

Interaction Notes

Note 430

July 1982

CLOSED CYLINDER

SINGULARITY EXPANSION DESCRIPTORS

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ABSTRACT

A collection of the dominant resonances for the Singularity Expansion Method (SEM) description of currents induced on a closed, cylindrical structure, is reported for cylindrical aspect ratios ranging from length/diameter of 3.3333 to length/diameter of 100, for both, the zeroth-order and first order azimuthal variation cases. In the case of zero azimuthal variation: poles, natural modes, and concomittant normalization constants are presented for the twelve dominant resonances of a cylinder, whereas, for the case of first order azimuthal variation these data are presented for the four dominant resonances; in both cases, data is provided for each of the aspect ratios: 100,50,20,10,5, 3.3333. All of the data is available in data-cassette format compatible with the Tektronix 4050-Series computer communications

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Mexico.

I. Introduction

The data presented in this report was extracted from a Pocklington-type integro-differential equation. A method of moments solution was employed to solve the Electric Field Integral Equation of a closed, capped, thin-walled, perfectly-conducting cylinder. Symmetry was exploited to improve the efficiency of the extraction of the singularity expansion parameters. A Body of Revolution (BOR) approach, like that of Glisson and Wilton [1], was then used to allow extraction of individual harmonic modes. The first part of this report presents the zeroth harmonic modes -- those whose current and charge distributions are invariant around the cylinder. The natural resonances, the natural modes, and the associated SEM normalization constants are presented here for each of the aspect ratios: 100, 50, 20, 10, 5, 3.3333. The aspect ratio is defined to be the quotient of the cylinder's length and its diameter.

II. SEM Quantities of Zeroth-Order Azimuthal Variation

A graphical representation of the real and imaginary parts of each current natural mode in the case of zeroth-order azimuthal variation is given. Each graph shows how the real part (solid line) and the imaginary part (dashed line) of the current natural mode vary over the extent of the structure. No azimuthal variation is shown,

because the modes are constant in the azimuthal direction. The pole index, the pole location, the normalization constant, the number of zones along the sidewall (NZ) on half of the structure, and the number of zones (NR) along the cap radius on half of the structure are printed across the top of each graph. The pole index can be interpreted as follows: the last two digits of the index designate the pole location in the file, while the preceding digits indicate the aspect ratio. The pole and normalization constant (B) are represented by their real and imaginary parts. The poles are normalized on a peak magnitude basis.

The reported data for a closed cylinder of aspect ratio 100 were checked against those of Tesche [2]. The poles nearly coincided, while the normalization constants agreed to within ten percent in magnitude. The current natural modes reported for the 100 aspect ratio cylinder also exhibited good agreement with Tesche's results.

A complete listing of each zeroth-order pole, normalization constant, and current natural mode is presented herein. A companion Tektronix 4050-Series tape is formatted in the following manner:

File 1	Table of Contents
File 2	Aspect Ratio 100
File 3	Aspect Ratio 50
File 4	Aspect Ratio 20
File 5	Aspect Ratio 10
File 6	Aspect Ratio 5
File 7	Aspect Ratio 3.3333

Each file contains all twelve poles, normalization constant, and current natural mode compiled for that particular aspect ratio. Each record for a particular natural frequency is stored in the following form. Pole Index, Pole Location, number of current zones in the z-direction (NZ) on half of the structure, number of current zones (NR) in the p-direction on half of the structure.

If the pole index is odd there are $2*(NZ+NR)-1$ current natural mode values; if the index is even, there are $2*(NZ+NR)+1$ values. In either case, for a given aspect ratio: the number of values that represent each current

natural mode is constant. The Fortran format statements that are needed to read the data in transferring it via the Tektronix communications interface are: for the POLE INDEX, POLE, BETA, NZ, NR

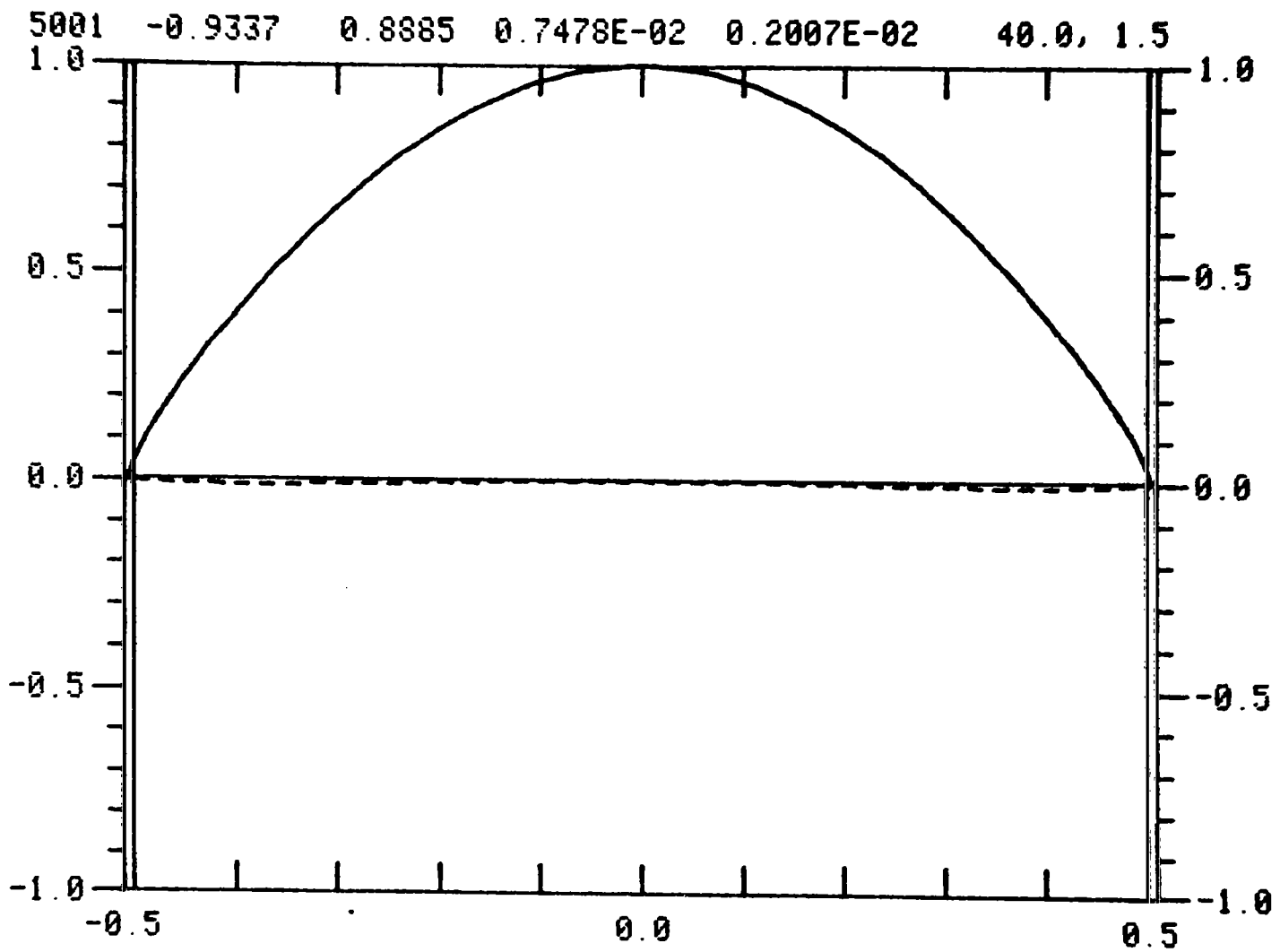
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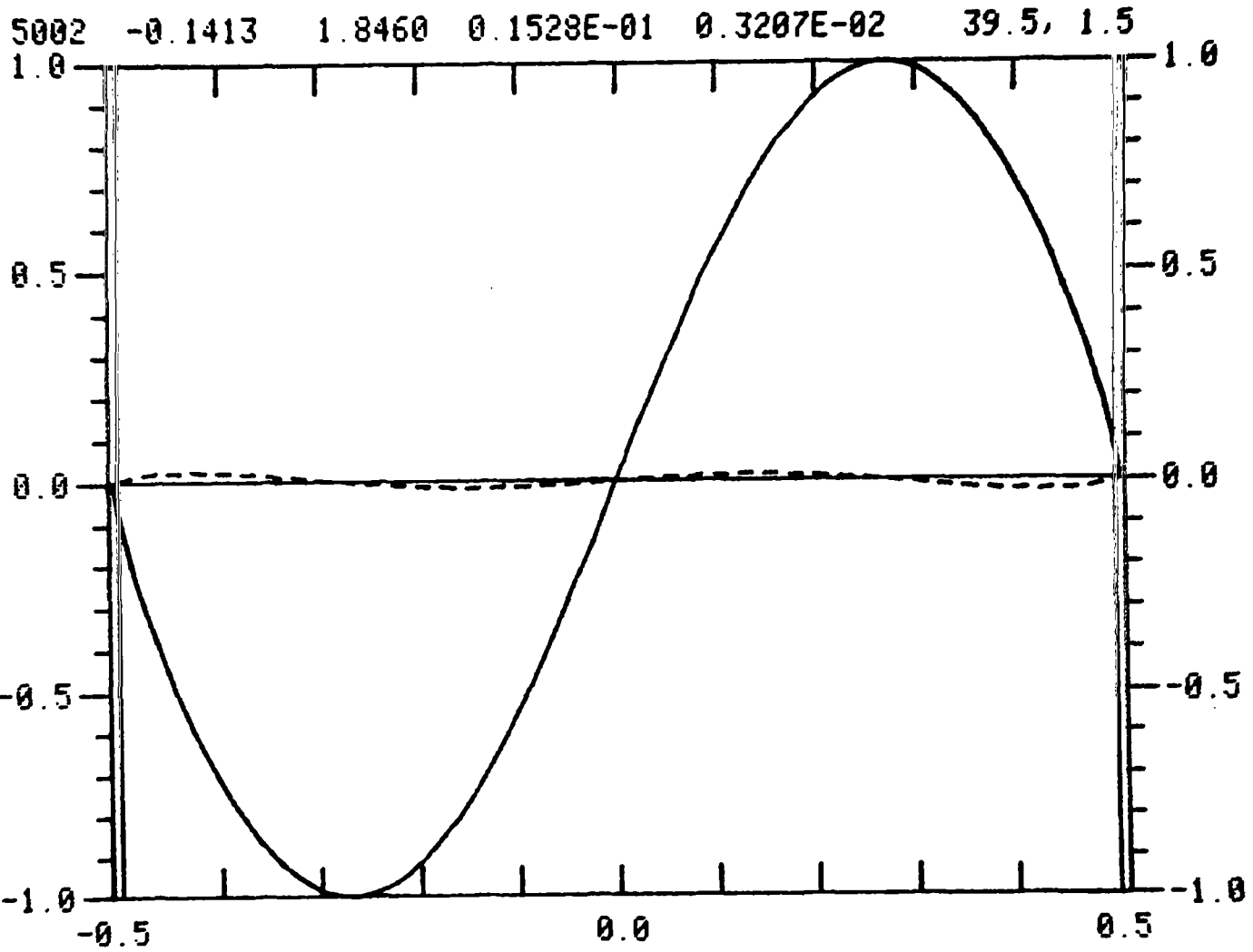
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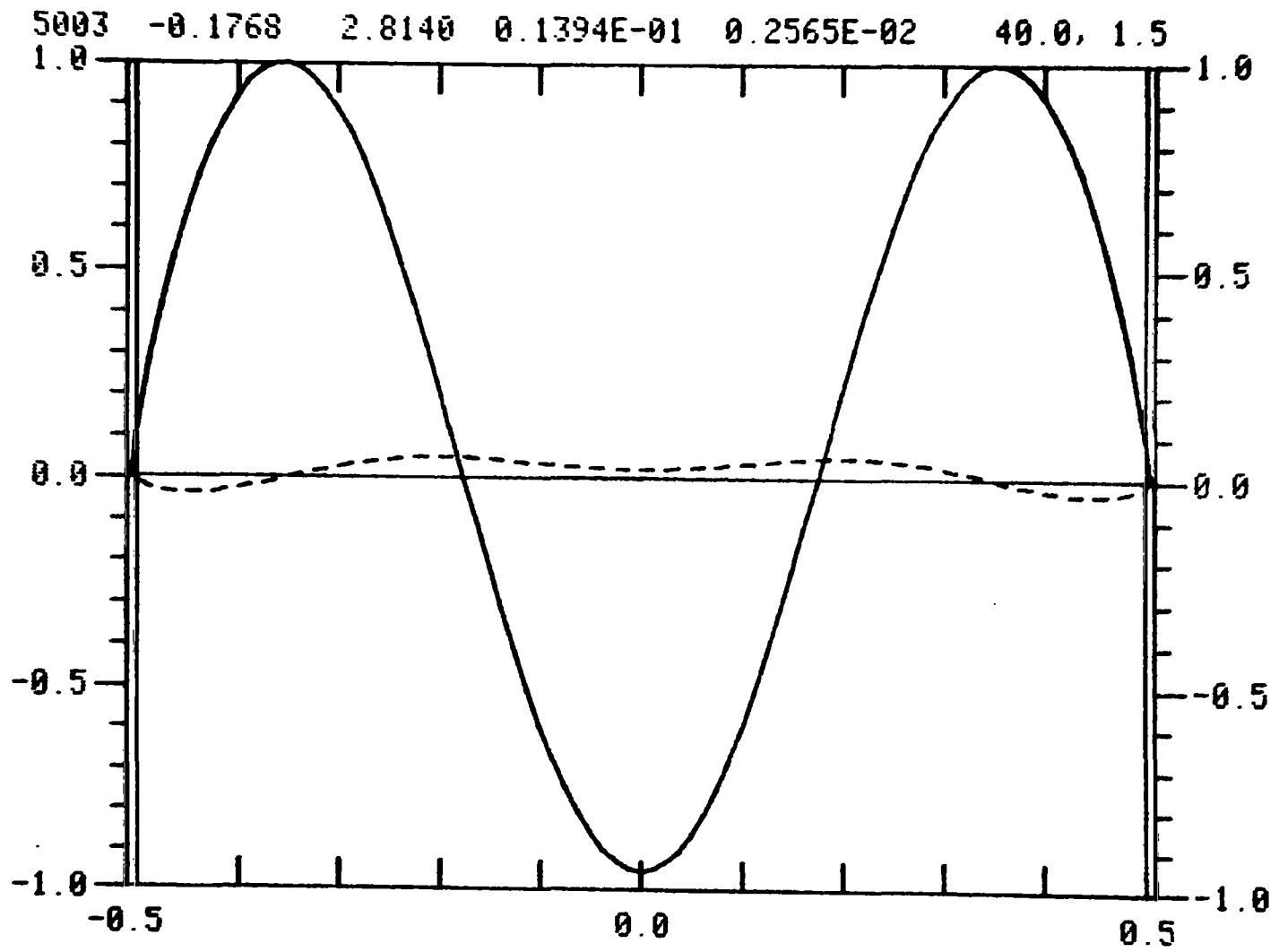
INDEX	POLE		NORMALIZATION		NZ	NR
10001	-0.8157E-01	0.9114	0.4081E-02	0.8080E-03	40.0	1.5
10002	-0.1203	1.881	0.1349E-01	0.2433E-02	39.5	1.5
10003	-0.1480	2.858	0.7181E-02	0.9771E-03	40.0	1.5
10004	-0.1703	3.838	0.1160	0.2636E-02	39.5	1.5
10005	-0.1894	4.818	0.3269E-01	0.7153E-02	40.0	1.5
10006	-0.2062	5.798	-0.8543E-01	-0.2189E-02	39.5	1.5
10007	-0.2211	6.776	0.4337E-01	0.9163E-02	40.0	1.5
10008	-0.2345	7.753	0.2471	-0.5576E-01	39.5	1.5
10009	-0.2467	8.727	-0.2537	0.2134	40.0	1.5
10010	-0.2576	9.699	0.9724E-01	0.2660	39.5	1.5
10011	-0.2674	10.67	0.1014	0.7896E-02	40.0	1.5
10012	-0.2761	11.63	-0.4387	1.434	39.5	1.5
5001	-0.9337	0.8885	0.7478E-02	0.2007E-02	40.0	1.5
5002	-0.1413	1.846	0.1528E-01	0.3207E-02	39.5	1.5
5003	-0.1768	2.814	0.1394E-01	0.2565E-02	40.0	1.5
5004	-0.2063	3.785	0.2820	-0.1488E-01	39.5	1.5
5005	-0.2319	4.759	0.5377E-01	0.1662E-01	40.0	1.5
5006	-0.2549	5.732	-0.1354	0.8306E-02	39.5	1.5
5007	-0.2758	6.705	0.6687E-01	0.1621E-01	40.0	1.5
5008	-0.2949	7.677	0.3250	-0.5285	39.5	1.5
5009	-0.3125	8.646	-0.3008	0.1933	40.0	1.5
5010	-0.3287	9.613	-0.1130	0.3467E-01	39.5	1.5
5011	-0.3436	10.58	0.1754	0.1124E-01	40.0	1.5
5012	-0.3517	11.54	-0.4537	0.3969E-01	39.5	1.5
2001	-0.1139	0.8434	-0.5844E-01	0.5867E-01	40.0	1.5
2002	-0.1811	1.776	0.1988E-01	0.5662E-02	39.5	1.5
2003	-0.2342	2.724	0.3306E-01	0.9734E-02	40.0	1.5
2004	-0.2805	3.680	2.356	-1.683	39.5	1.5
2005	-0.3225	4.639	0.1213	0.7183E-01	40.0	1.5
2006	-0.3616	5.601	-0.2634	0.3763E-01	39.5	1.5
2007	-0.3986	6.564	-0.1101	0.1480	40.0	1.5
2008	-0.4339	7.527	-0.7727E-01	-0.4578	39.5	1.5
2009	-0.4679	8.488	-0.5486	0.7362	40.0	1.5
2010	-0.5007	9.449	-0.1496	0.1101	39.5	1.5
2011	-0.5326	10.41	1.033	-0.1303	40.0	1.5
2012	-0.5636	11.36	-0.4538	-0.1605	39.5	1.5
1001	-0.1341	0.7924	0.2398E-01	0.1591E-01	40.0	3.5
1002	-0.2255	1.695	0.2076E-01	0.8318E-02	39.5	3.5
1003	-0.3031	2.620	0.4805E-01	0.1760E-01	40.0	3.5
1004	-0.3745	3.557	0.4295	0.6887	39.5	3.5
1005	-0.4432	4.502	0.1482E-01	0.4874	40.0	3.5
1006	-0.5111	5.451	-0.4387	0.4079E-01	39.5	3.5
1007	-0.5800	6.404	-0.1494	0.9251E-01	40.0	3.5
1008	-0.6519	7.358	0.1518	0.1643	39.5	3.5
1009	-0.7293	8.315	0.1243	-0.6895E-01	40.0	3.5
1010	-0.8166	9.274	1.371	-0.7252E-01	39.5	3.5
1011	-0.9218	10.24	-0.2053	-1.365	40.0	3.5
1012	-1.065	11.21	-0.2053	-1.365	39.5	3.5

INDEX		POLE	NORMALIZATION		NZ	NR
501	-0.1596	0.7224	0.2737E-01	0.3646E-01	40.0	7.5
502	-0.2910	1.580	0.3245E-01	0.2004E-01	39.5	7.5
503	-0.4153	2.472	0.1045	0.5661E-01	40.0	7.5
504	-0.5438	3.380	0.9640	0.2571	39.5	7.5
505	-0.6884	4.294	0.3136	0.3731	40.0	7.5
506	-0.8740	5.197	-0.3783	-0.1854	39.5	7.5
507	-1.132	6.005	3.440	-2.626	40.0	7.5
508	-1.305	6.609	-1.557	-0.7530	39.5	7.5
509	-1.203	7.150	1.165	-0.3044	40.0	7.5
510	-0.9707	7.952	0.6514	0.2945	39.5	7.5
511	-0.8471	8.845	0.2842	-0.6162	40.0	7.5
512	-0.7842	9.741	3.394	1.076	39.5	7.5
301	-0.1769	0.6706	0.1117E-01	0.9659E-02	40.0	11.5
302	-0.3441	1.491	0.4982E-01	0.2421E-01	39.5	11.5
303	-0.5191	2.347	-0.4889	0.7989	40.0	11.5
304	-0.7266	3.196	-0.4904	-1.452	39.5	11.5
305	-0.9701	3.939	-2.265	2.150	40.0	11.5
306	-1.076	4.520	-0.3767	-0.2222	39.5	11.5
307	-0.9502	5.137	-0.1696	-0.3479	40.0	11.5
308	-0.7881	5.926	0.3434	0.2081	39.5	11.5
309	-0.7134	6.767	-1.473	-0.7943	40.0	11.5
310	-0.7134	7.611	-0.1859	-0.3687	39.5	11.5
311	-0.7806	8.461	0.9392	0.1686	40.0	11.5
312	-0.9196	9.319	0.2919	-0.4981	39.5	11.5

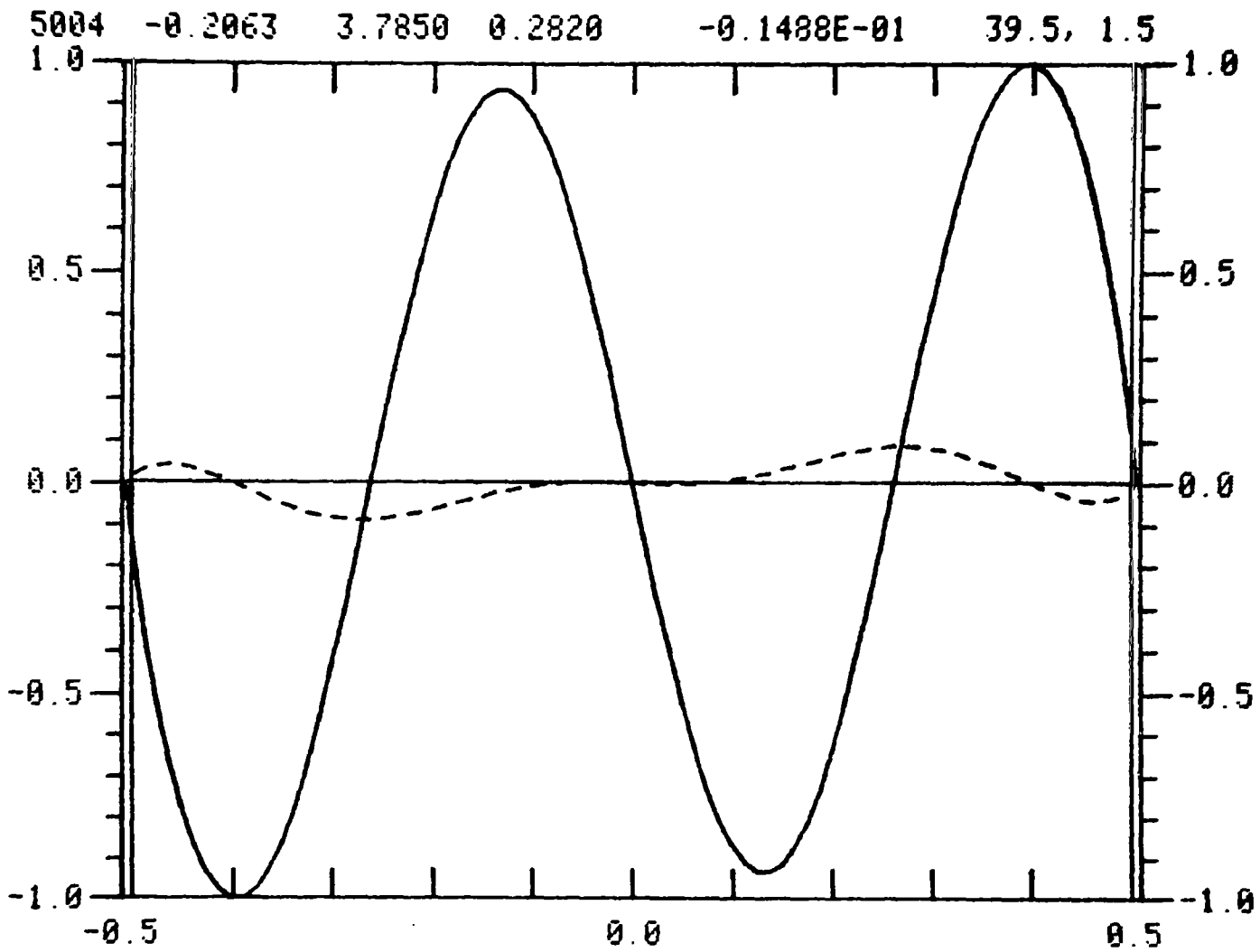


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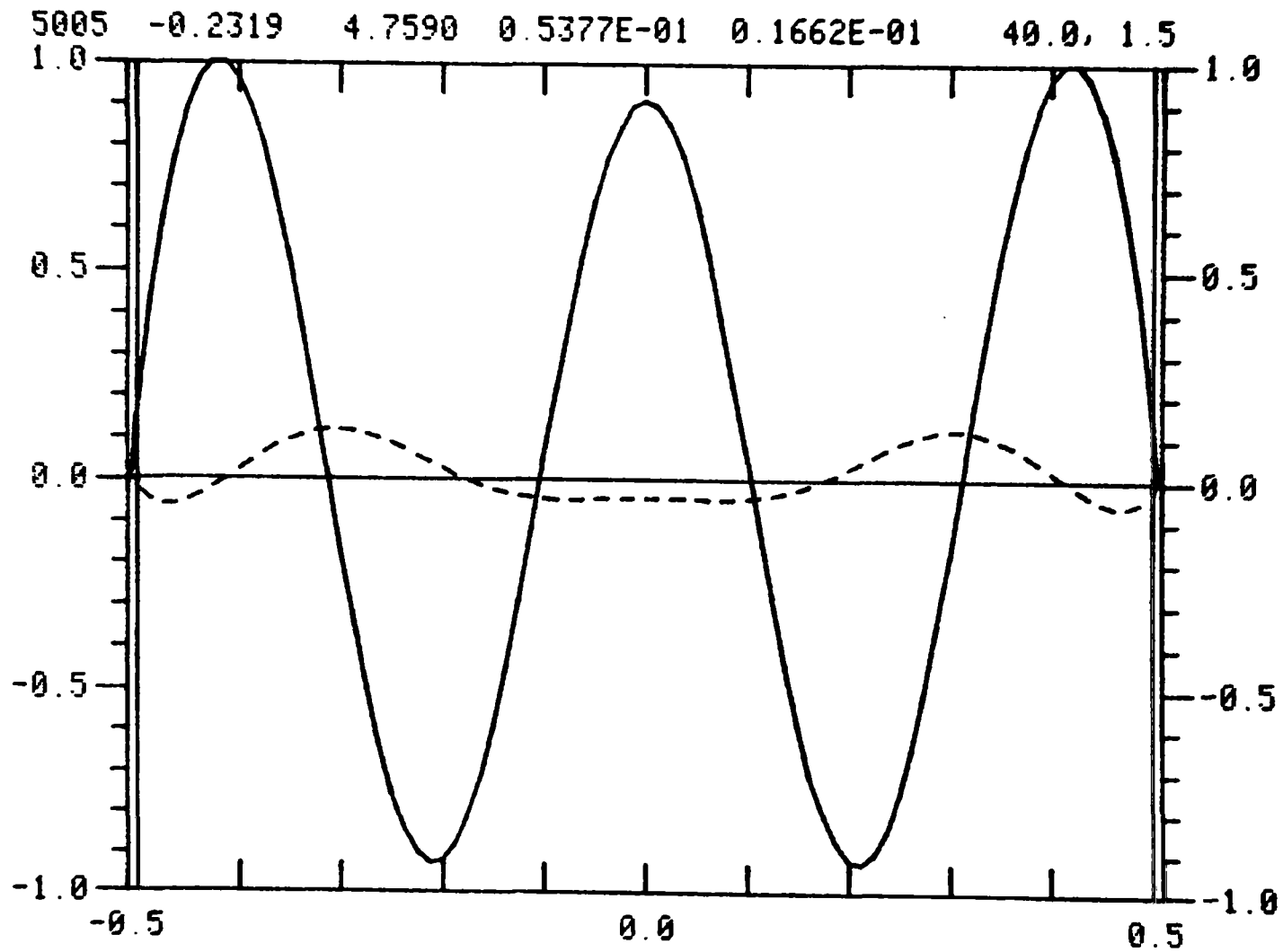


