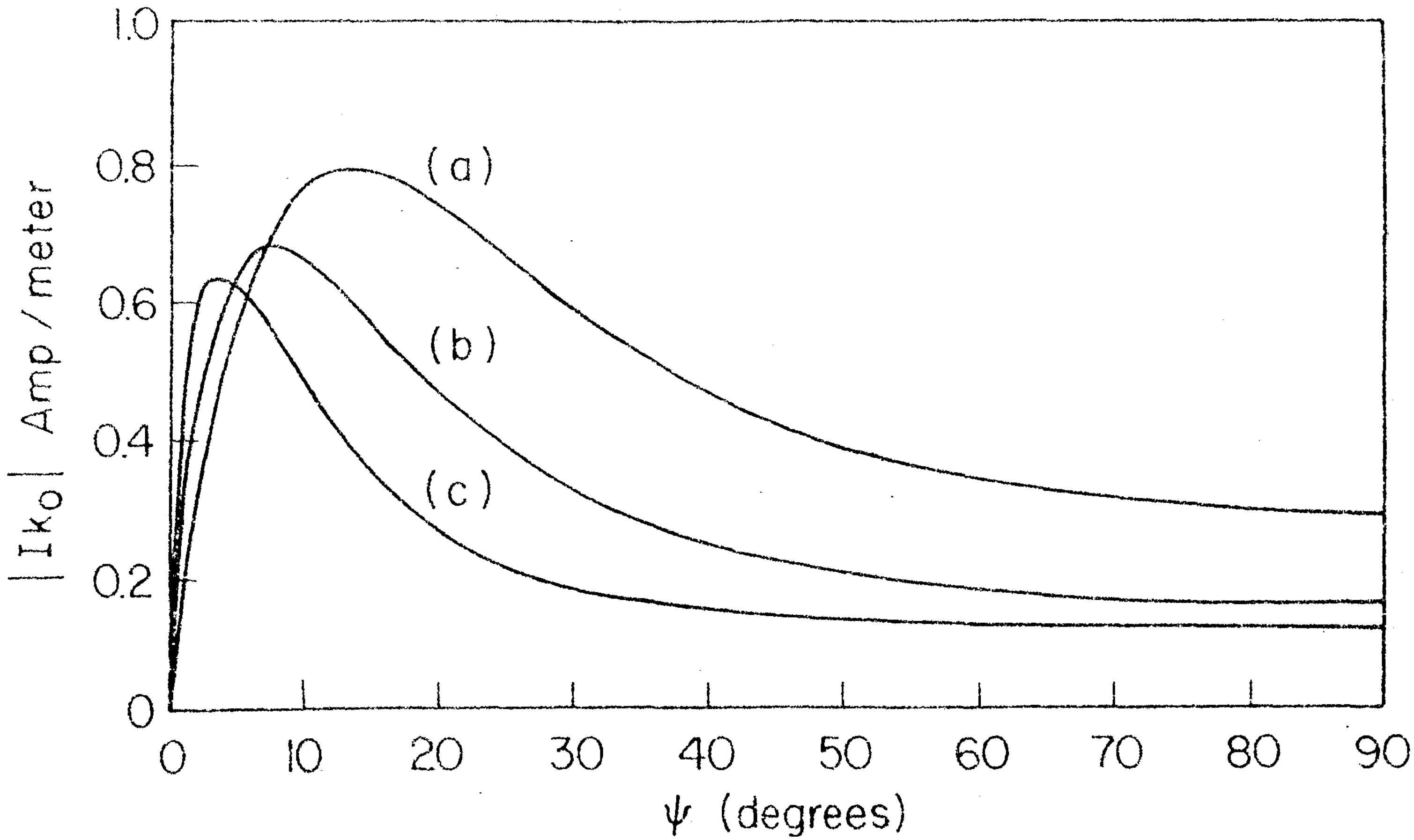
$$Z = \frac{\omega\mu_o}{2\pi j} \left\{ \frac{v_o^2}{k_o^2} \left[\ell n\left(0.89jv_o r_w\right) - \int_o^{\infty} \frac{dt}{u_o} e^{-2hu_o} R\right] + \left(\frac{v_o^2}{k_o^2} - \frac{v_c^2}{k_c^2}\right) \ell n \frac{r_c}{r_w} - \frac{1-j}{\sqrt{2\sigma_w \mu_o \omega r_w^2}} \right\} \tag{27}$$

Here we have simplified the expressions containing Bessel functions by using expansions which are valid for wavelengths large compared with r_c and for high conductivity of the wire. The result (27) shows that the current is obtained from the external field by dividing by the impedance Z which contains the following four terms: (a) The impedance of the long wire without the ground, and assuming that it is perfectly conducting; (b) The contribution due to the effect of the ground; (c) The contribution of the coating; (d) The internal impedance of the wire.

Olsen and Chang [4] have calculated the current induced on a bare, perfectly conducting wire above the ground, using a different method. We have found that for $r_c = r_w$ (i.e., no coating) and $\sigma_w \to \infty$ our results agree with those of Ref. [4].