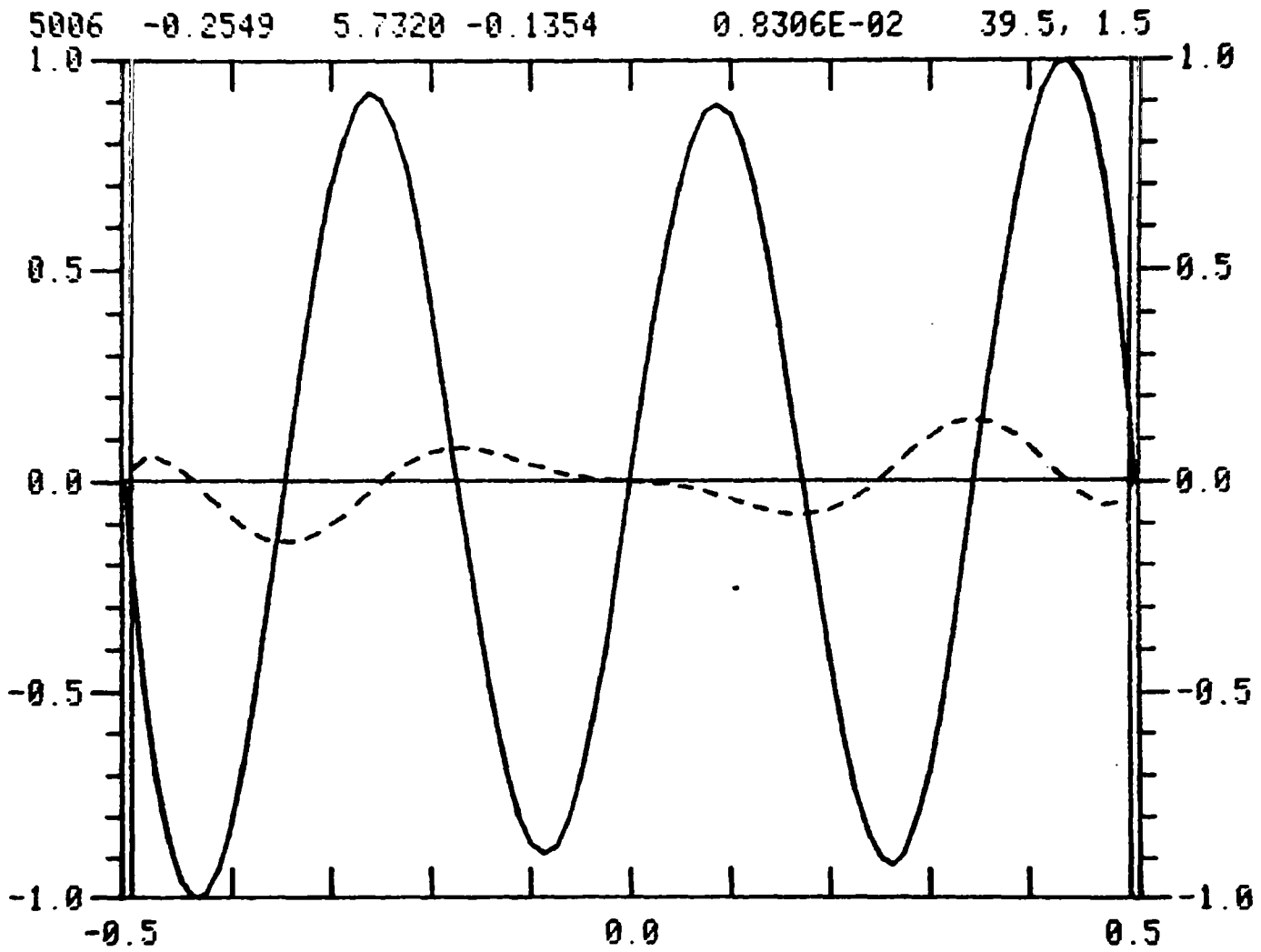


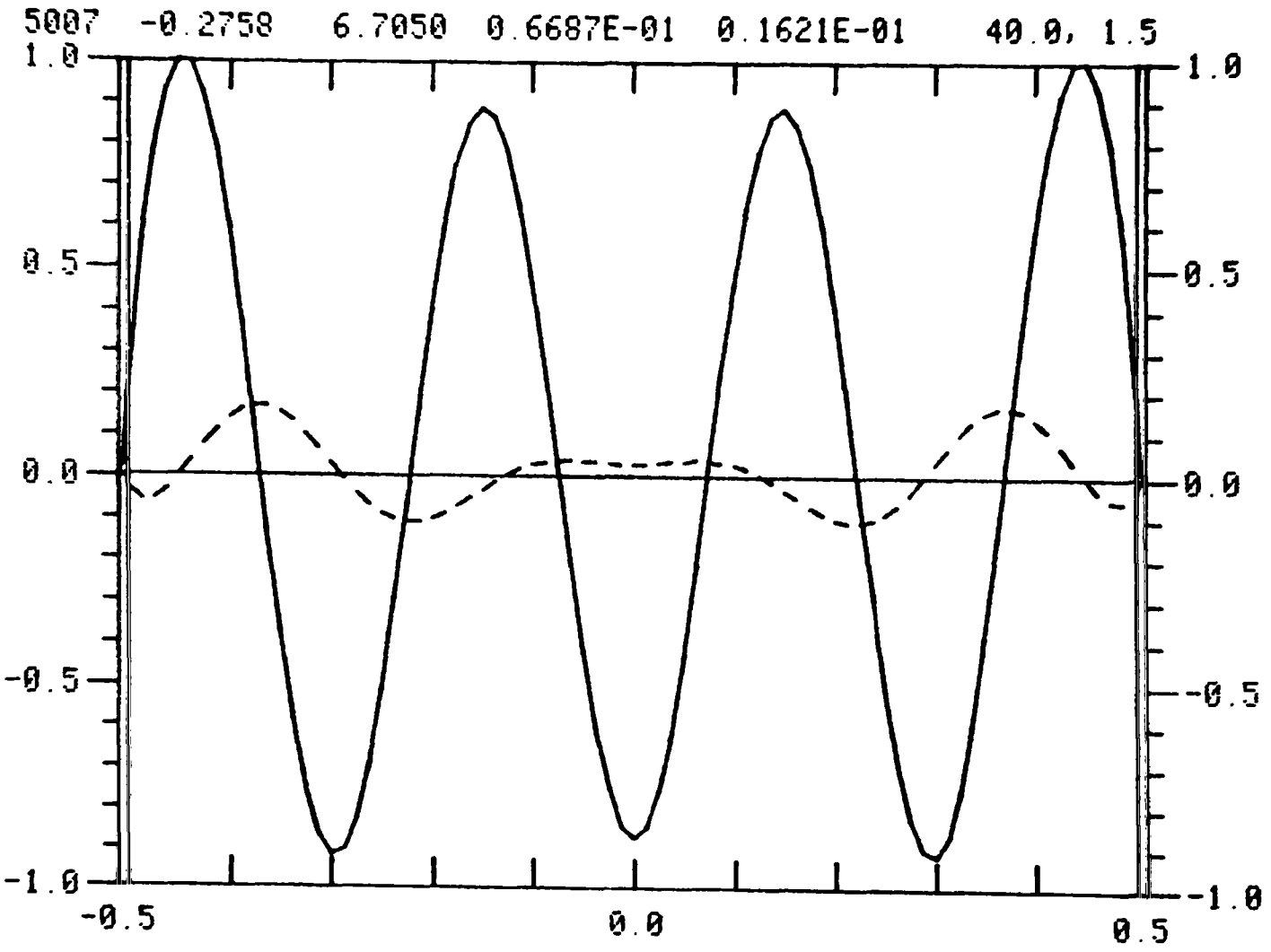
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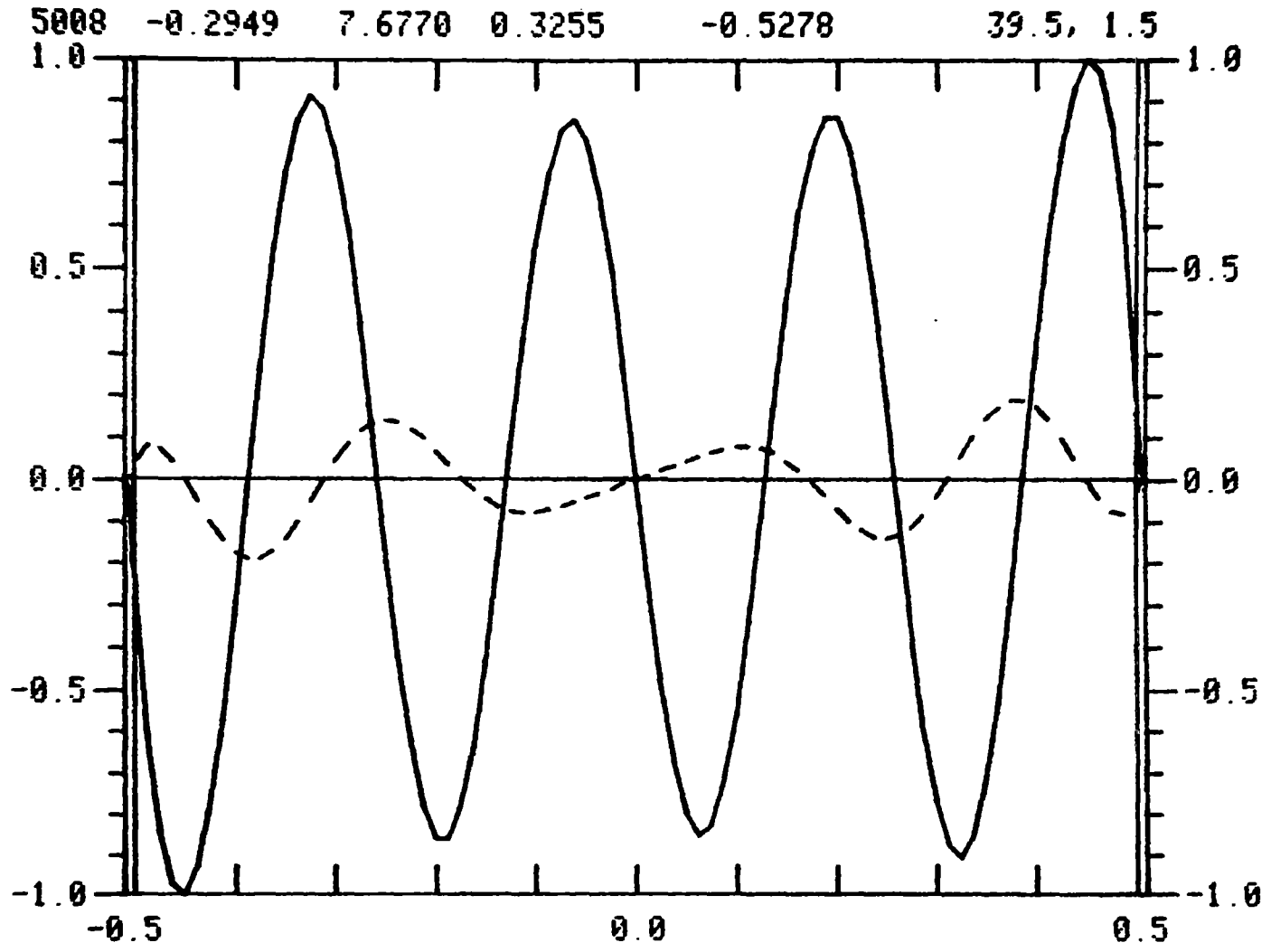


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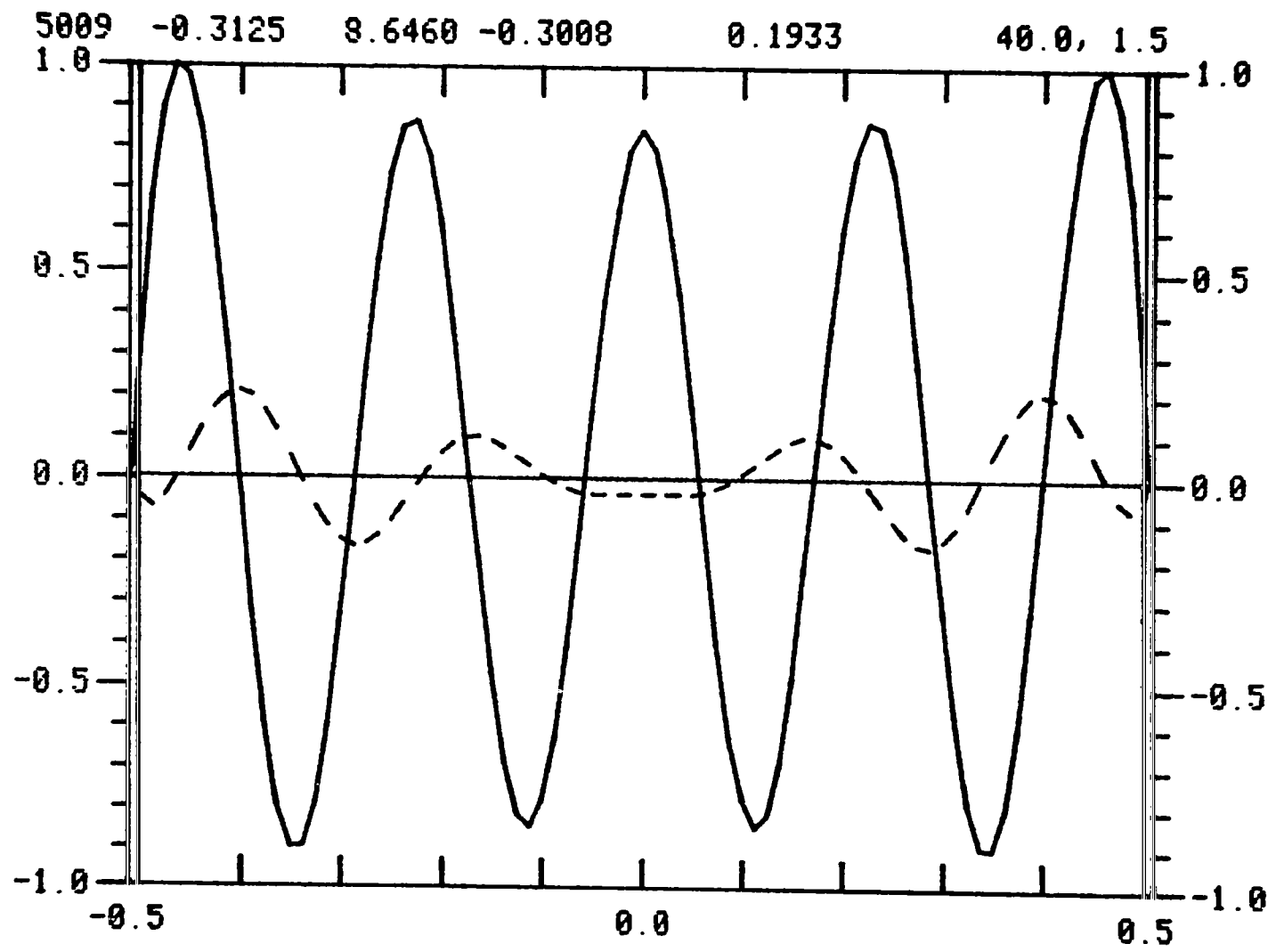


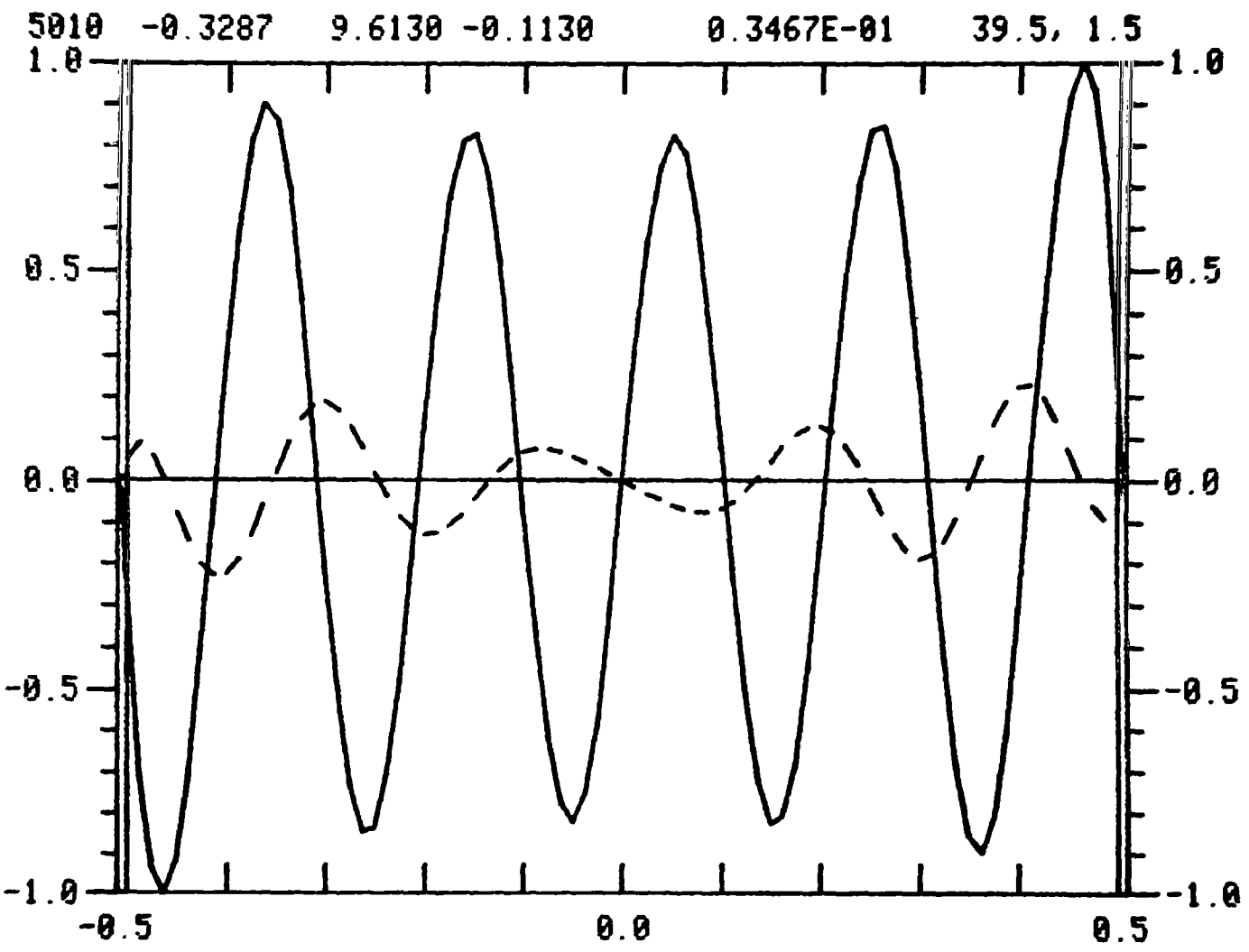




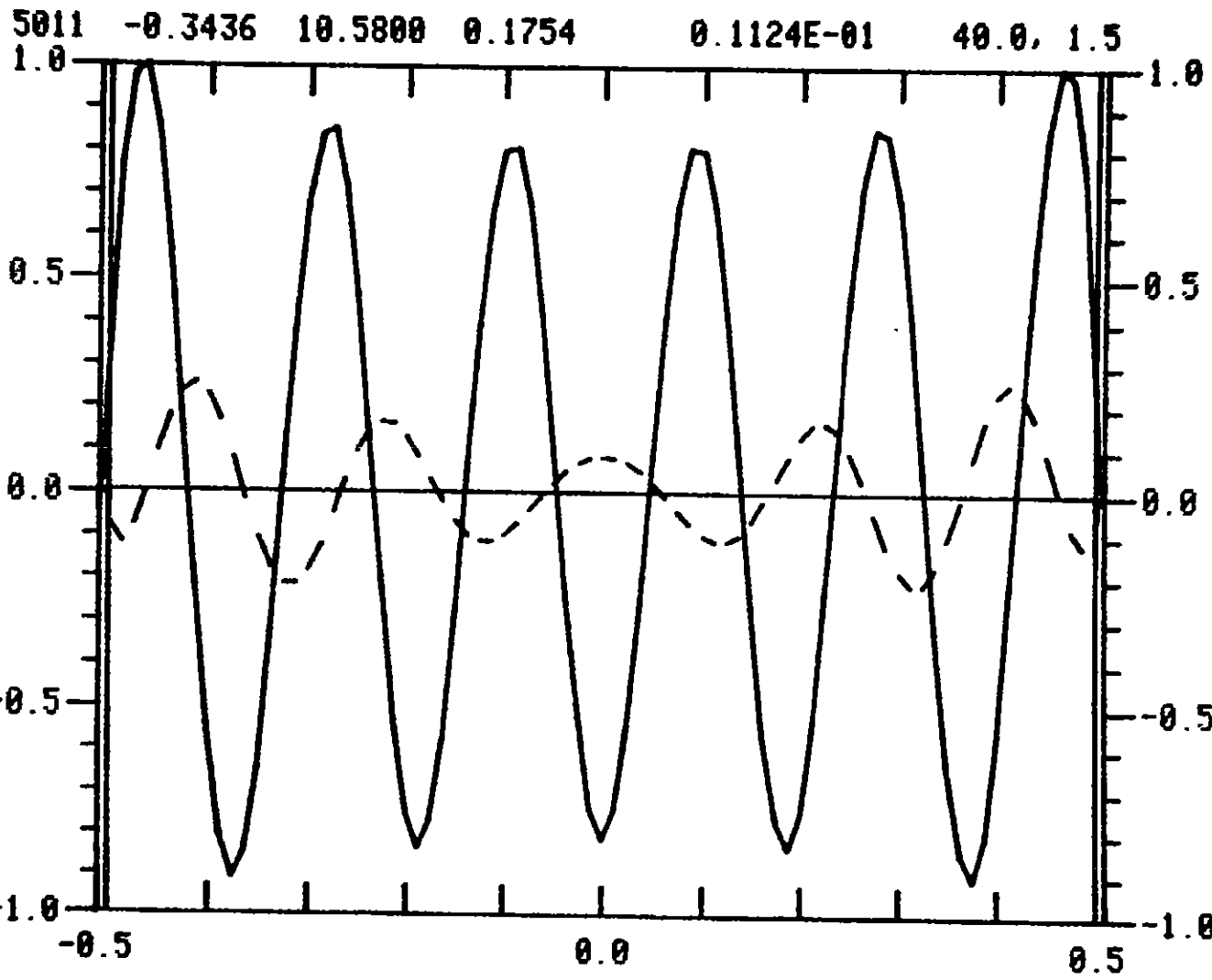


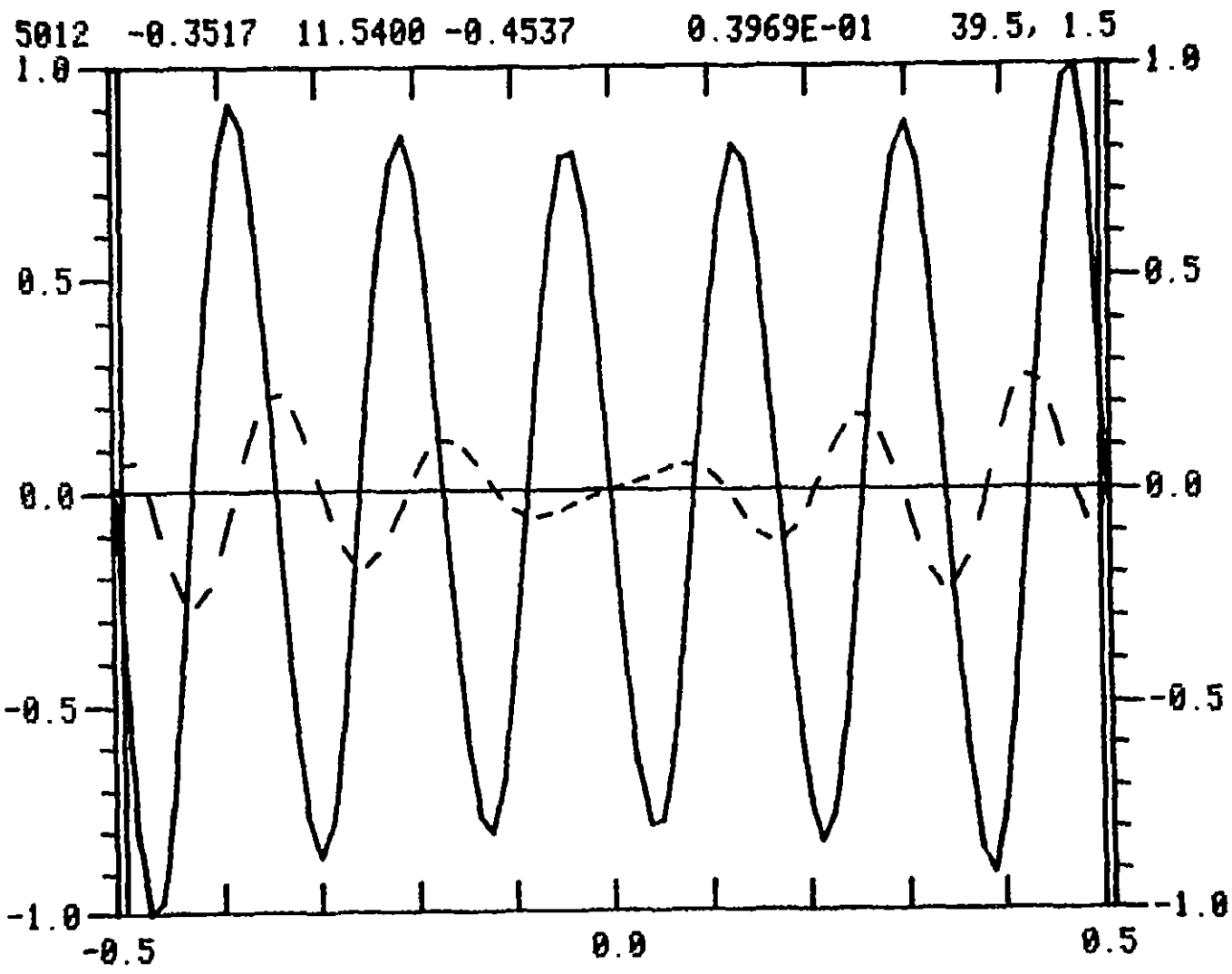
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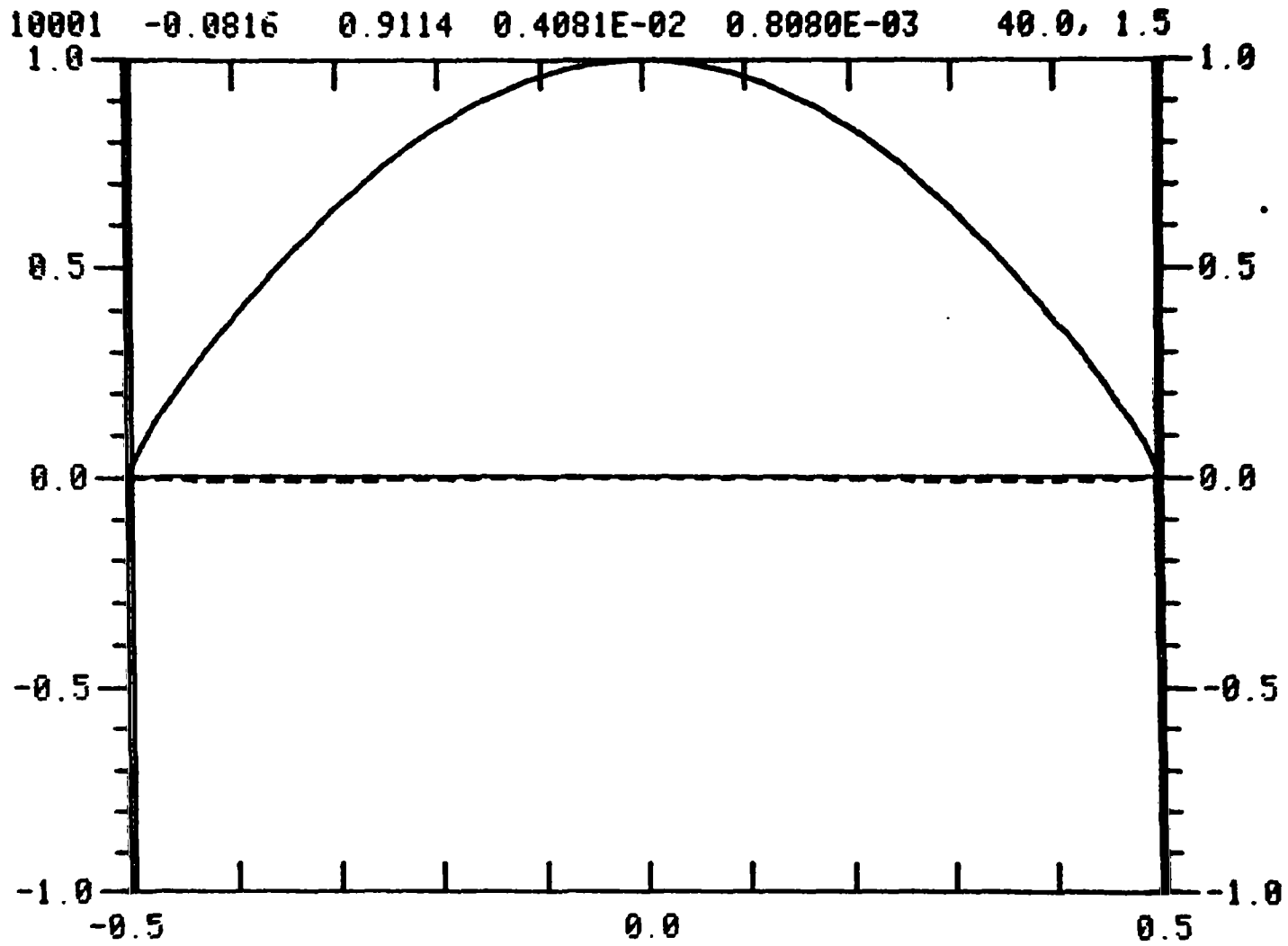


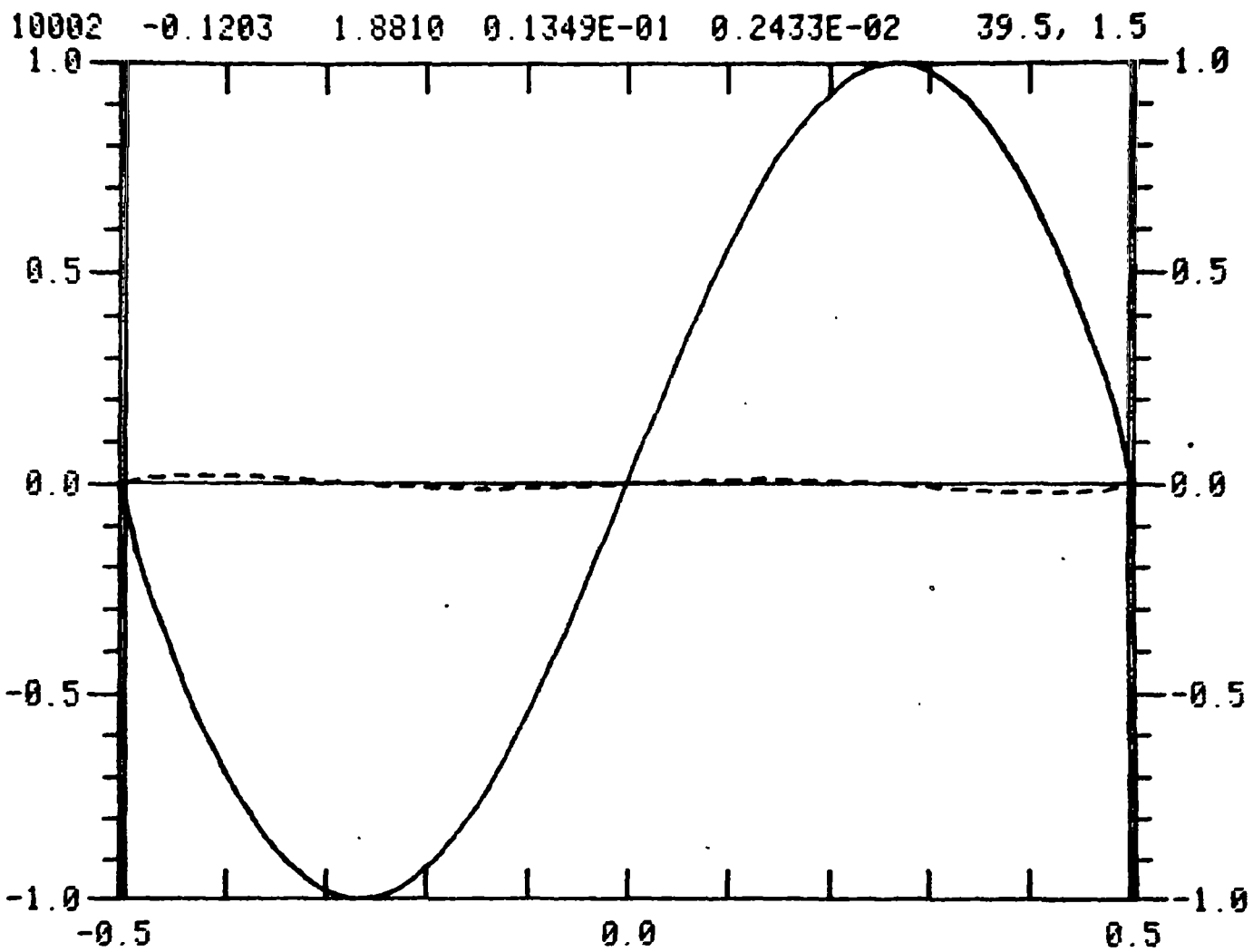


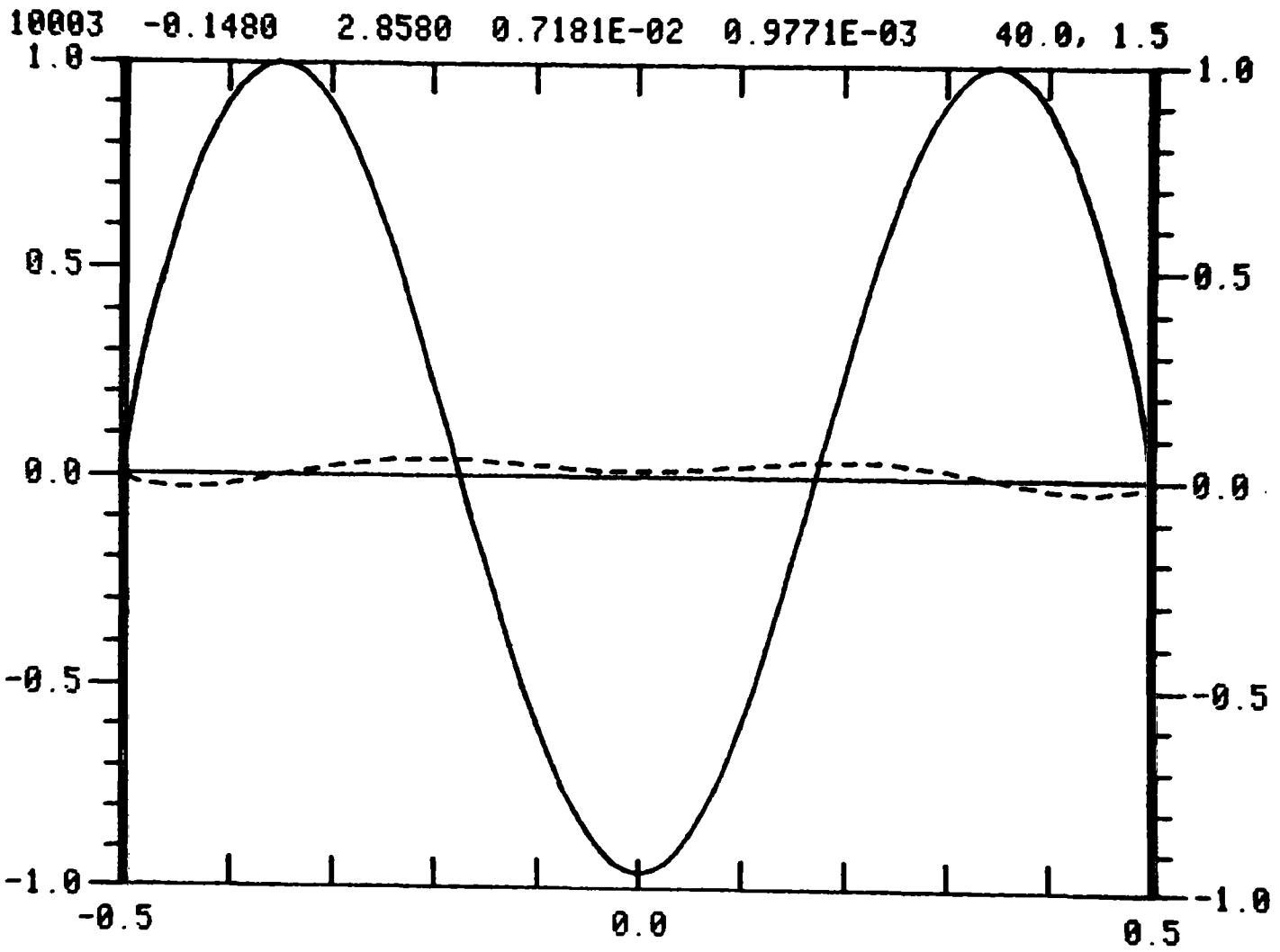
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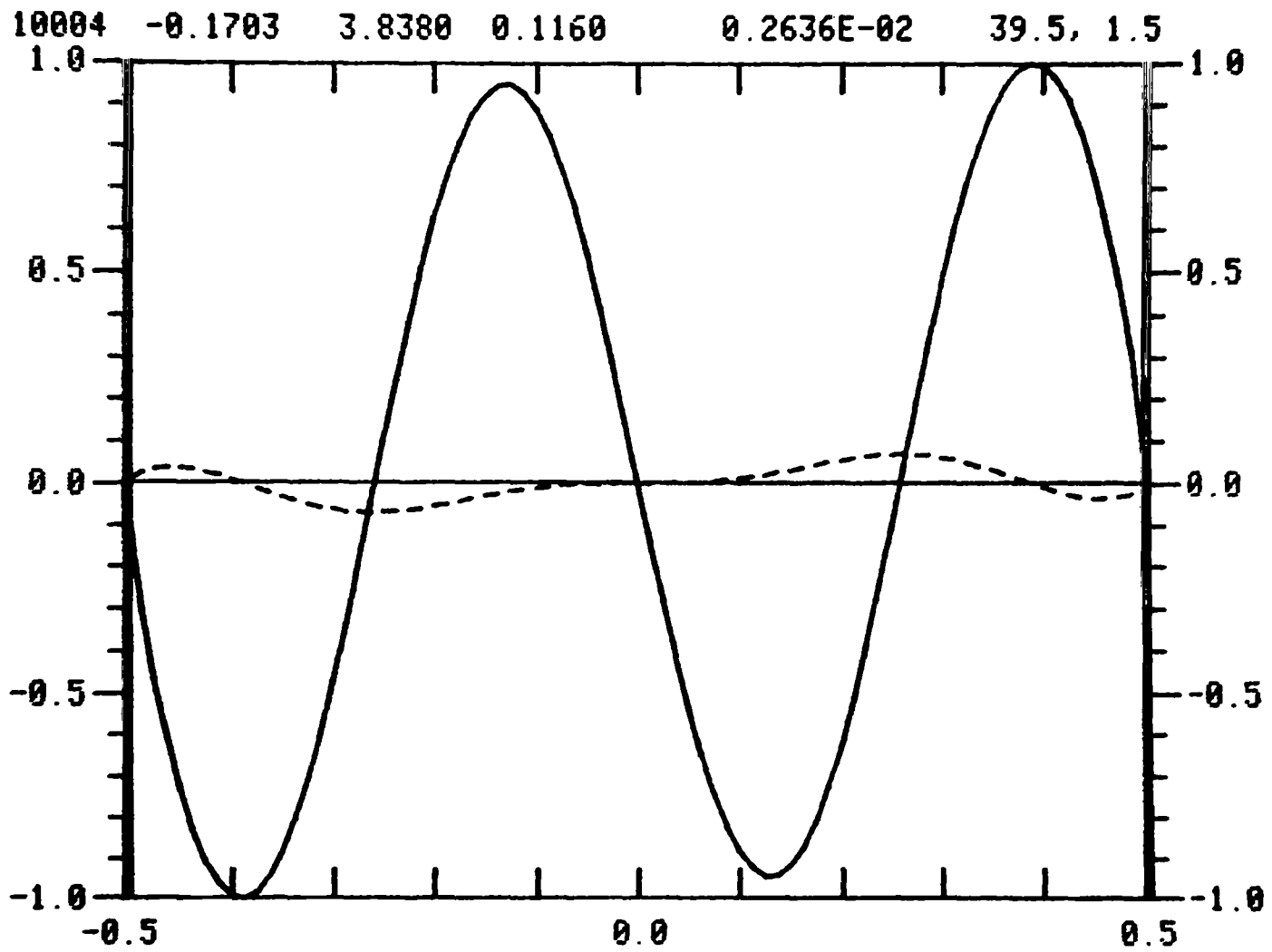