III. Numerical results and discussion

We present the results of some calculations for the case of a wire of radius $r_w = 0.01$ m and conductivity $\sigma_w = 5.8\text{x}10^7$ mho/m at a height of h=6 m. All calculations were performed for an incident plane wave of unit amplitude having the frequency f = 500 kHz. For the relative dielectric constant of the ground the value $\varepsilon_g = 10$ has been employed. Figs. 2 and 3 show the dependence of the current on the angle ψ, for the case of a bare wire, and for three different values of the ground conducitivity. For the H polarization (Fig. 2) there appears a maximum at small ψ. This maximum tends to $\psi=0°$ with increasing conductivity of the ground. For the E polarization (Fig. 3)



Fig, 2. Induced current on a bare wire for the case of H polarization, calculated with the parameters h=6 m r_w=0.01 m, f=500kHz, θ=0, and for three ground conductivities: (a) σ_g=0.001 mho/m; (b) σ_g=0.01 mho/m; (c) σ_g=0.1 mho/m.

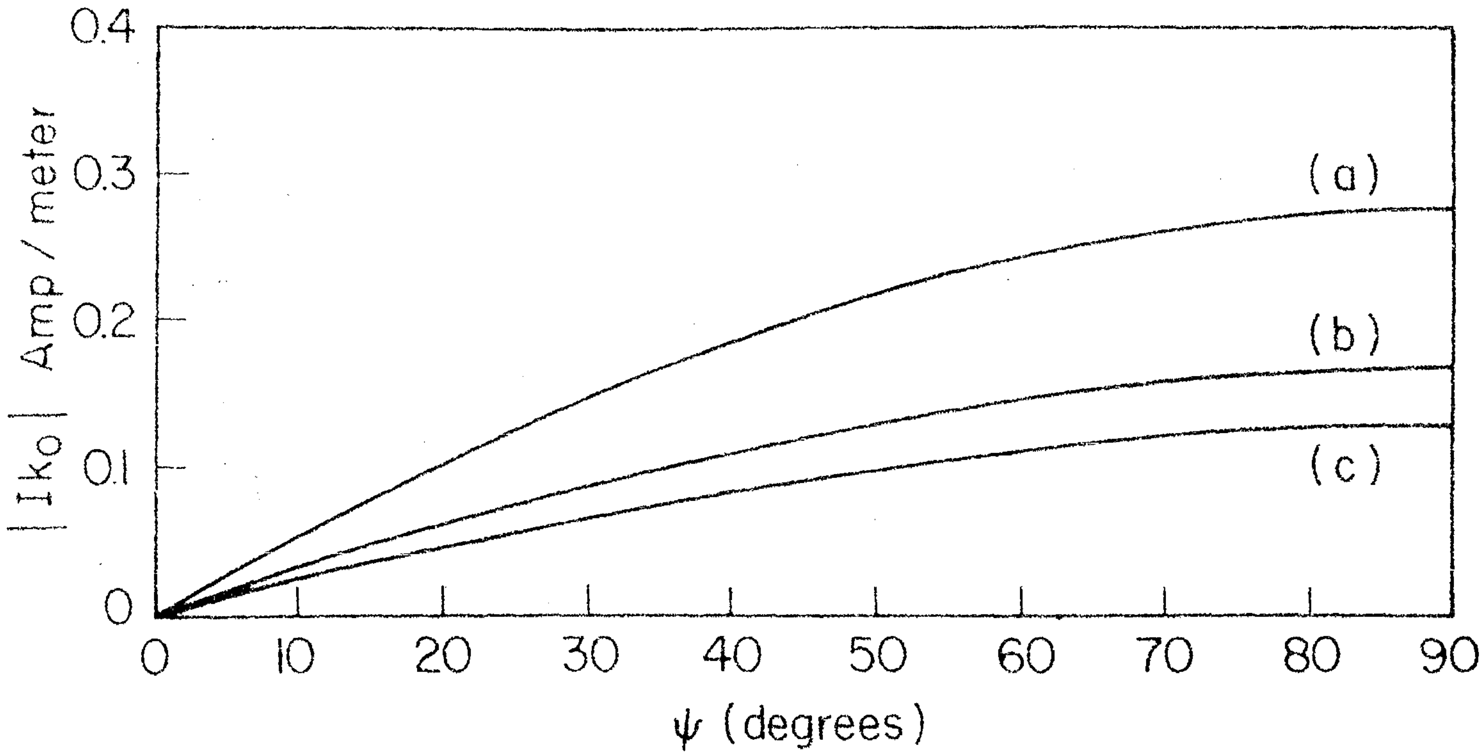


Fig. 3. Induced current on a bare wire for the case of E polarization, calculated with the parameters $h=6m$, $r_w=0.01$ m, $f=500kHz$, $\theta=90°$, and for three ground conductivities: (a) $\sigma_g=0.001$ mho/m; (b) $\sigma_g=0.01$ mho/m; (c) $\sigma_g=0.1$ mho/m.

the current increases monotonically with ψ. In Fig. 4 the effect of a polyethylene coating ($\varepsilon_c = 2.31$) is demonstrated for the H polarization. The effect of the coating is significant only for angles ψ smaller than about 45°. As could be expected, the induced current decreases when the thickness of the coating increases.