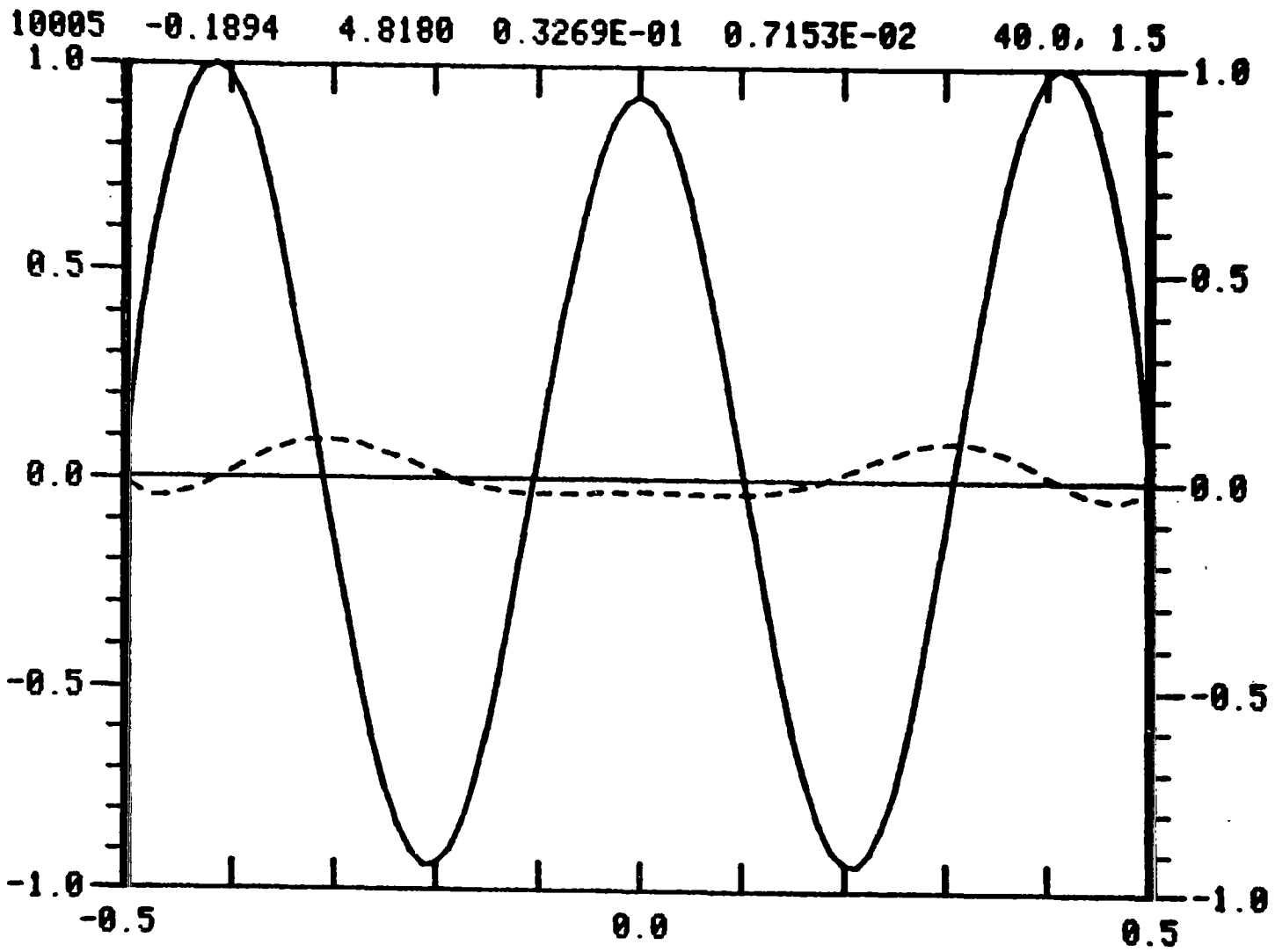


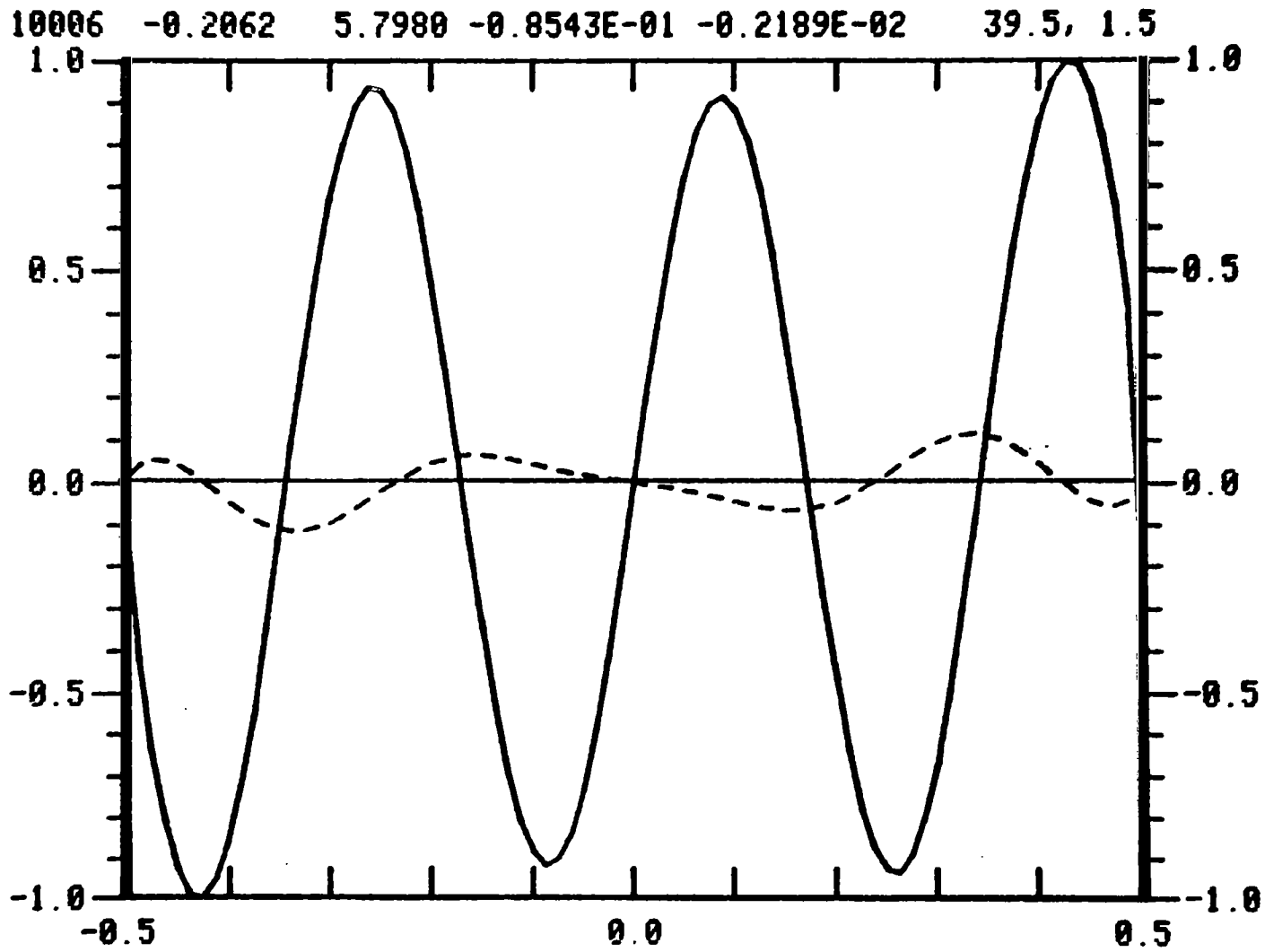


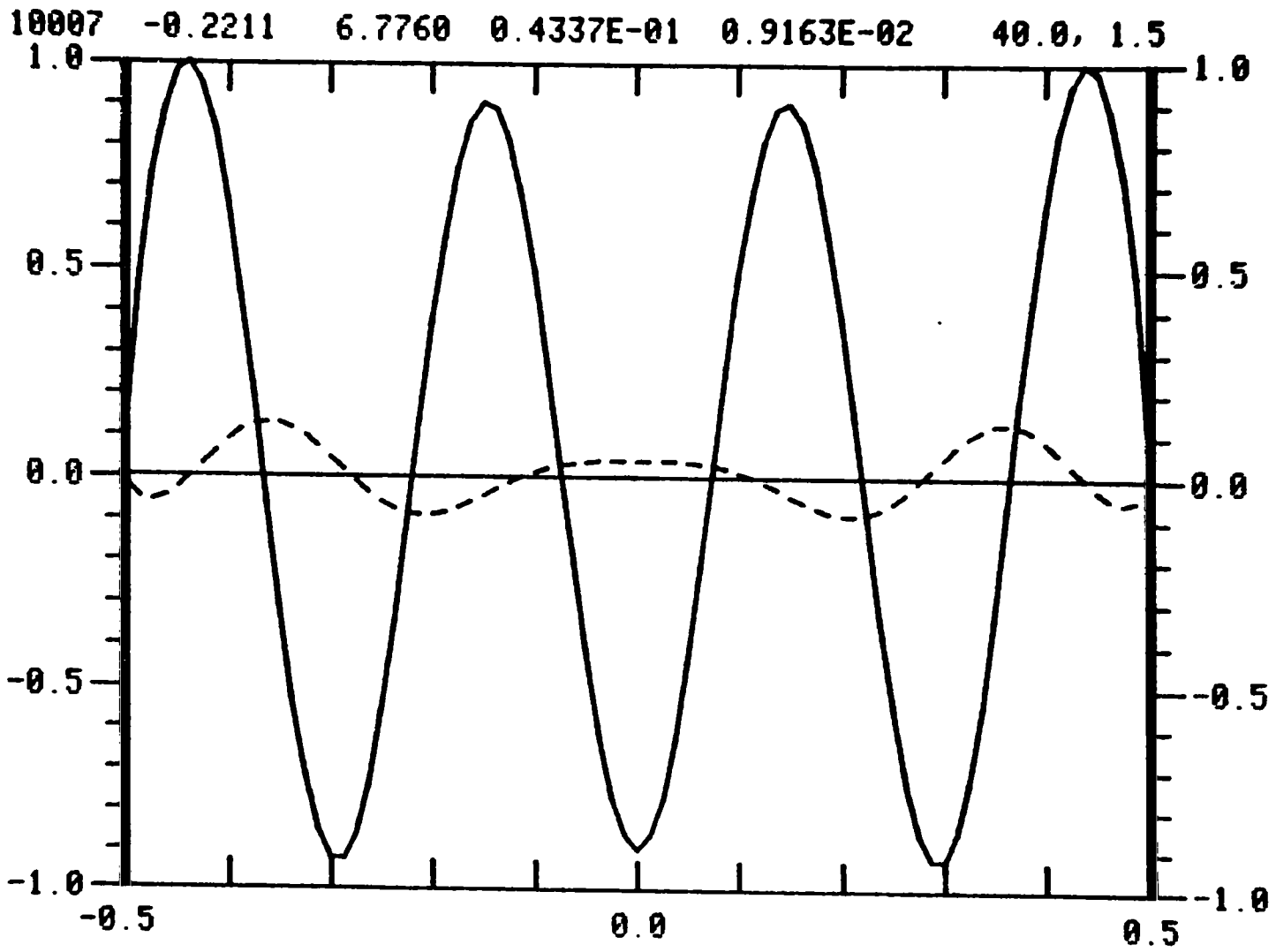


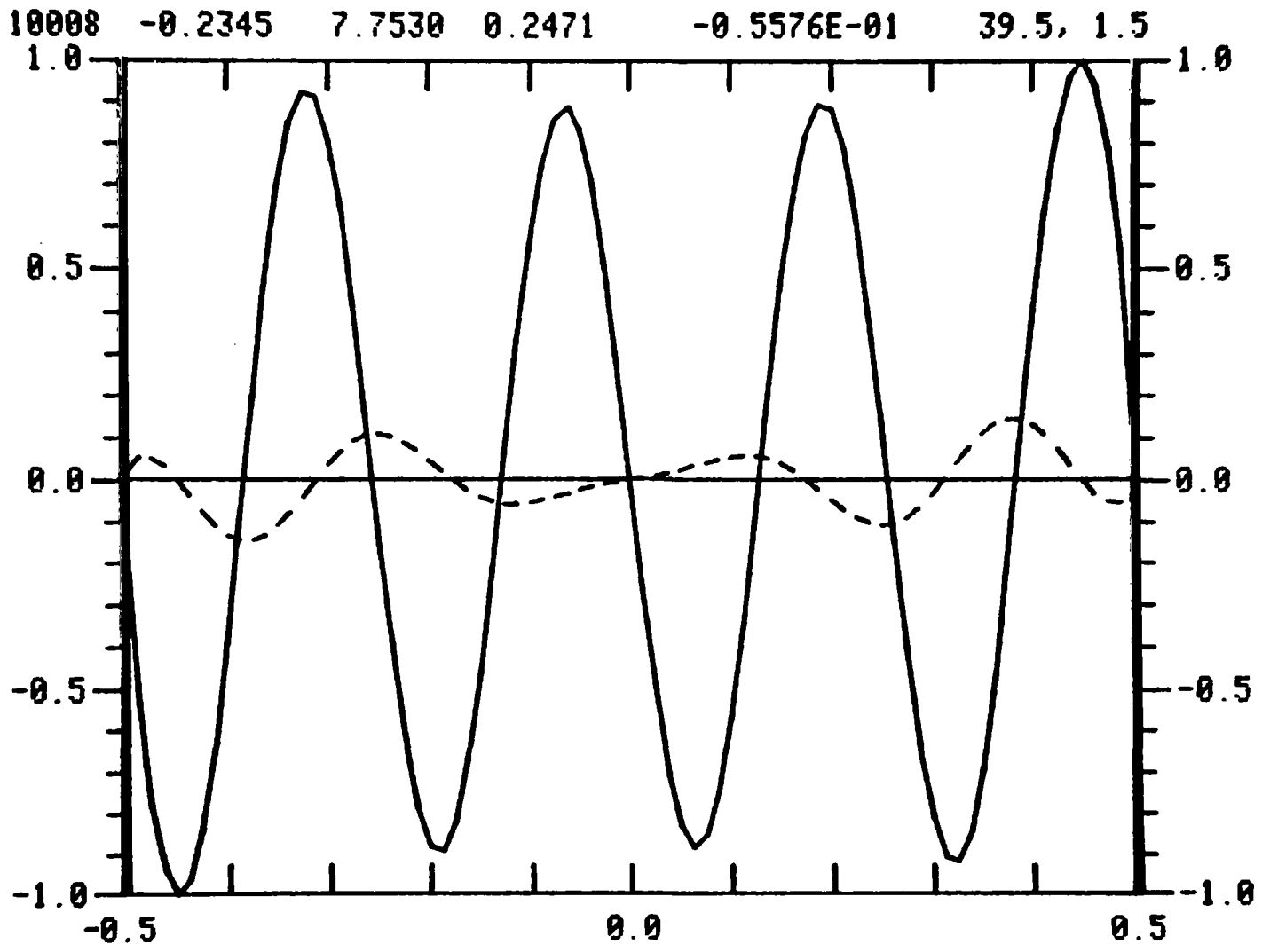


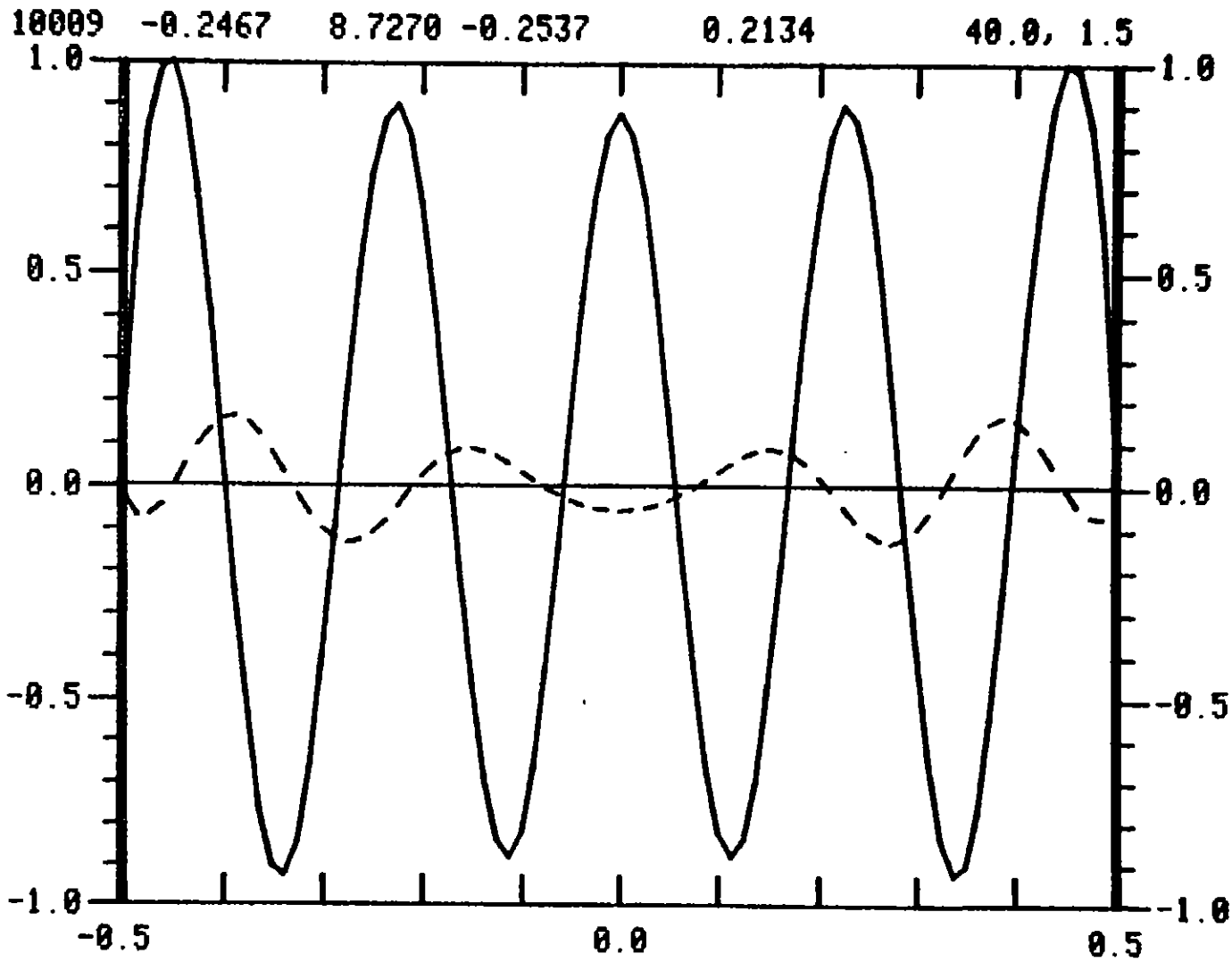


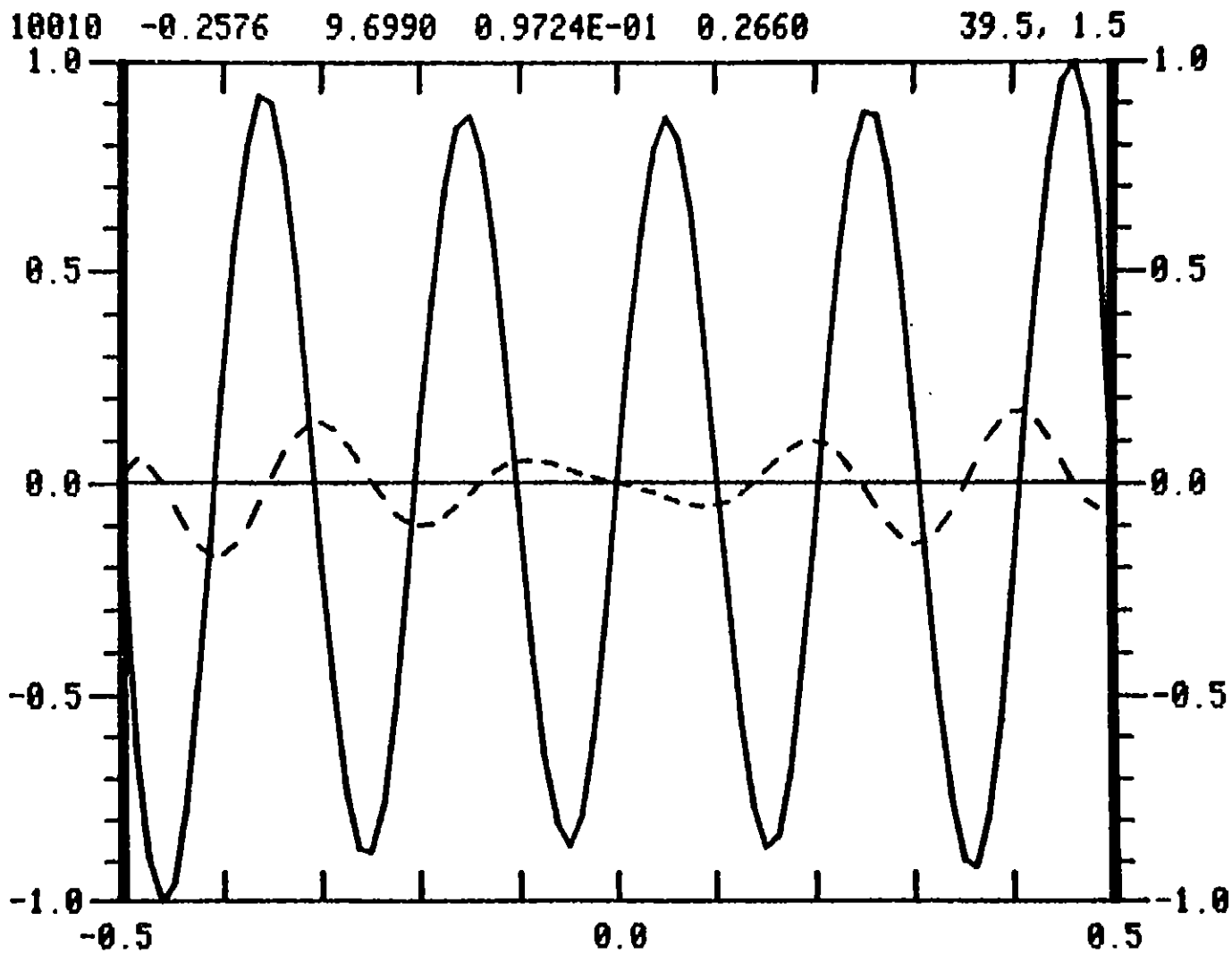


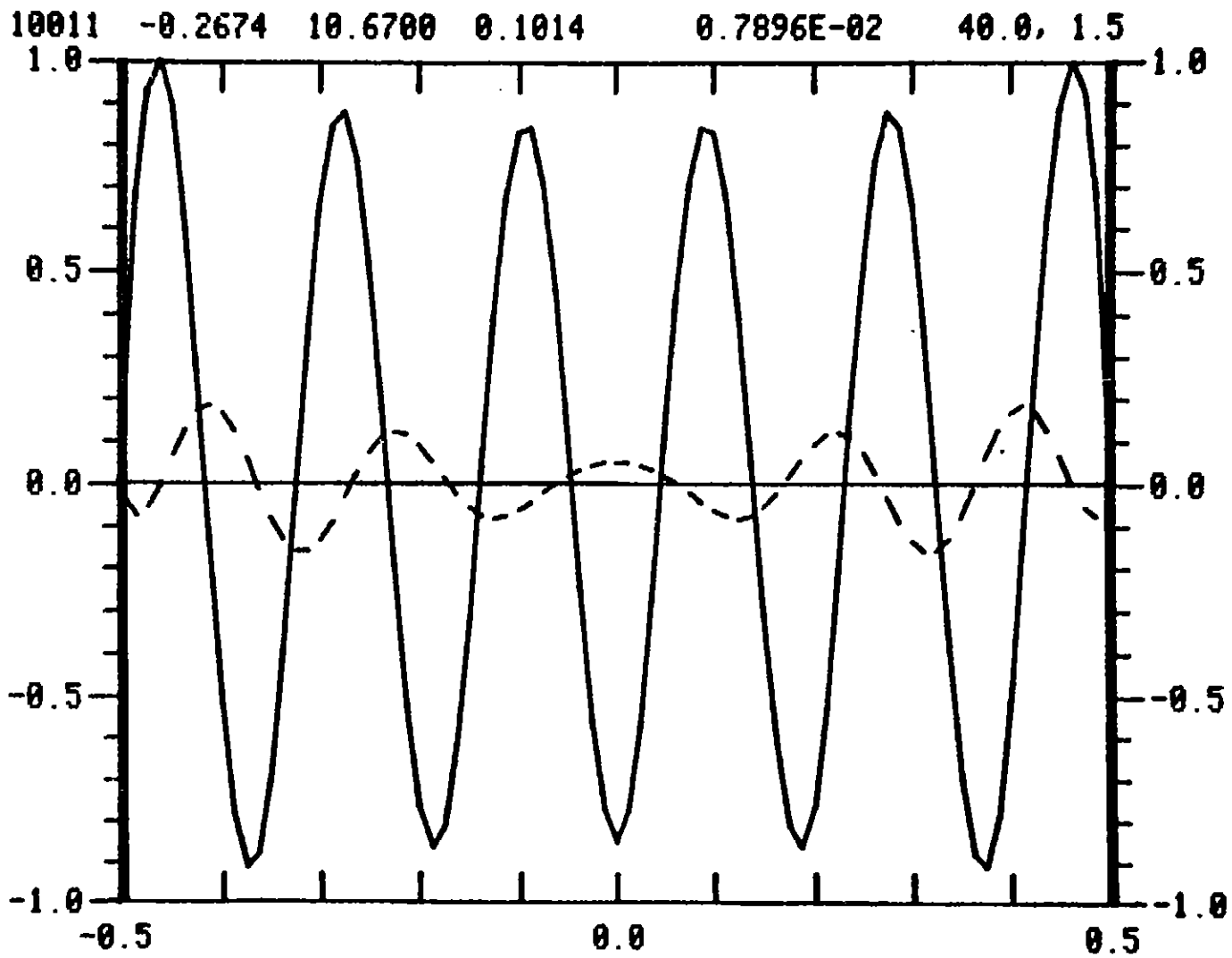


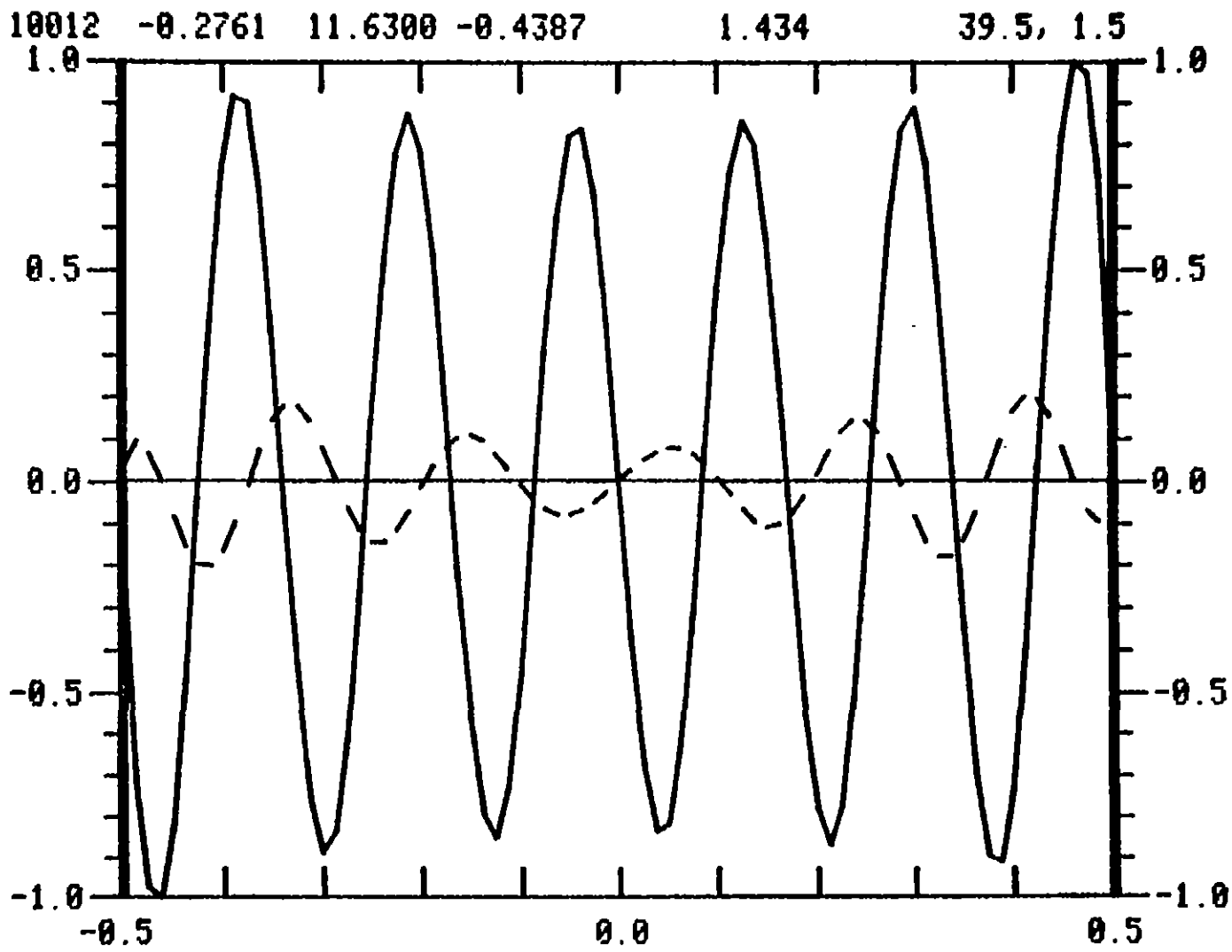




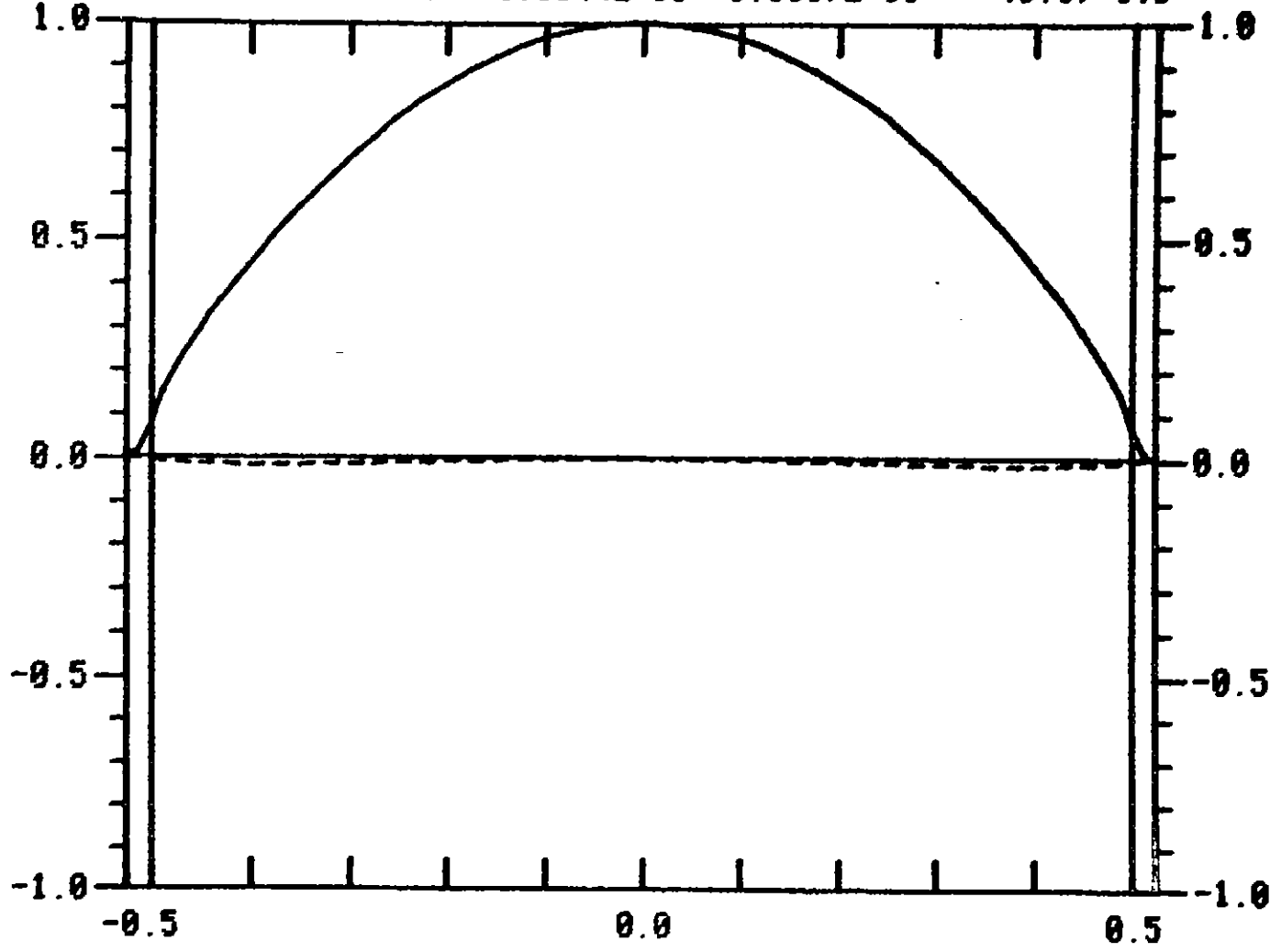


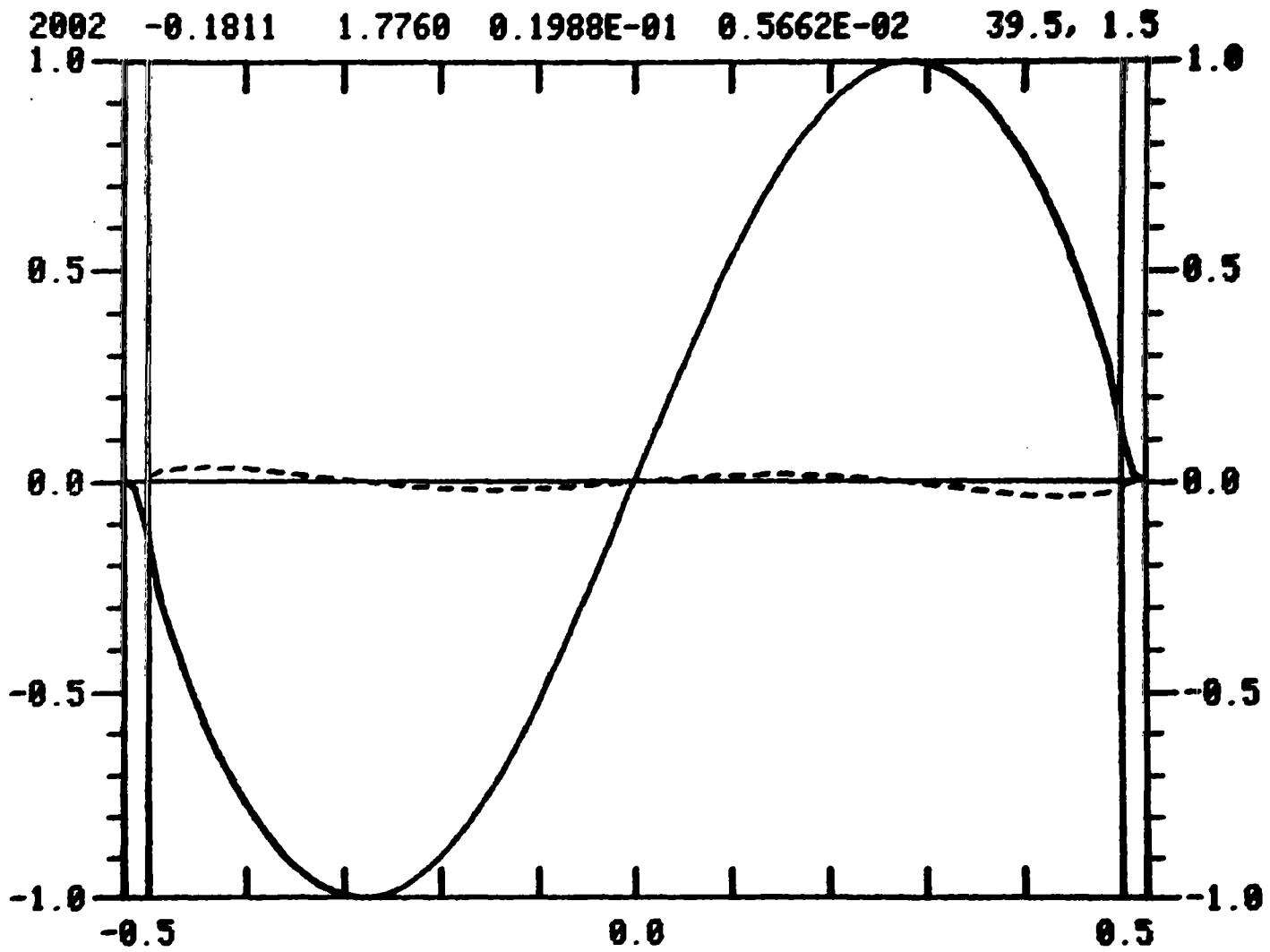


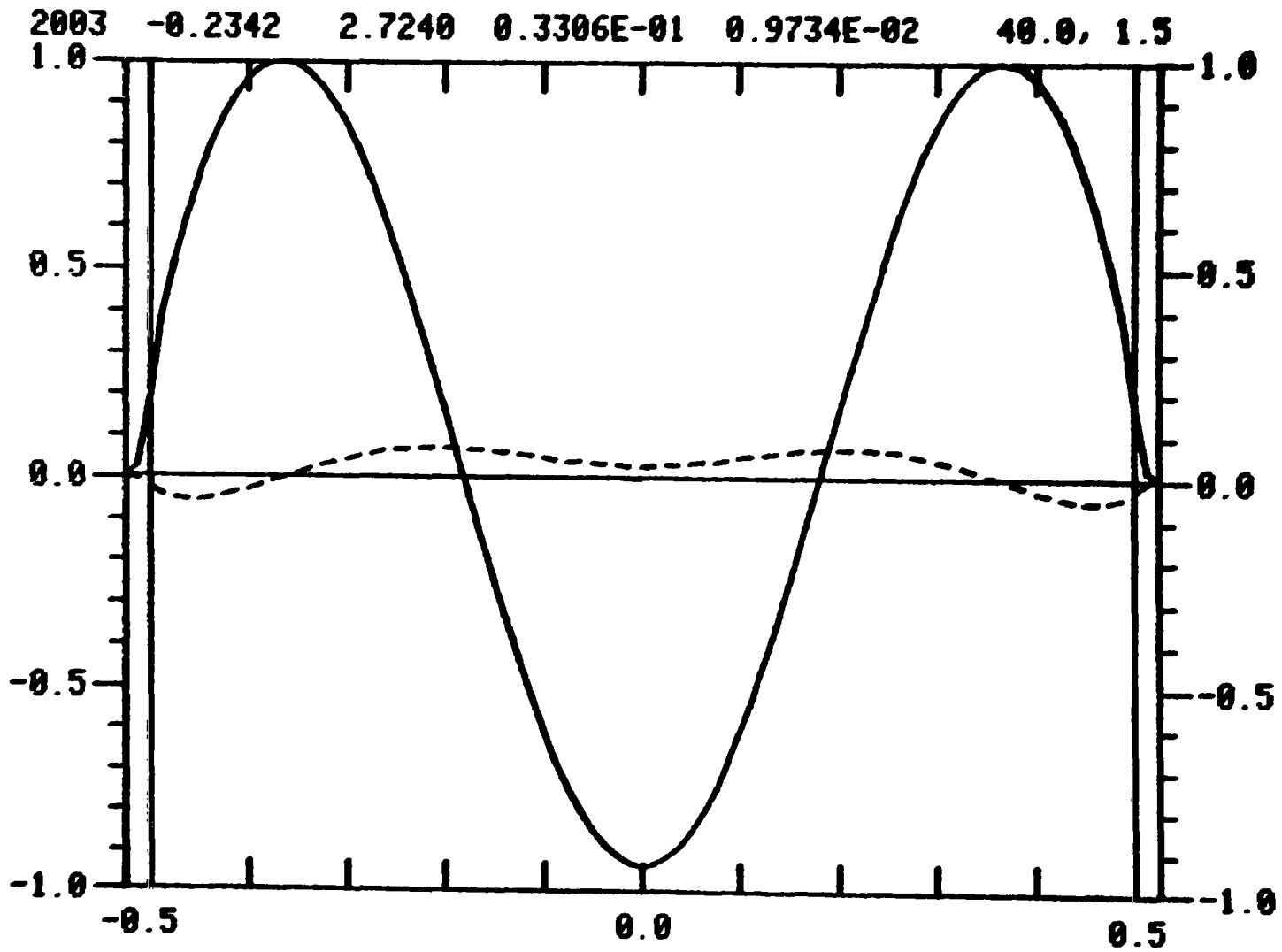


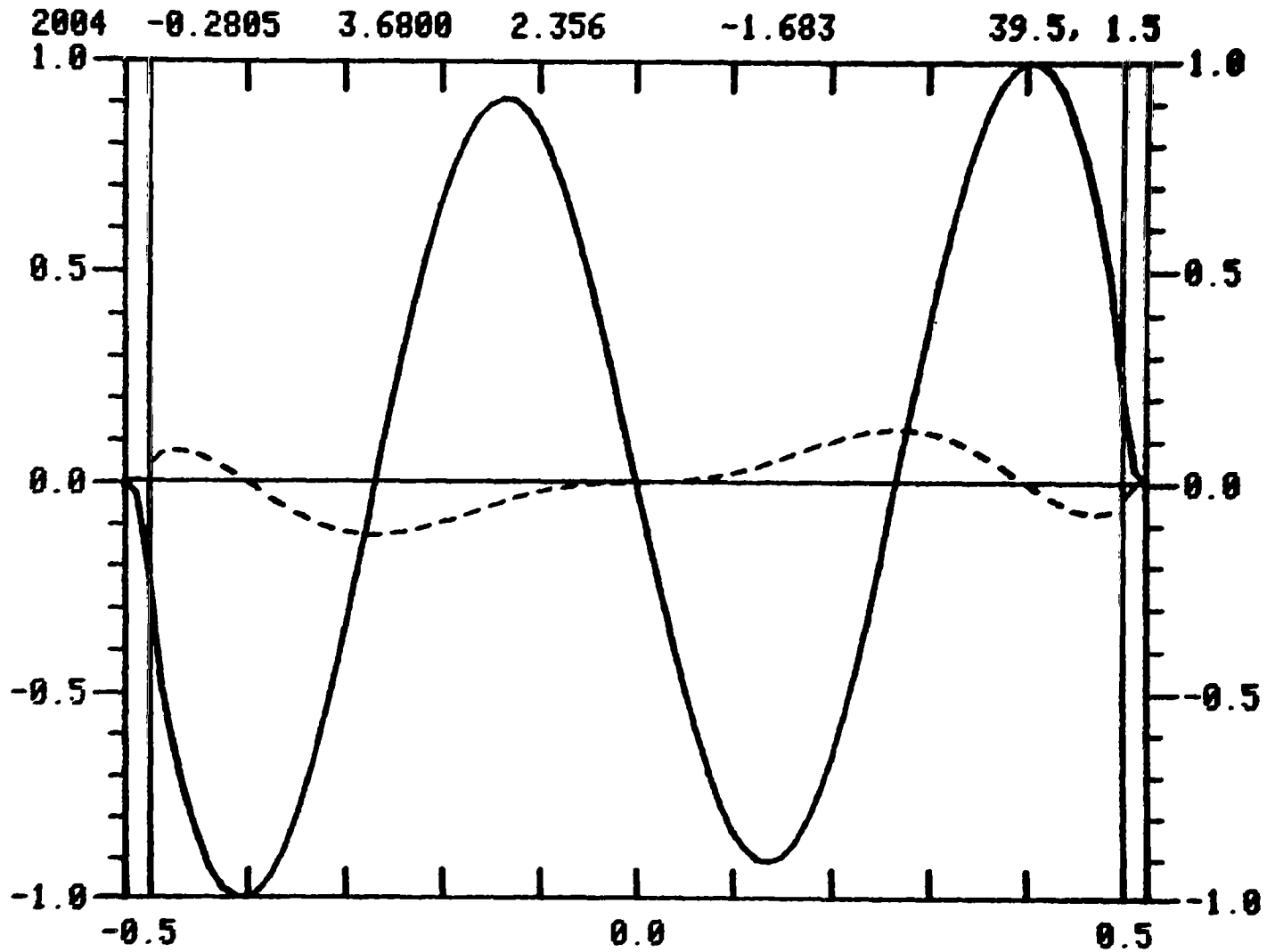


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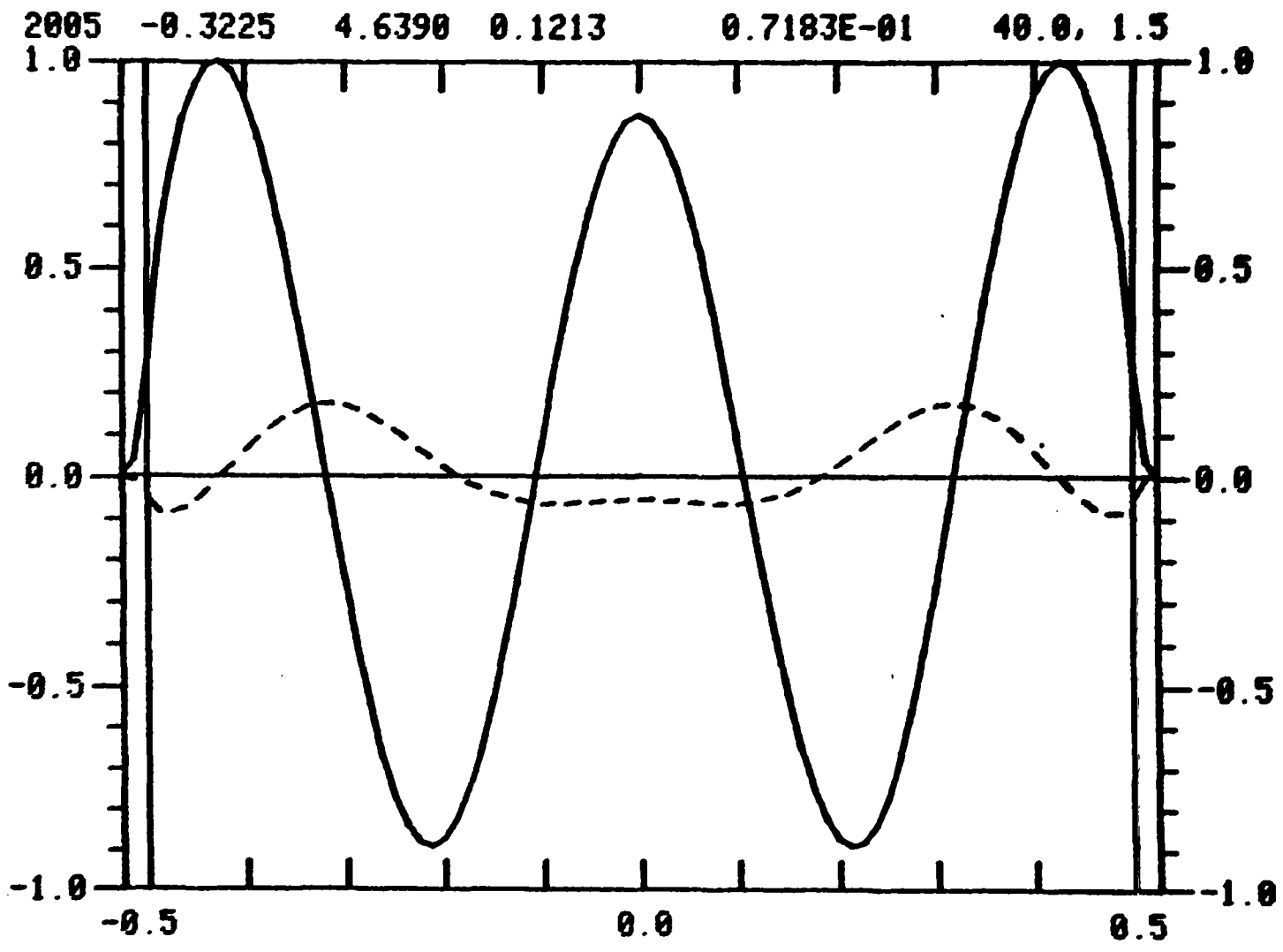


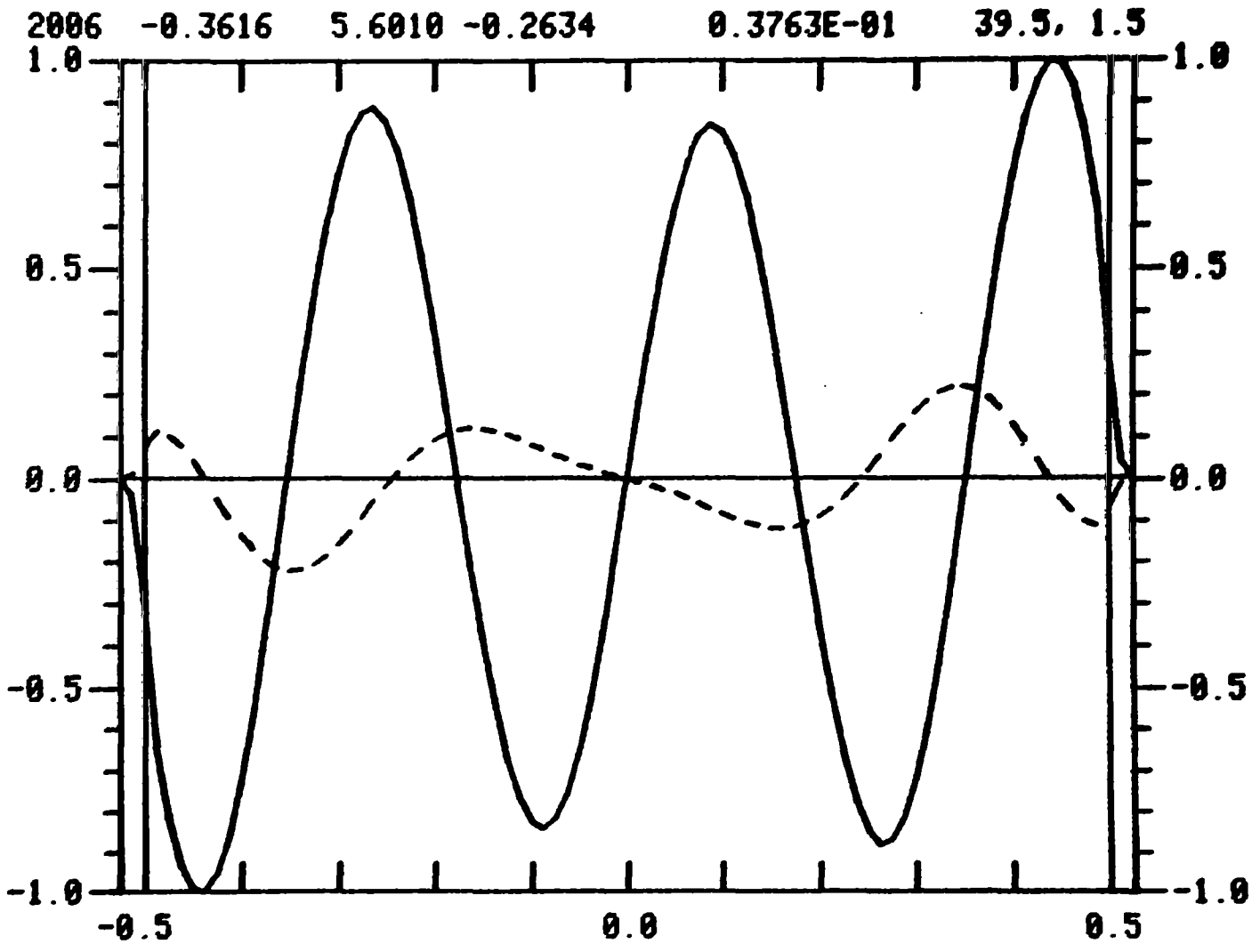


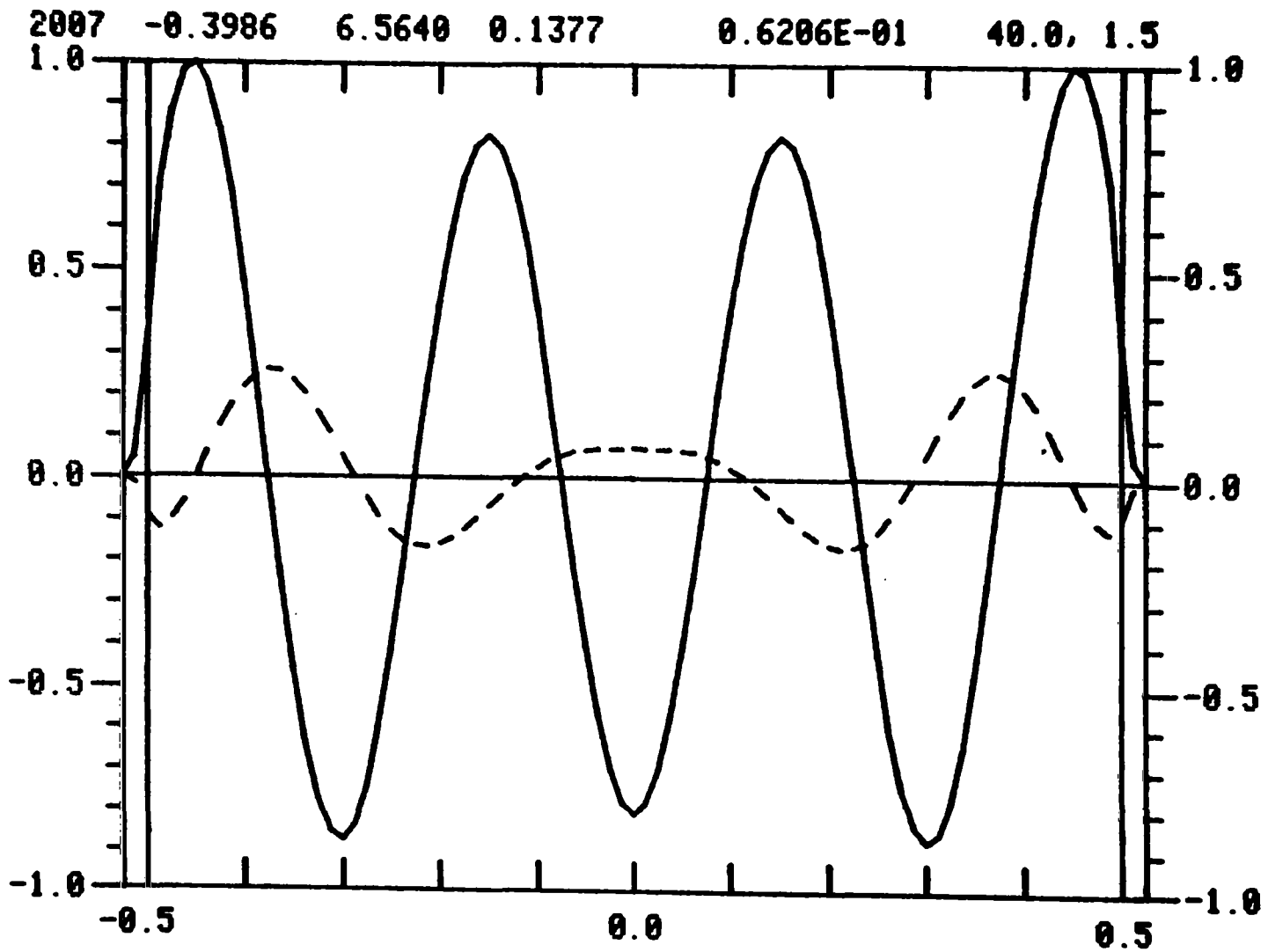


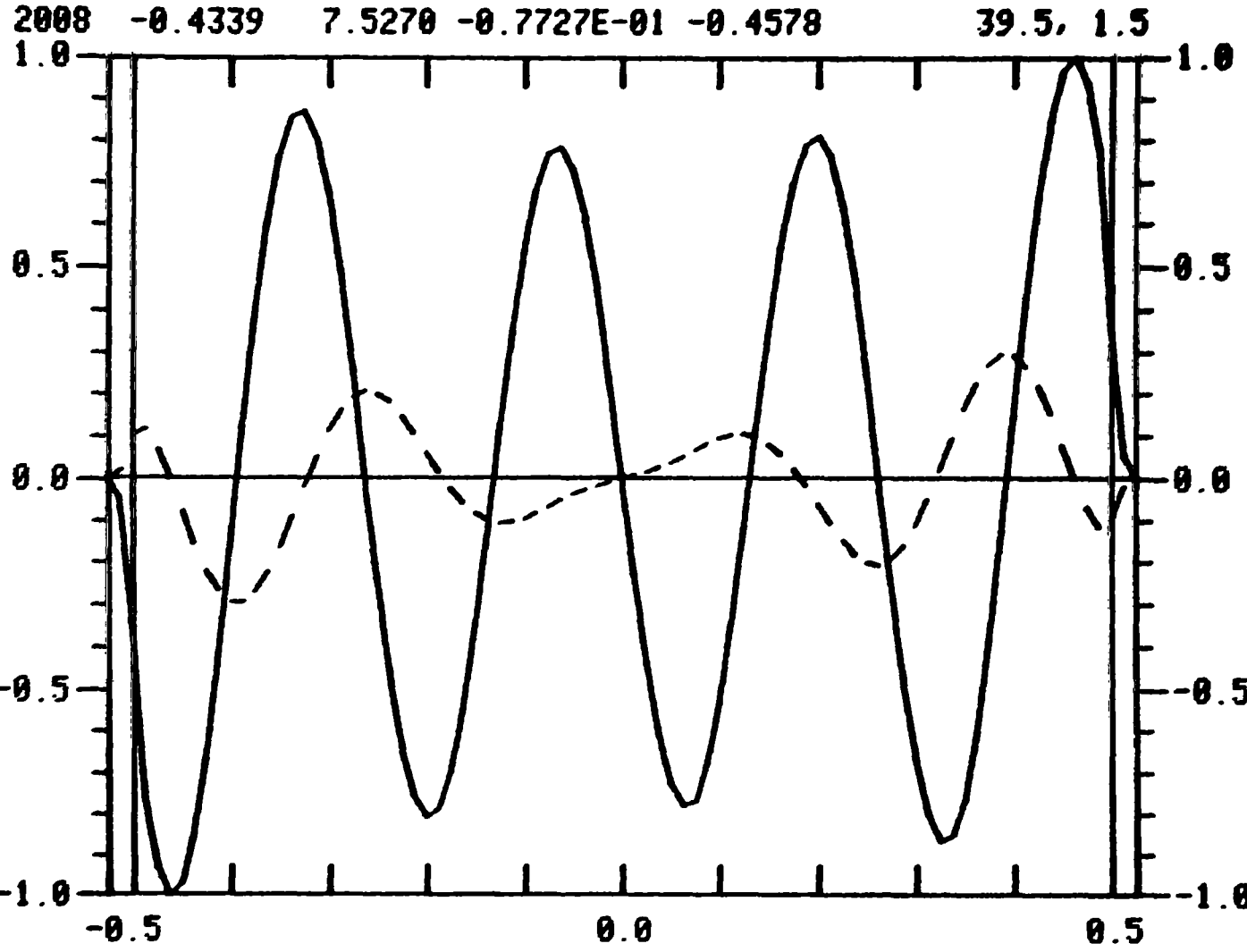


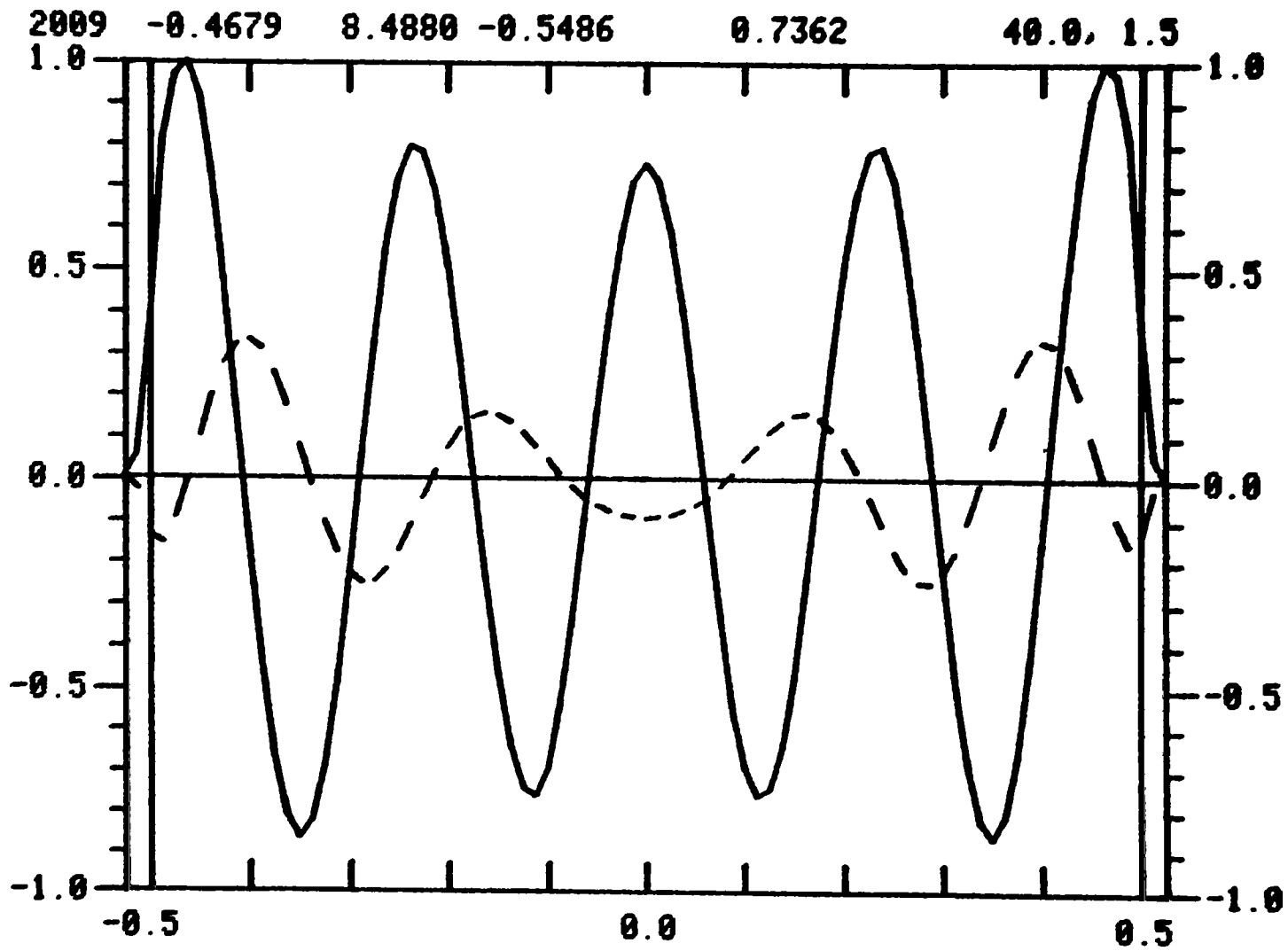
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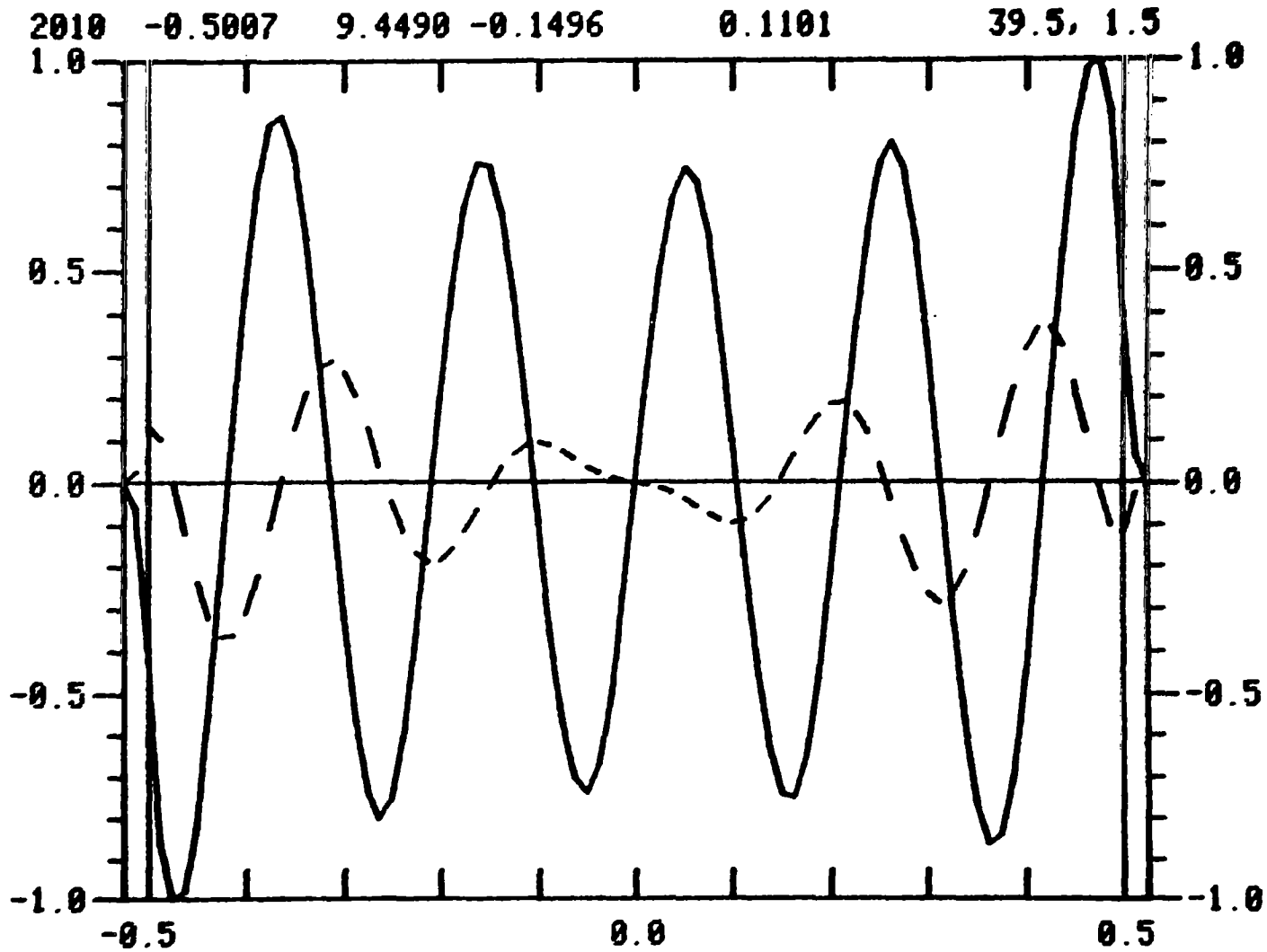