In conclusion, we have derived a convenient formula for the induced current on a coated infinite thin wire above the ground. The final result, eq. (27), involves a simple integral which can be readily evaluated numerically. It is therefore useful for investigating the effects of various parameters, such as the ground conductivity and the coating thickness, on the induced current.

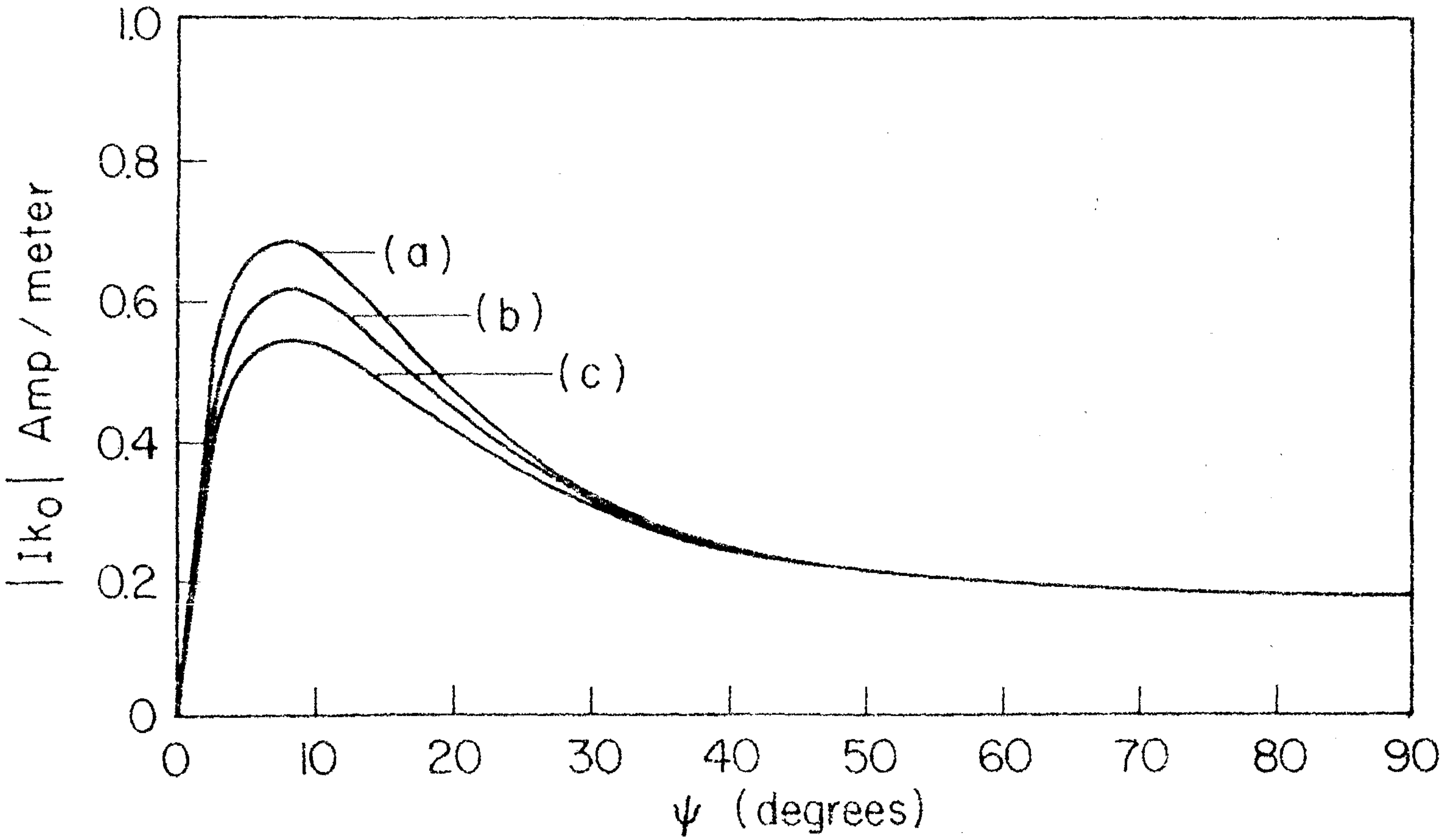


Fig. 4. Induced current for the case of H polarization, calculated with the parameters $h=6$ m, $r_w=0.01$ m, $f=500$kHz, $\sigma_g=0.01$ mho/m, $\theta=0$ and with: (a) $r_c=0.01$ m (no coating); (b) $r_c=0.012$ m; (c) $r_c=0.015$ m.

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