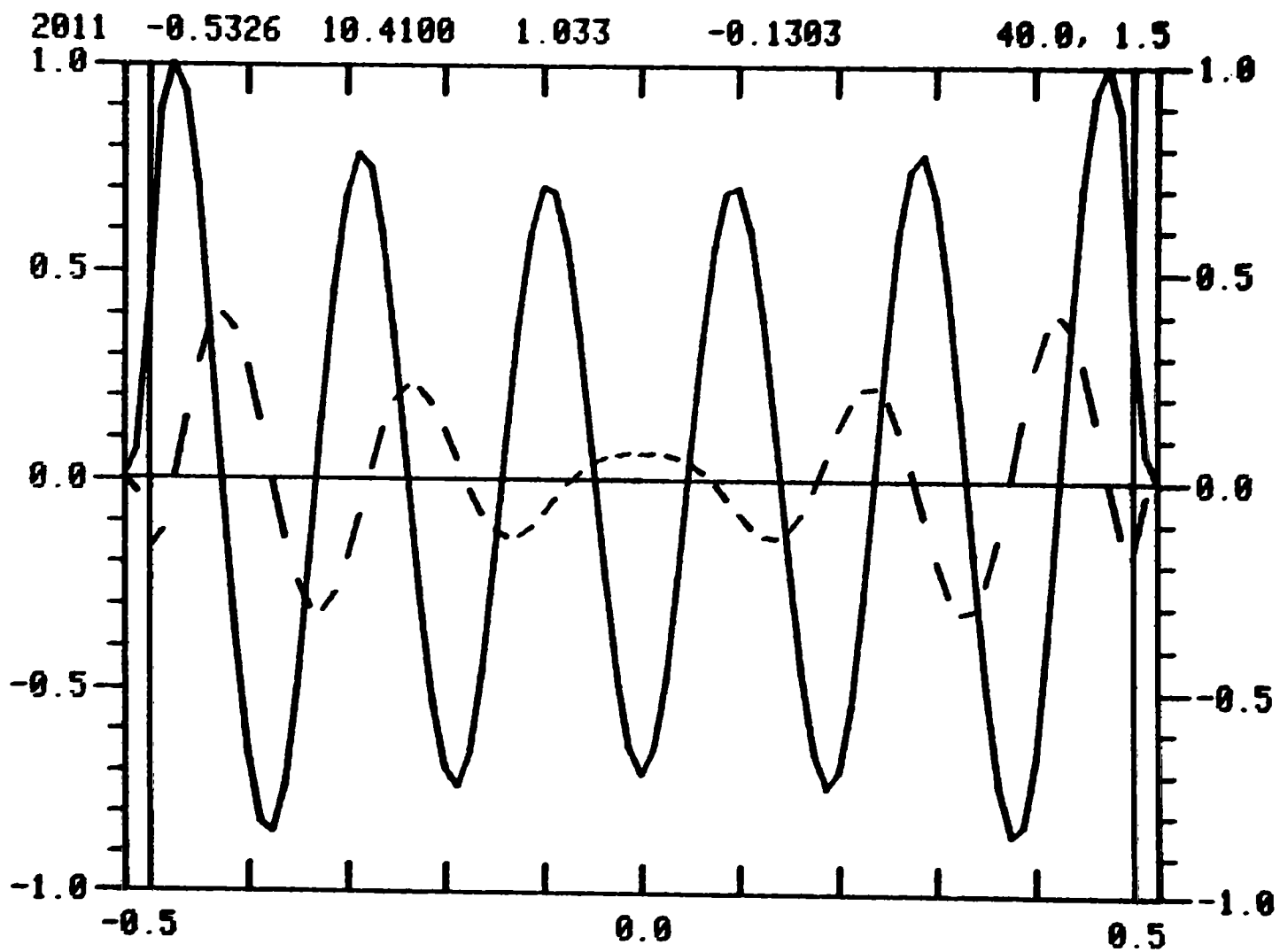


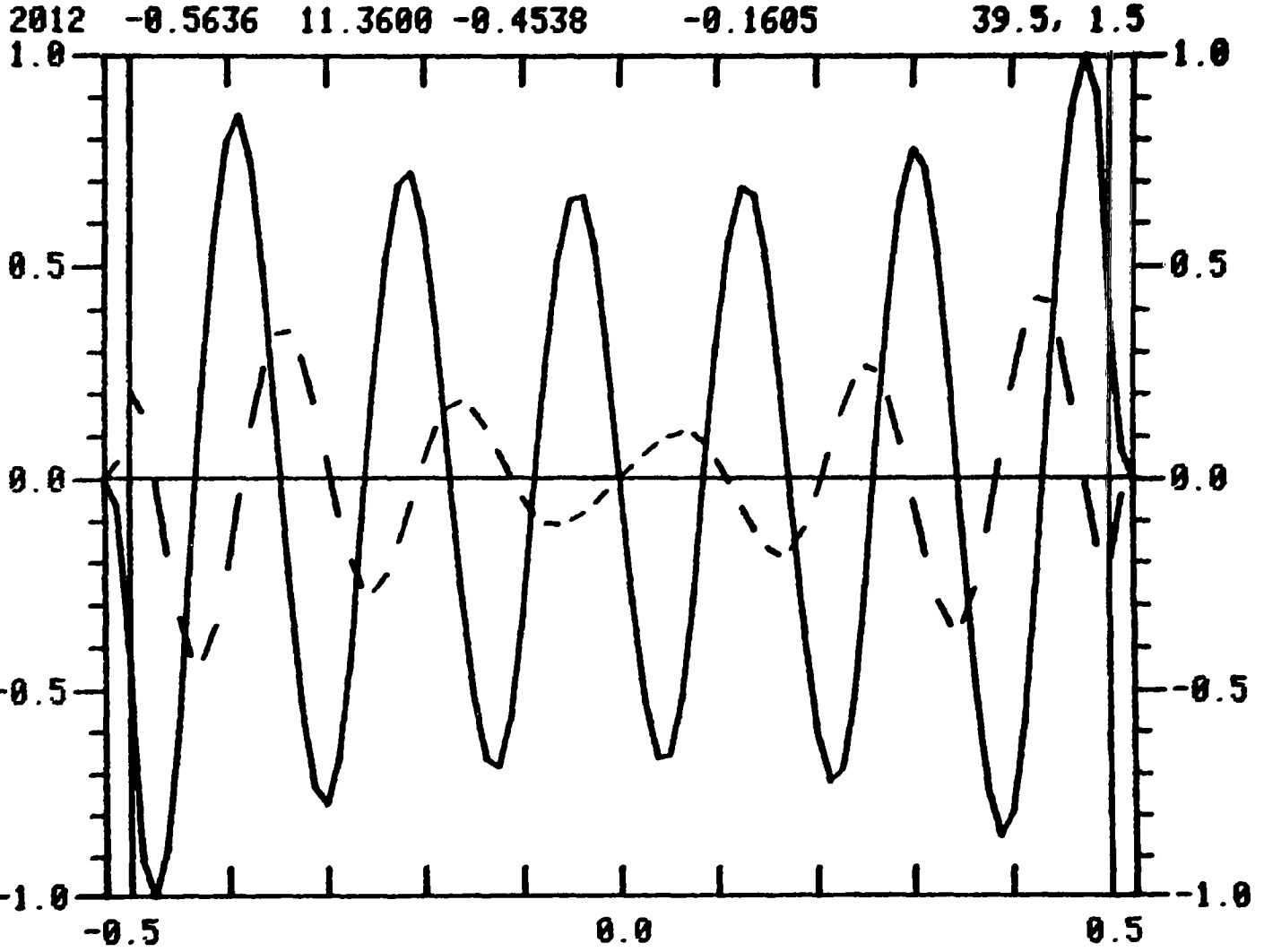




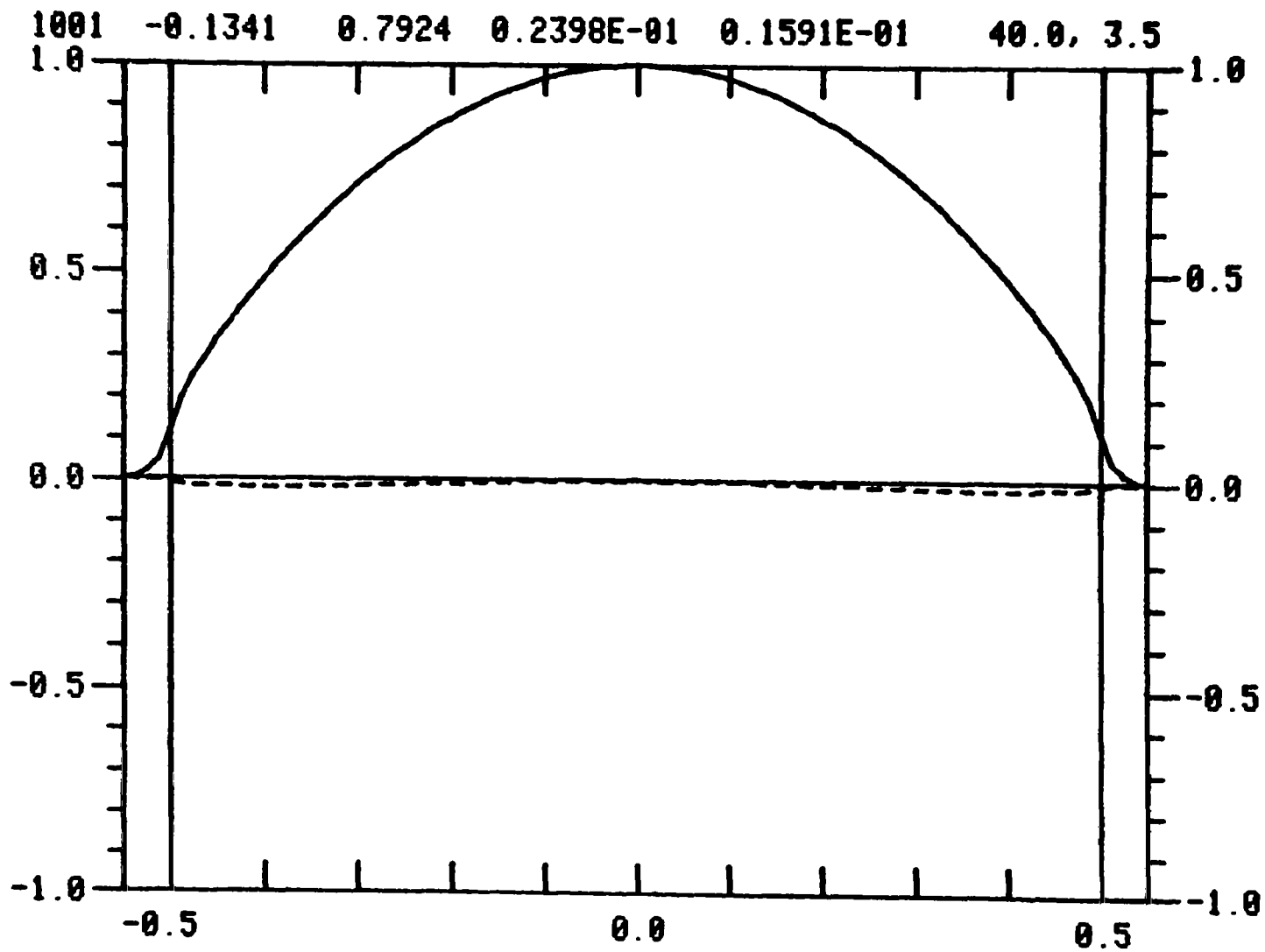


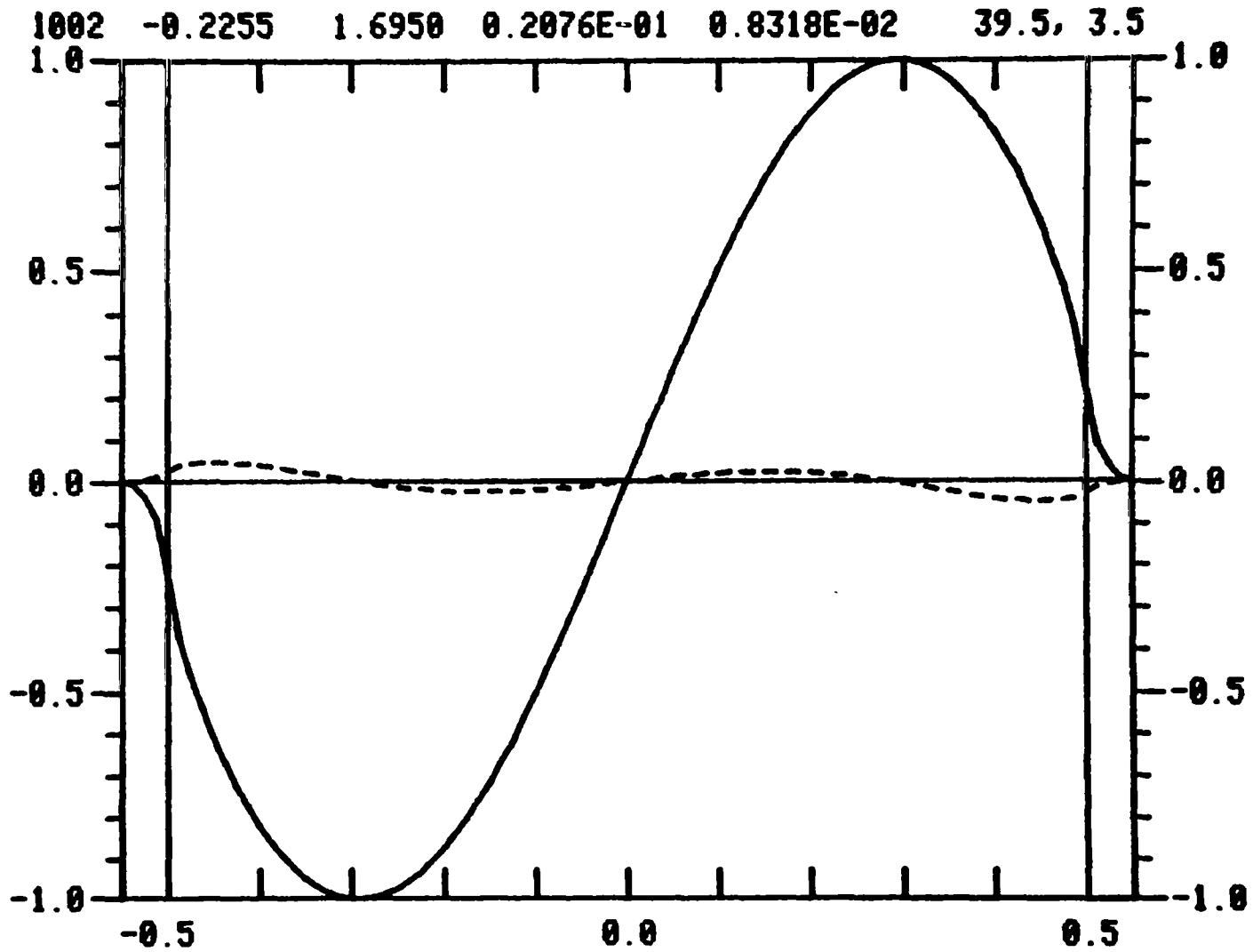


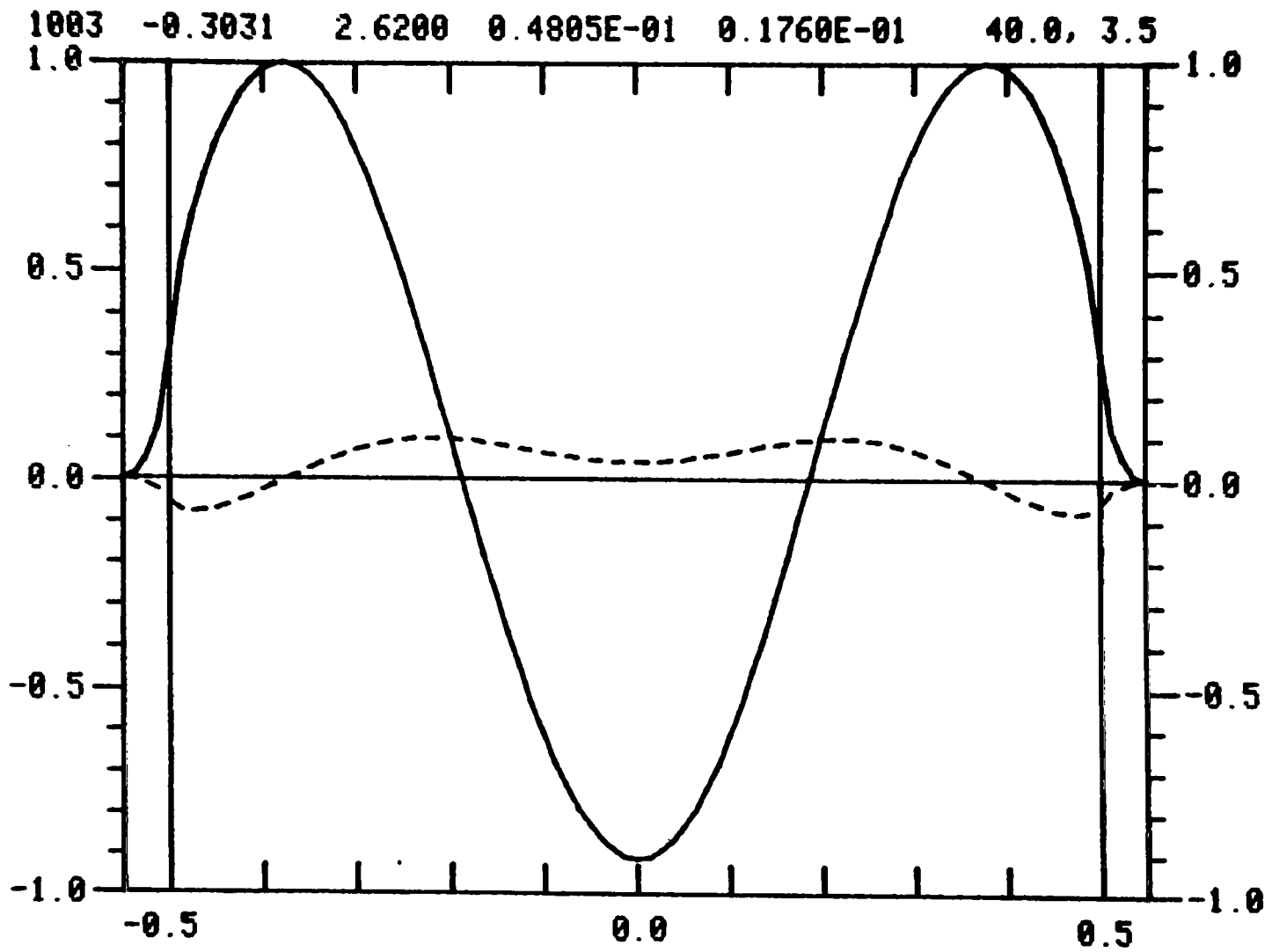


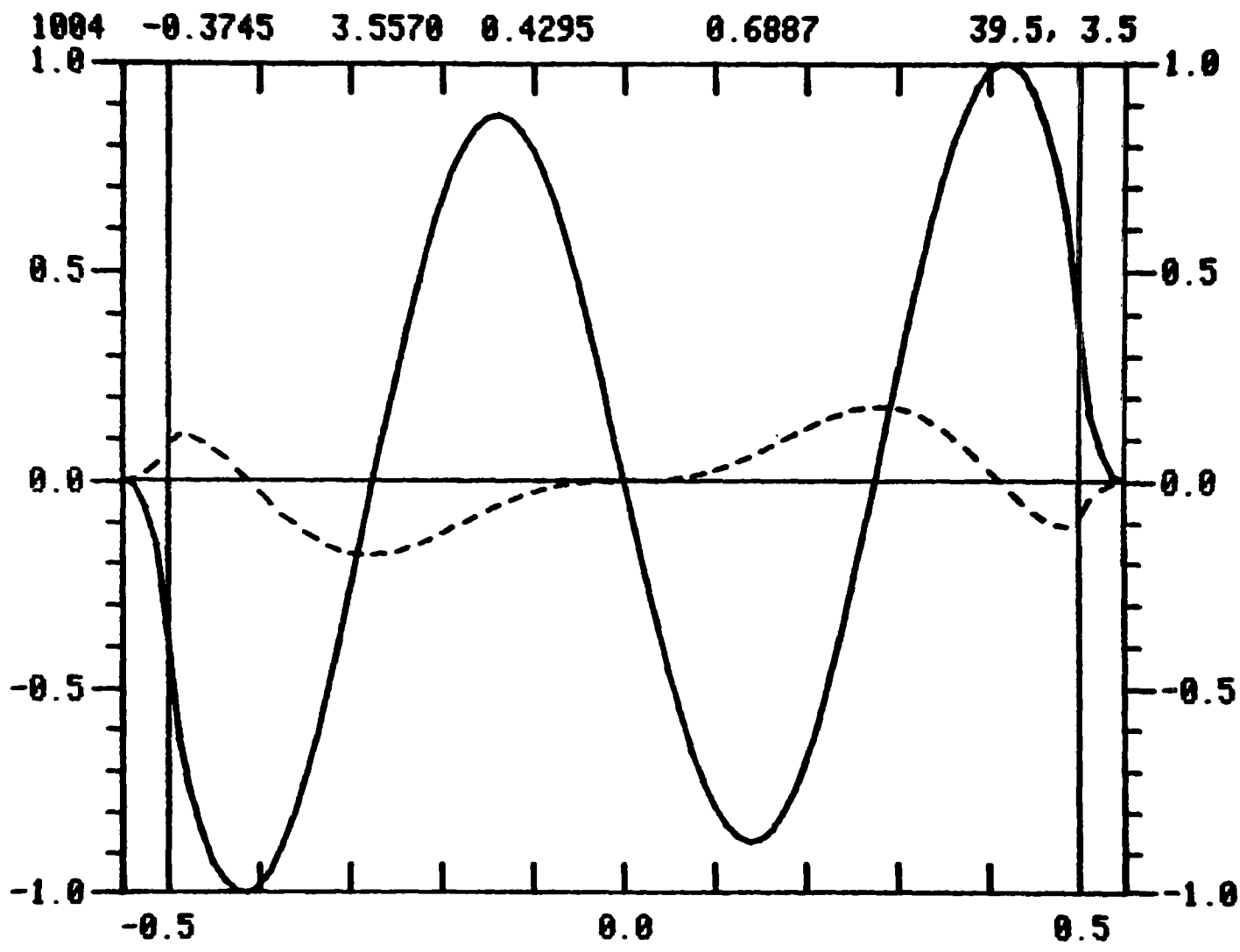


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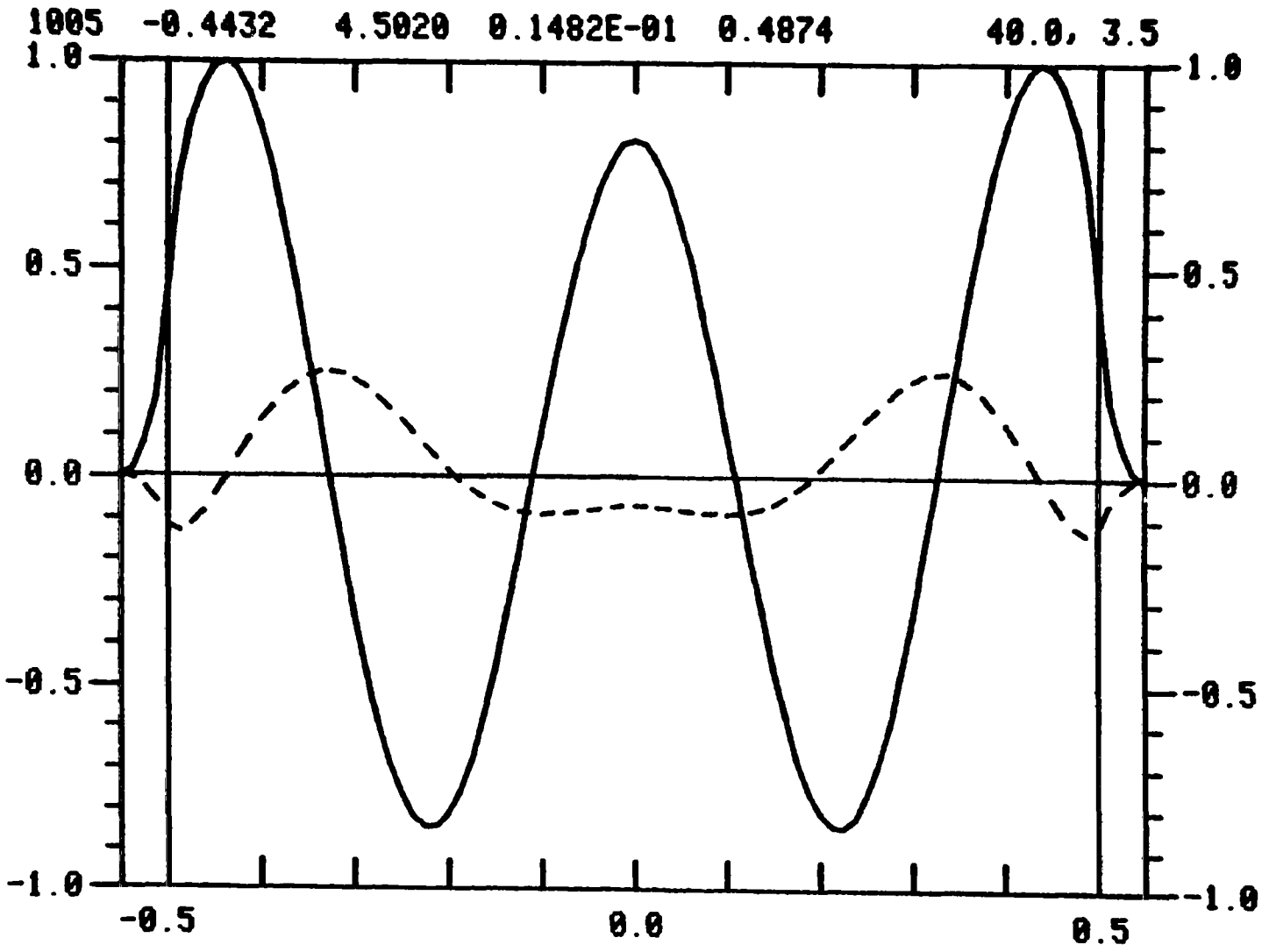


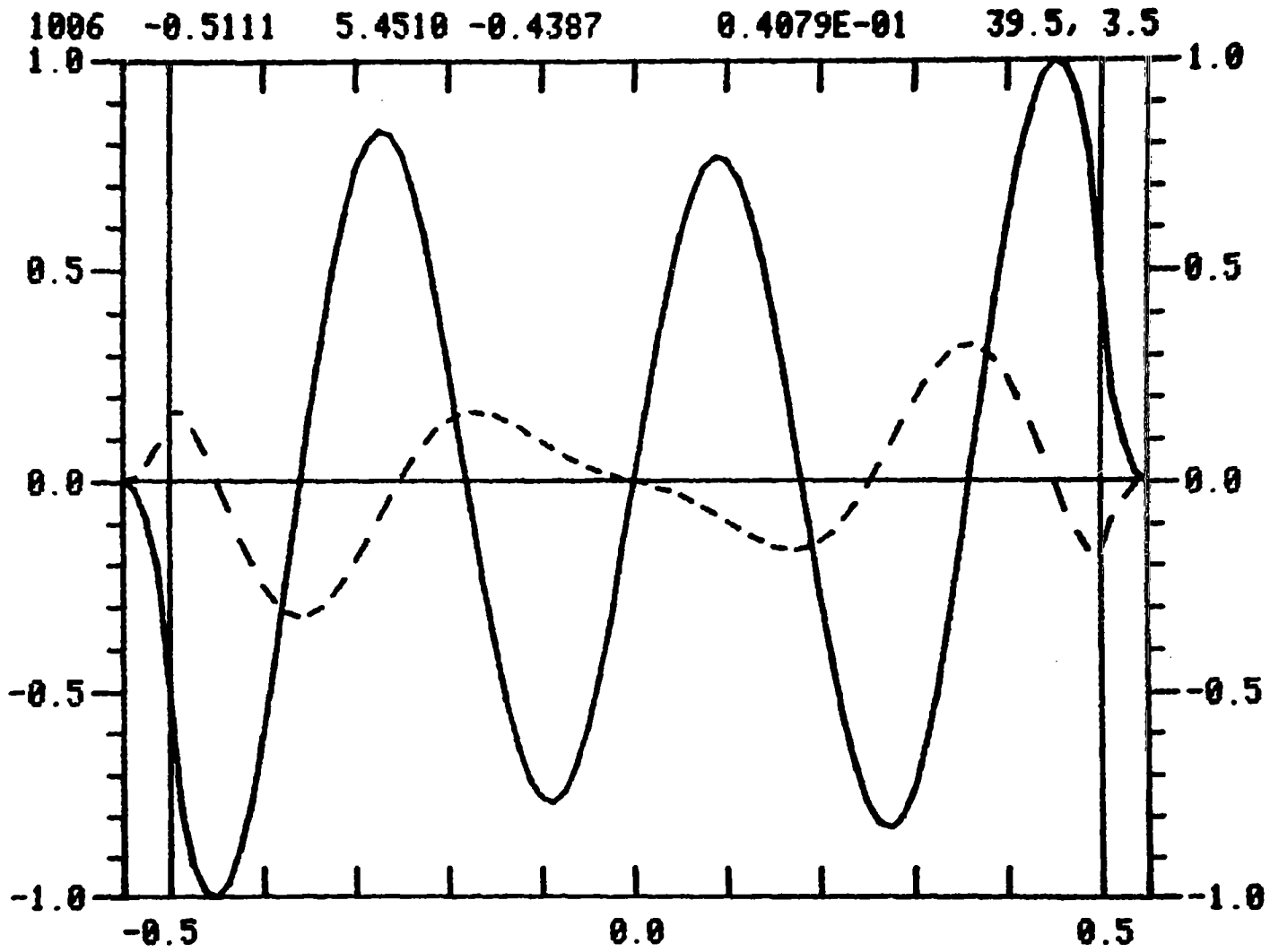




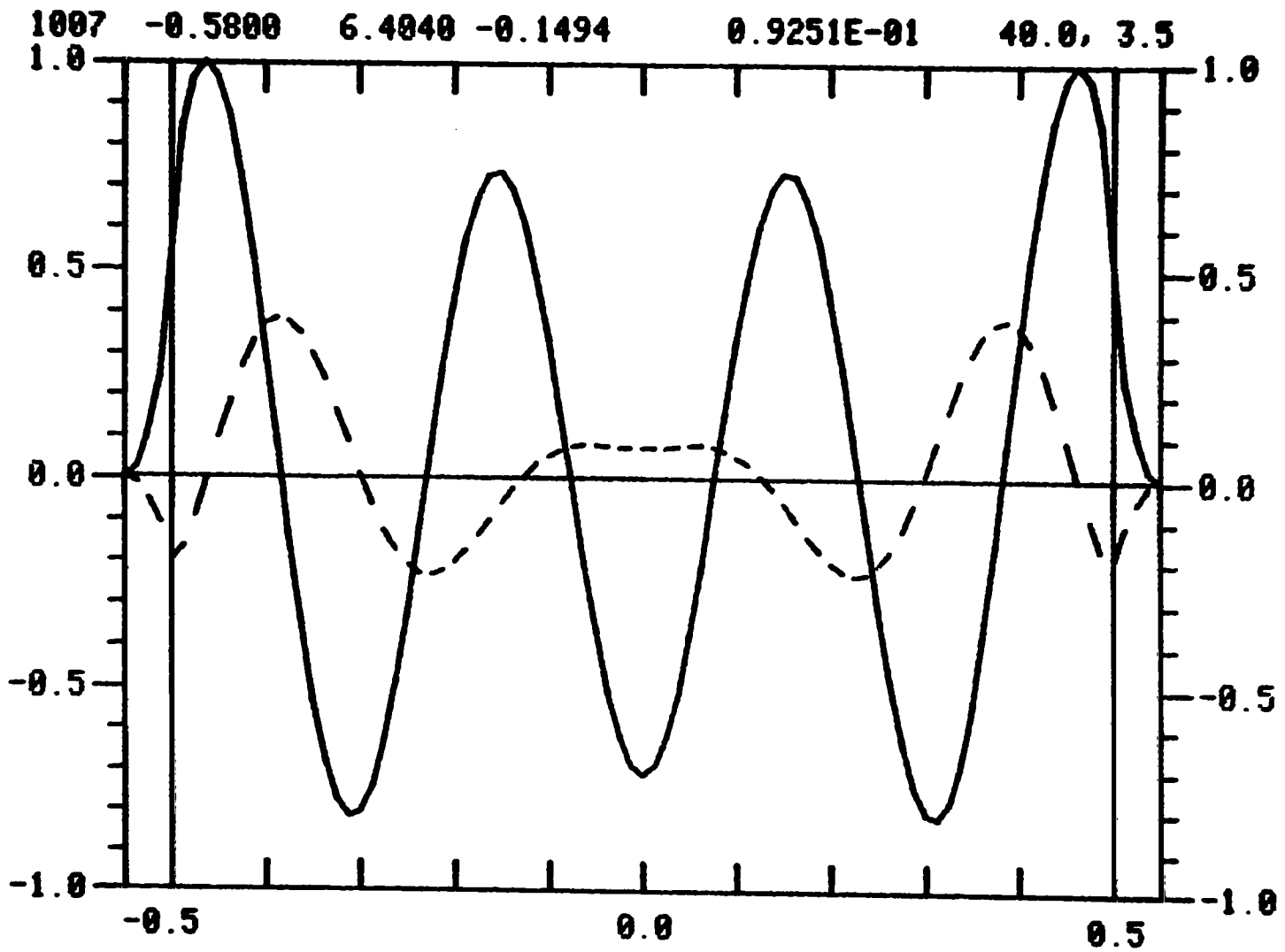


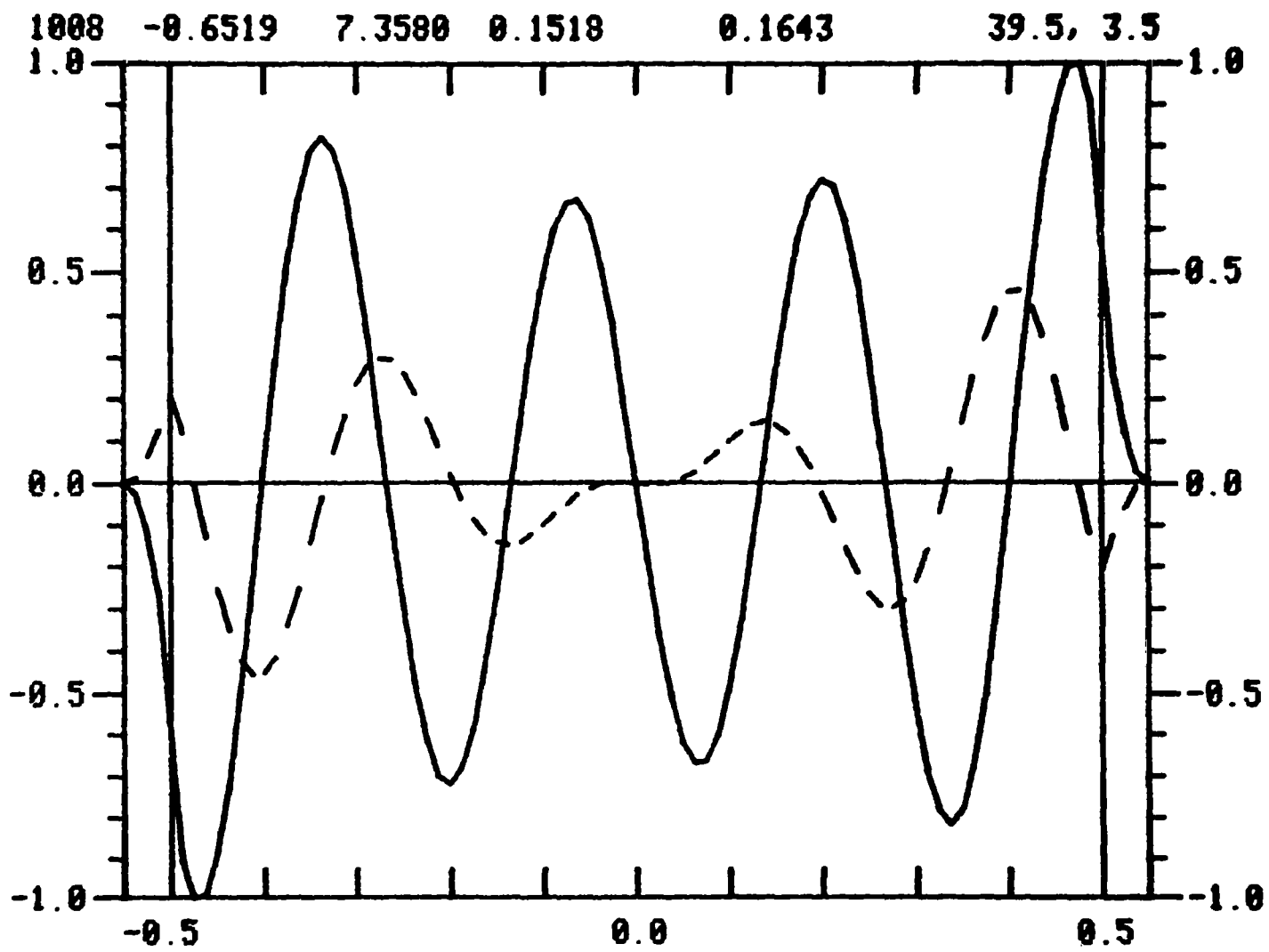
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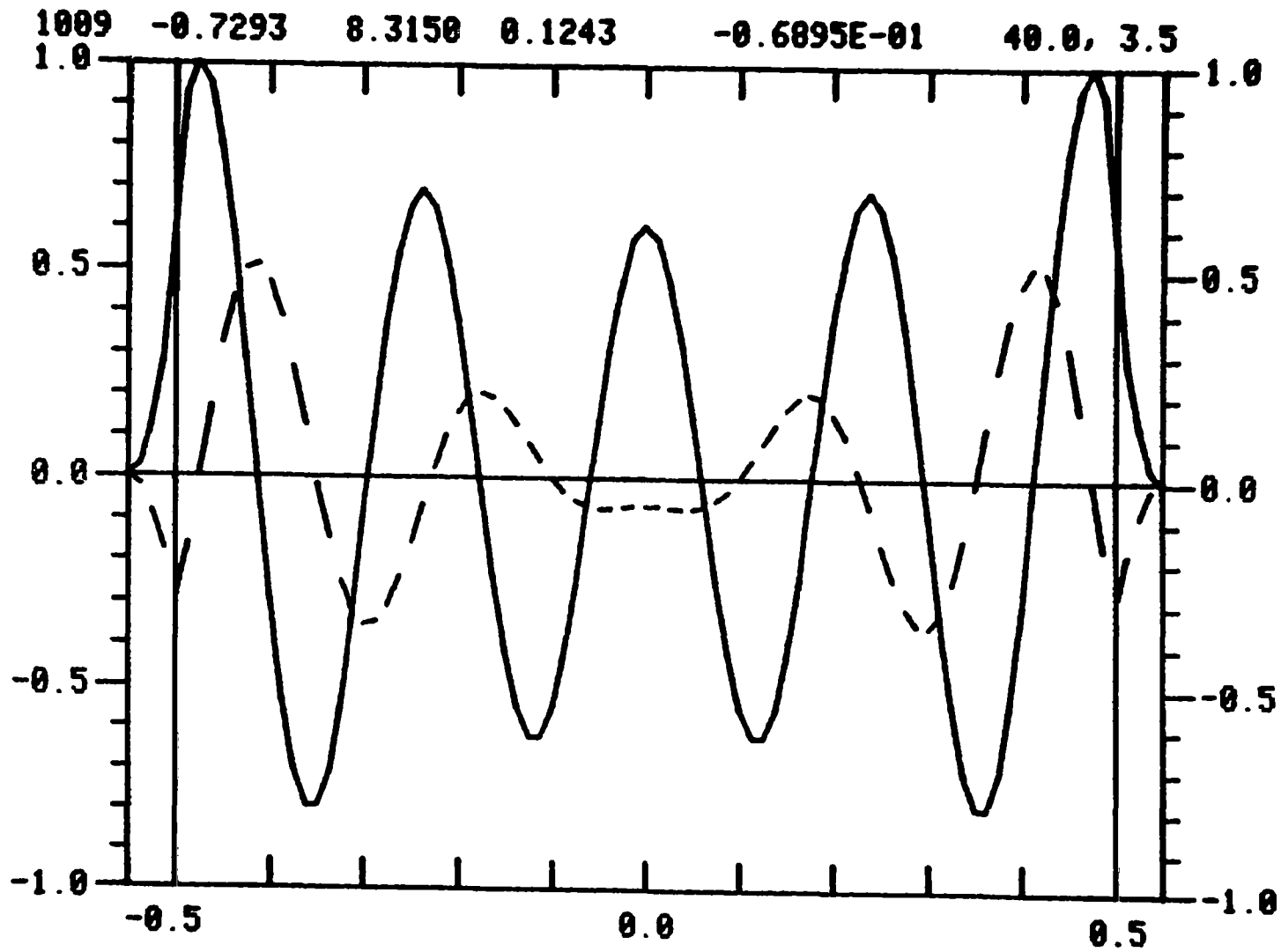


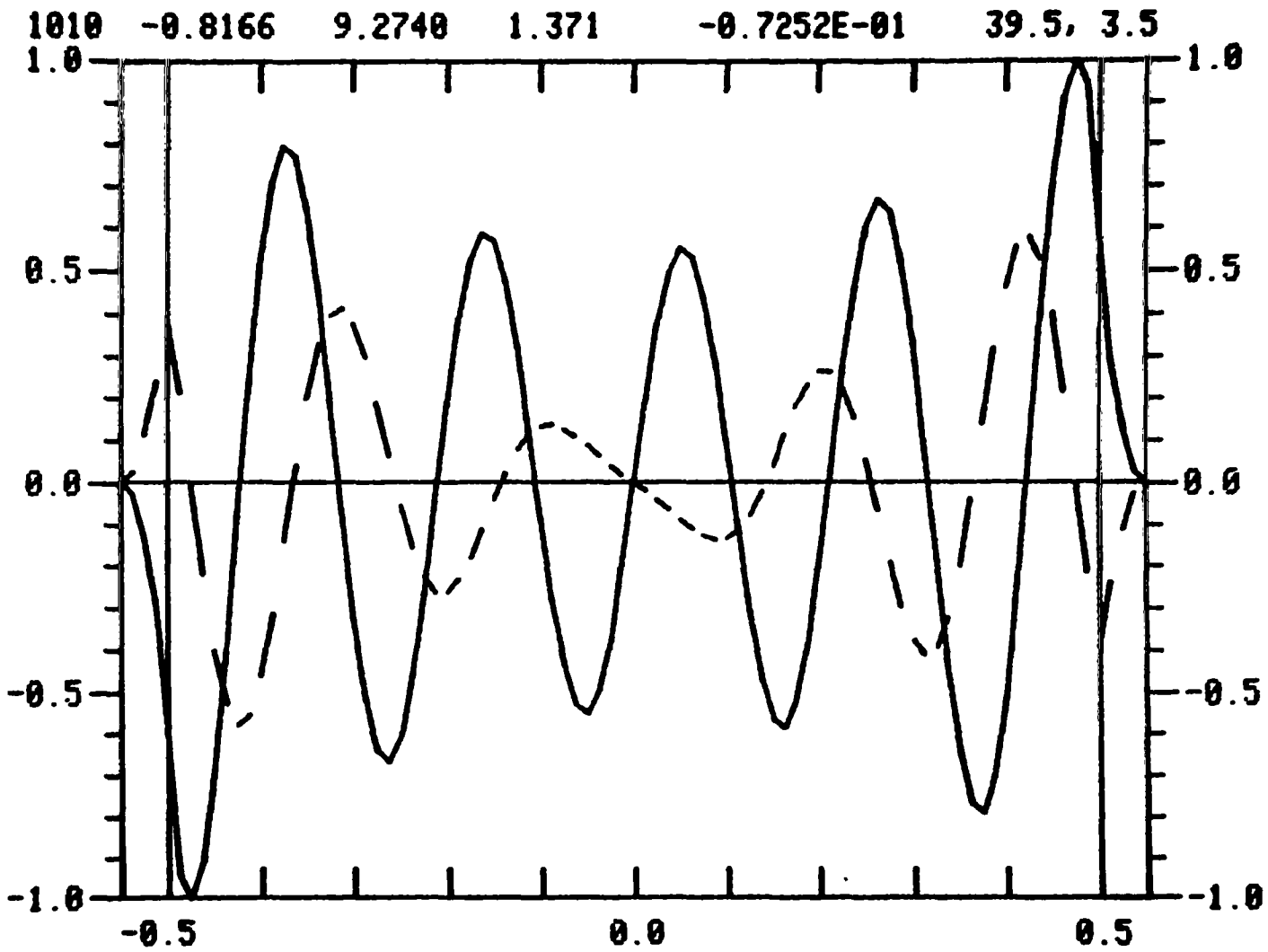
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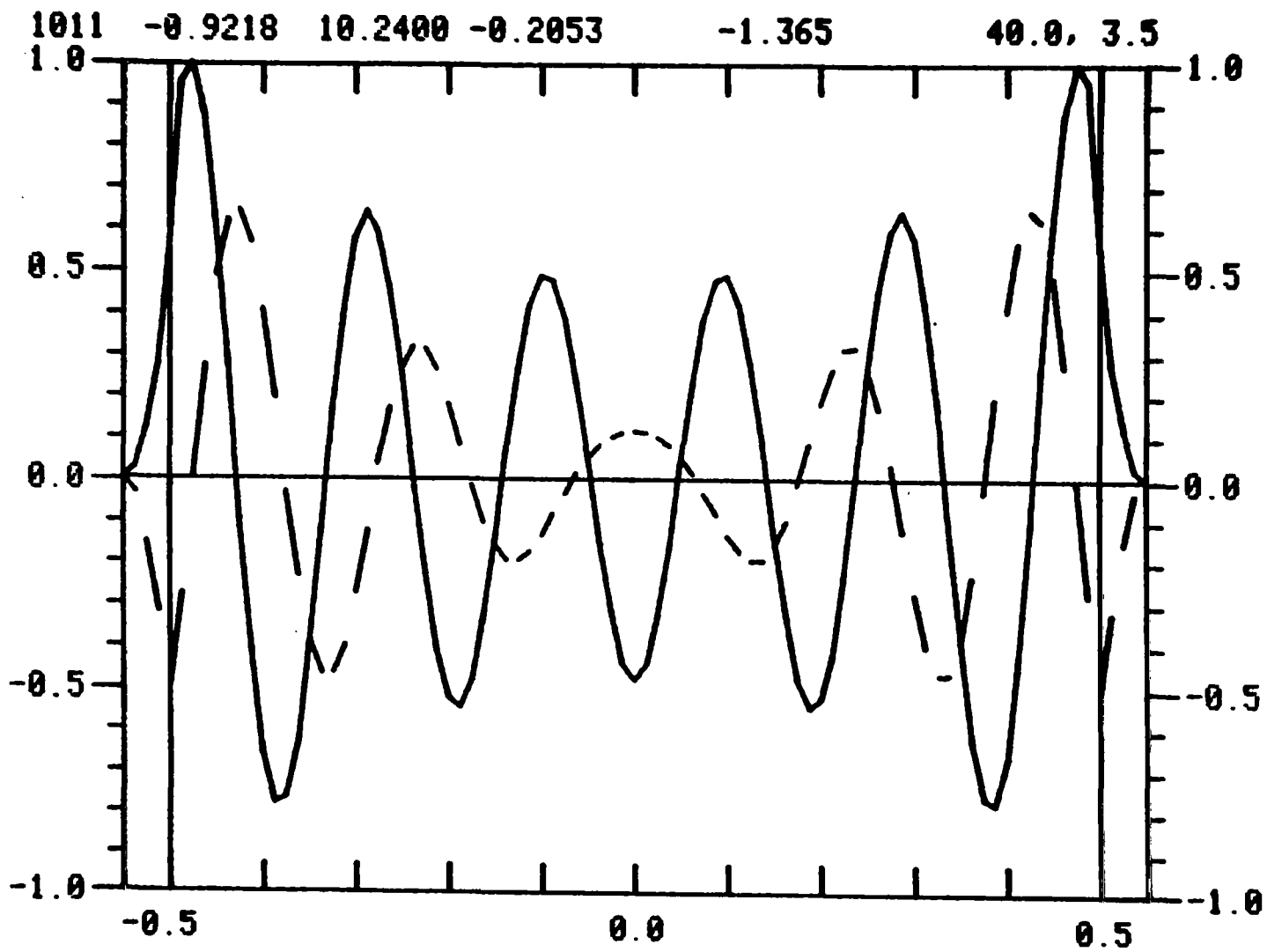


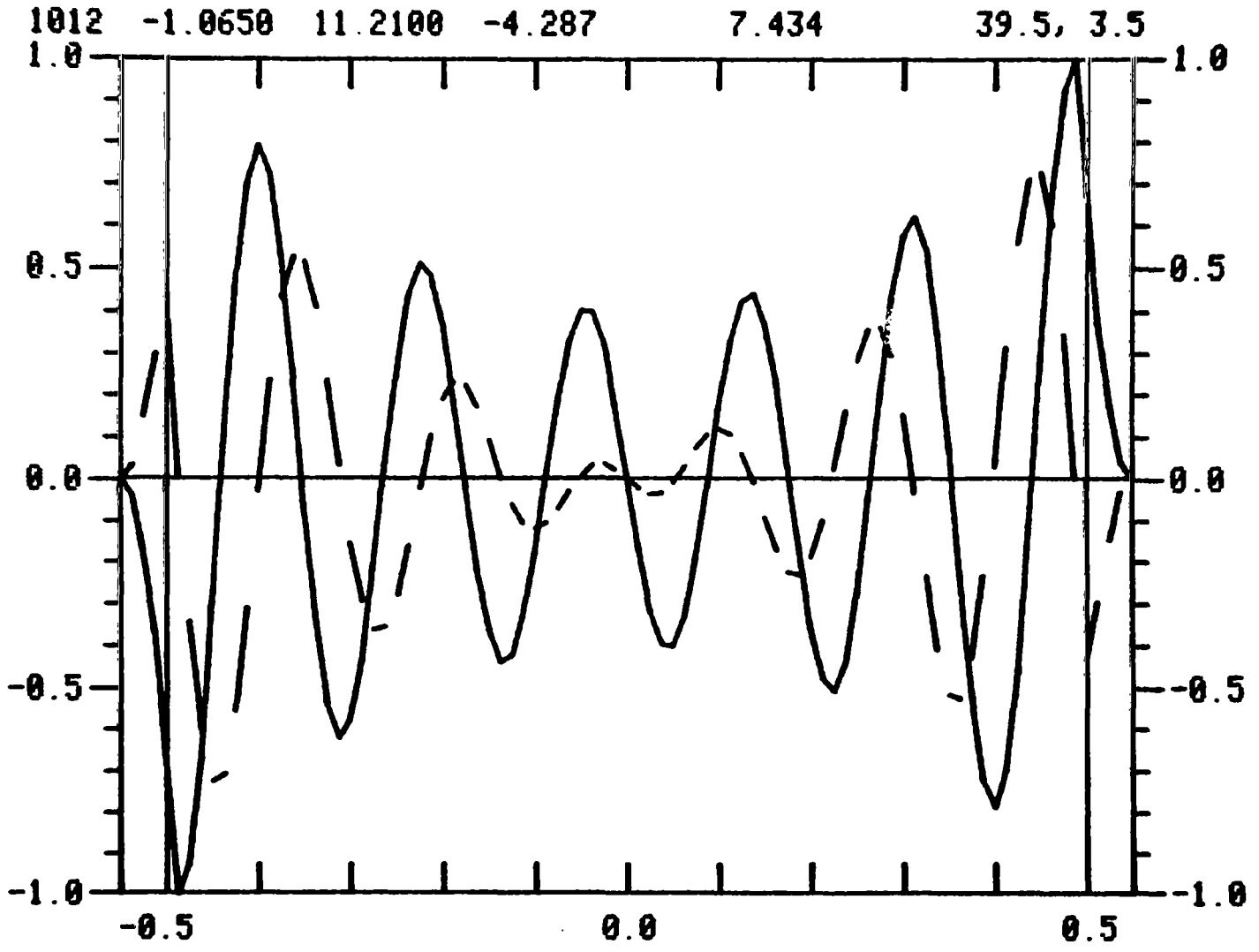


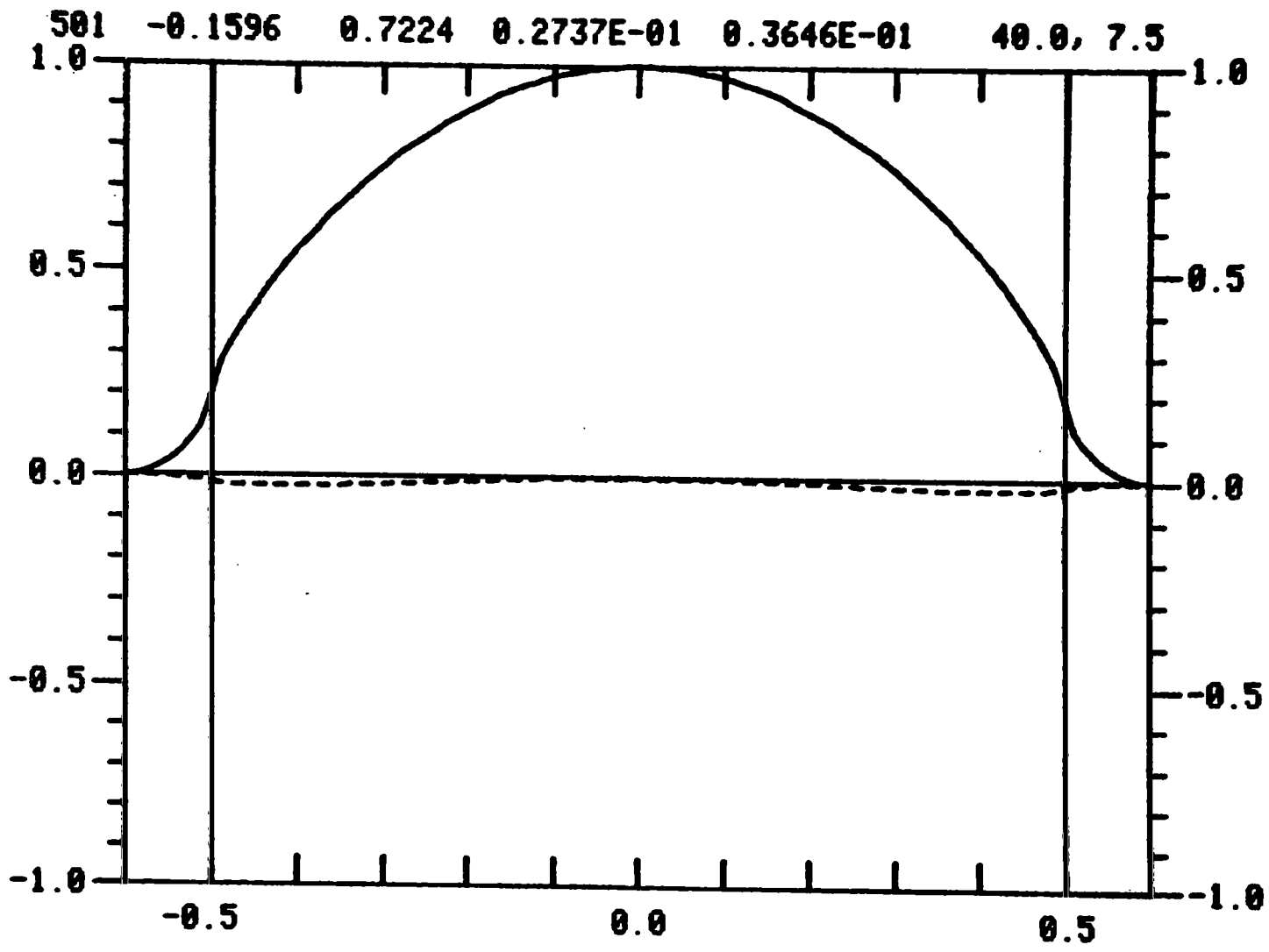
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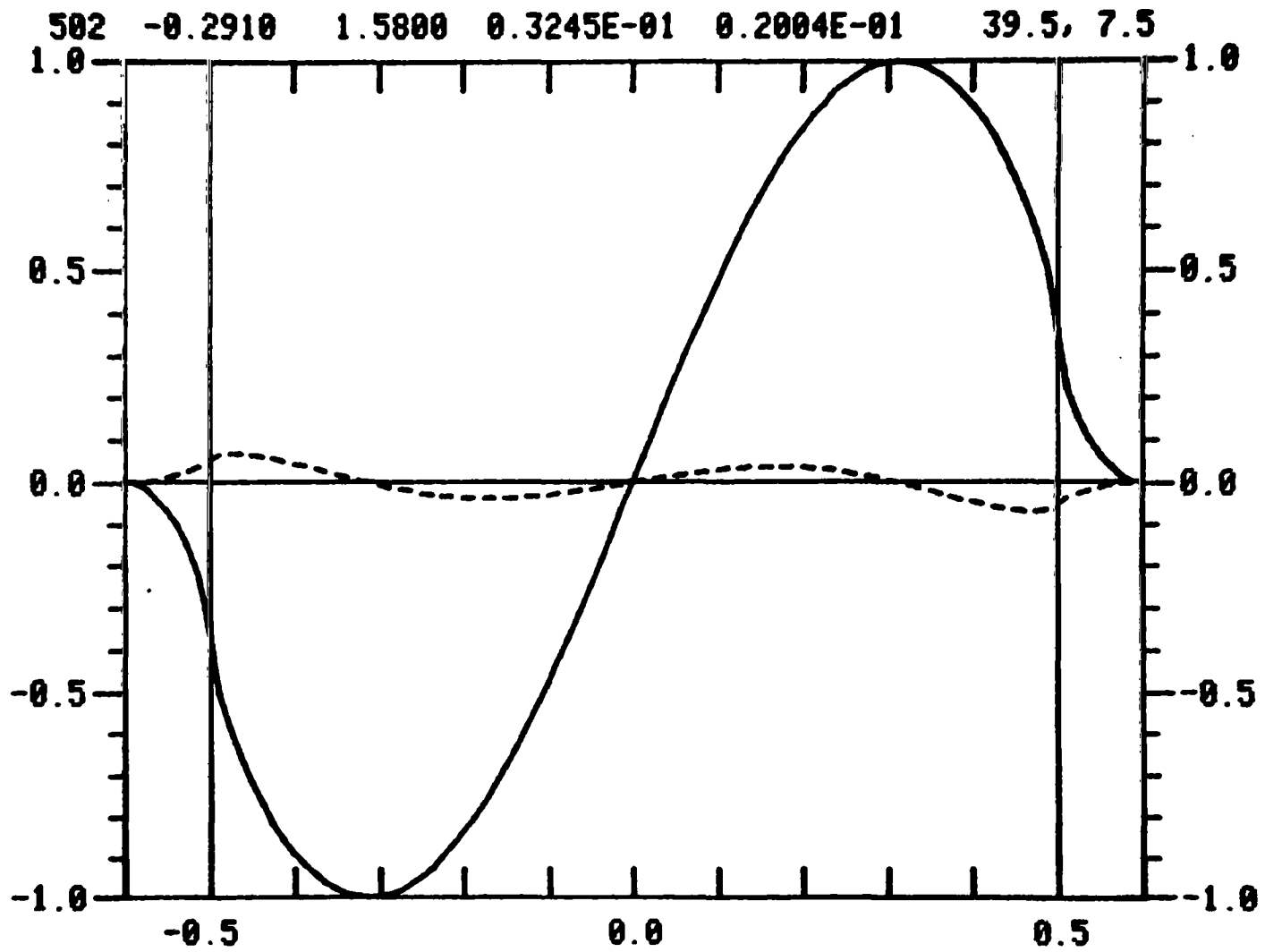


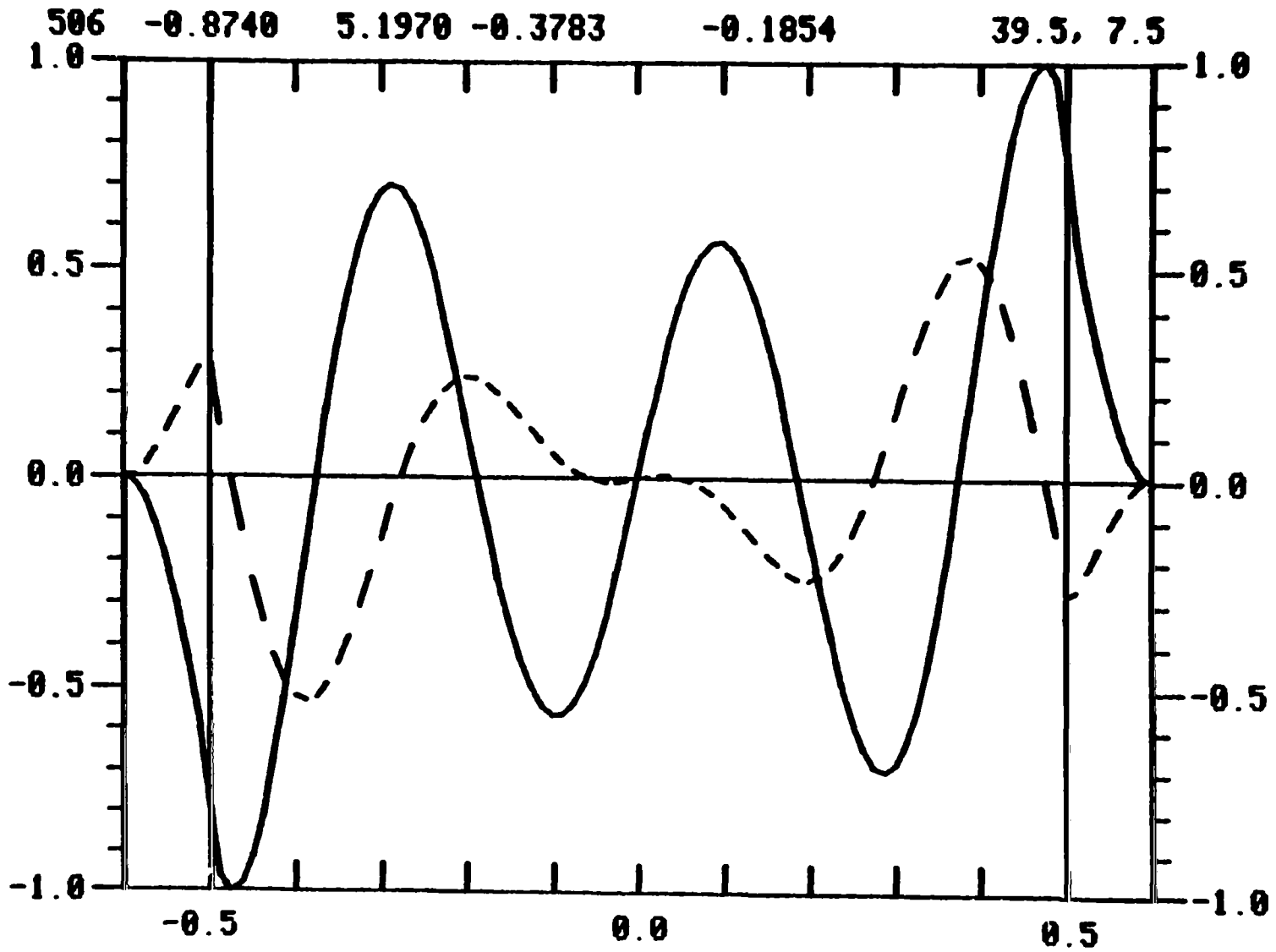


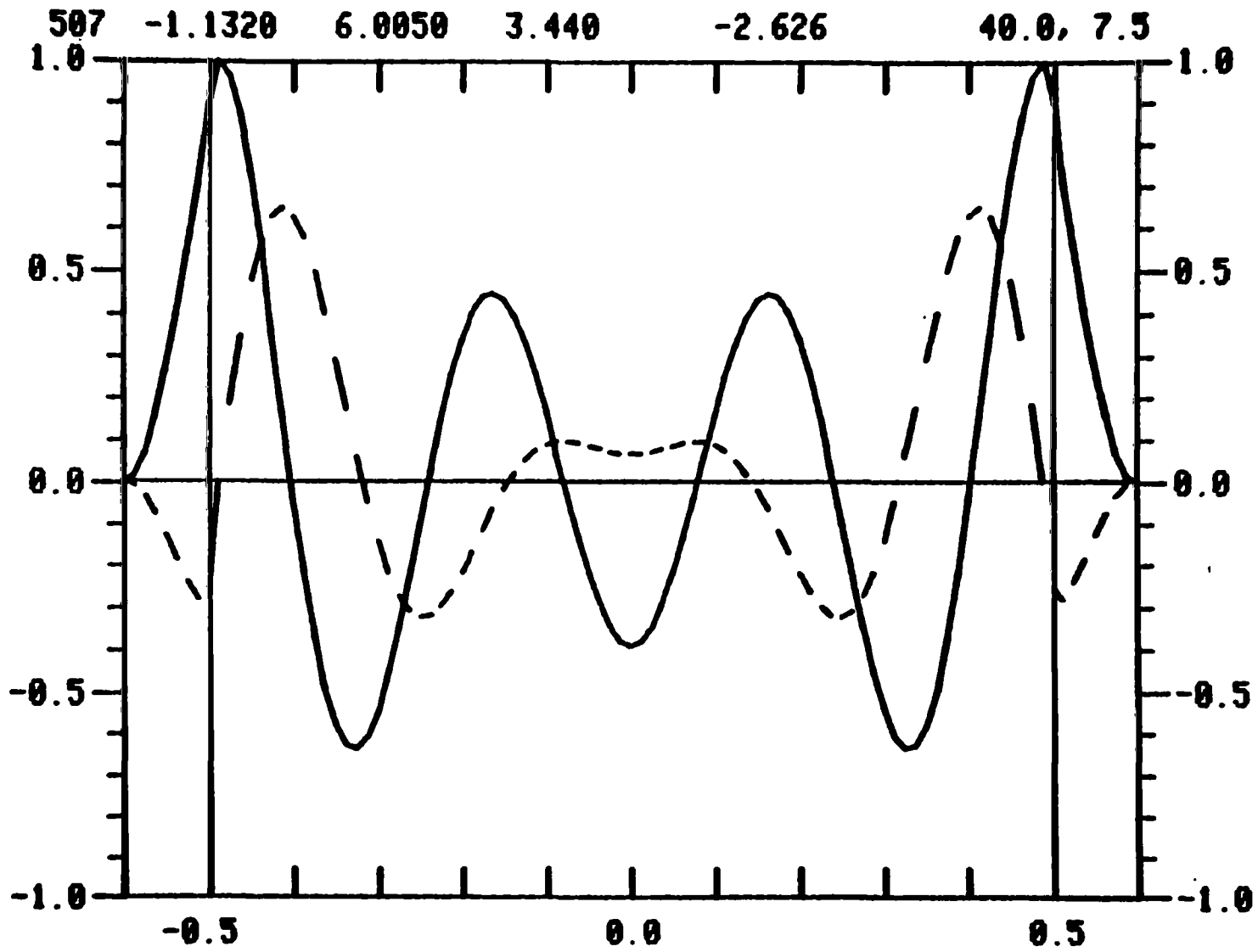


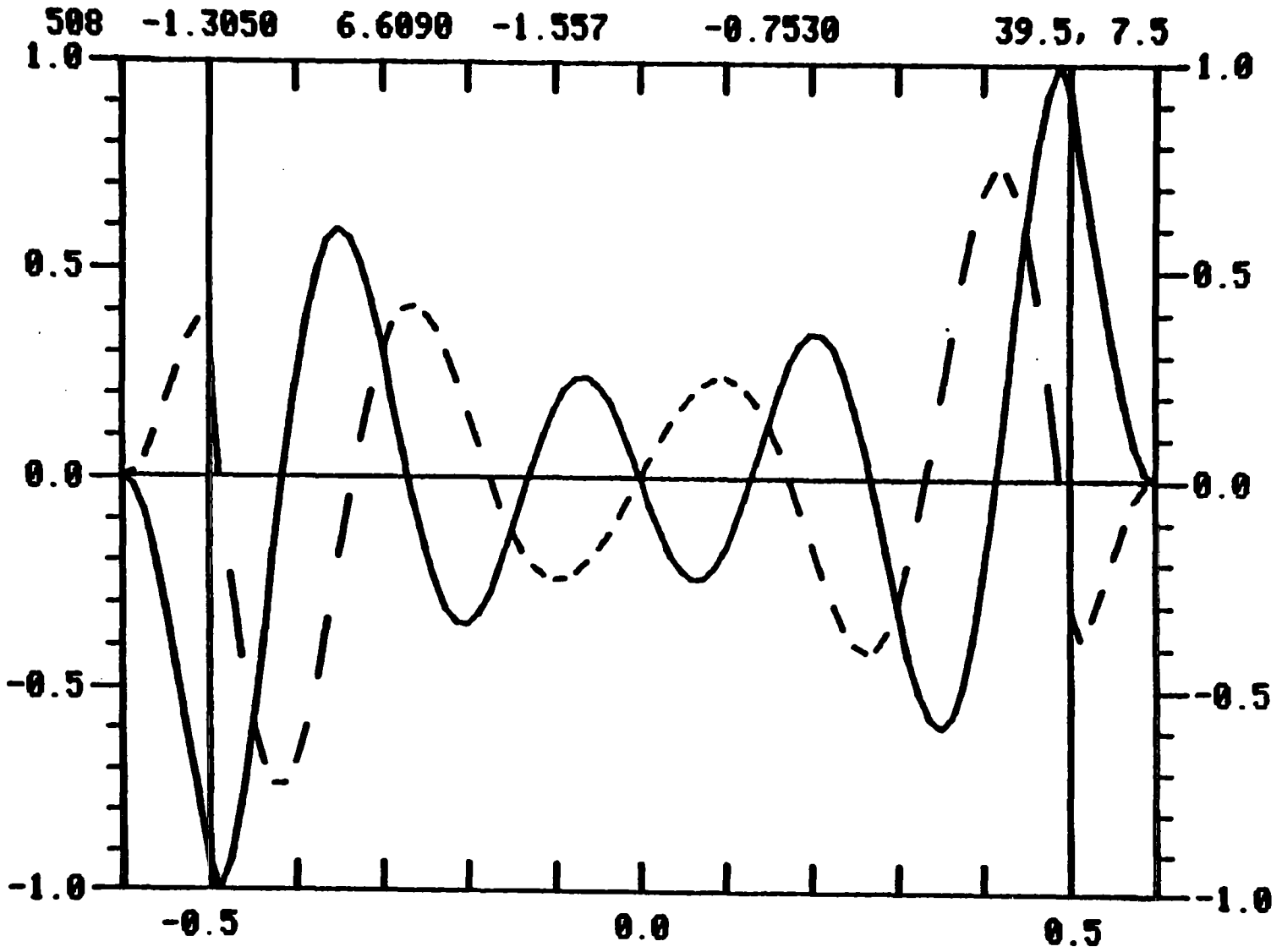


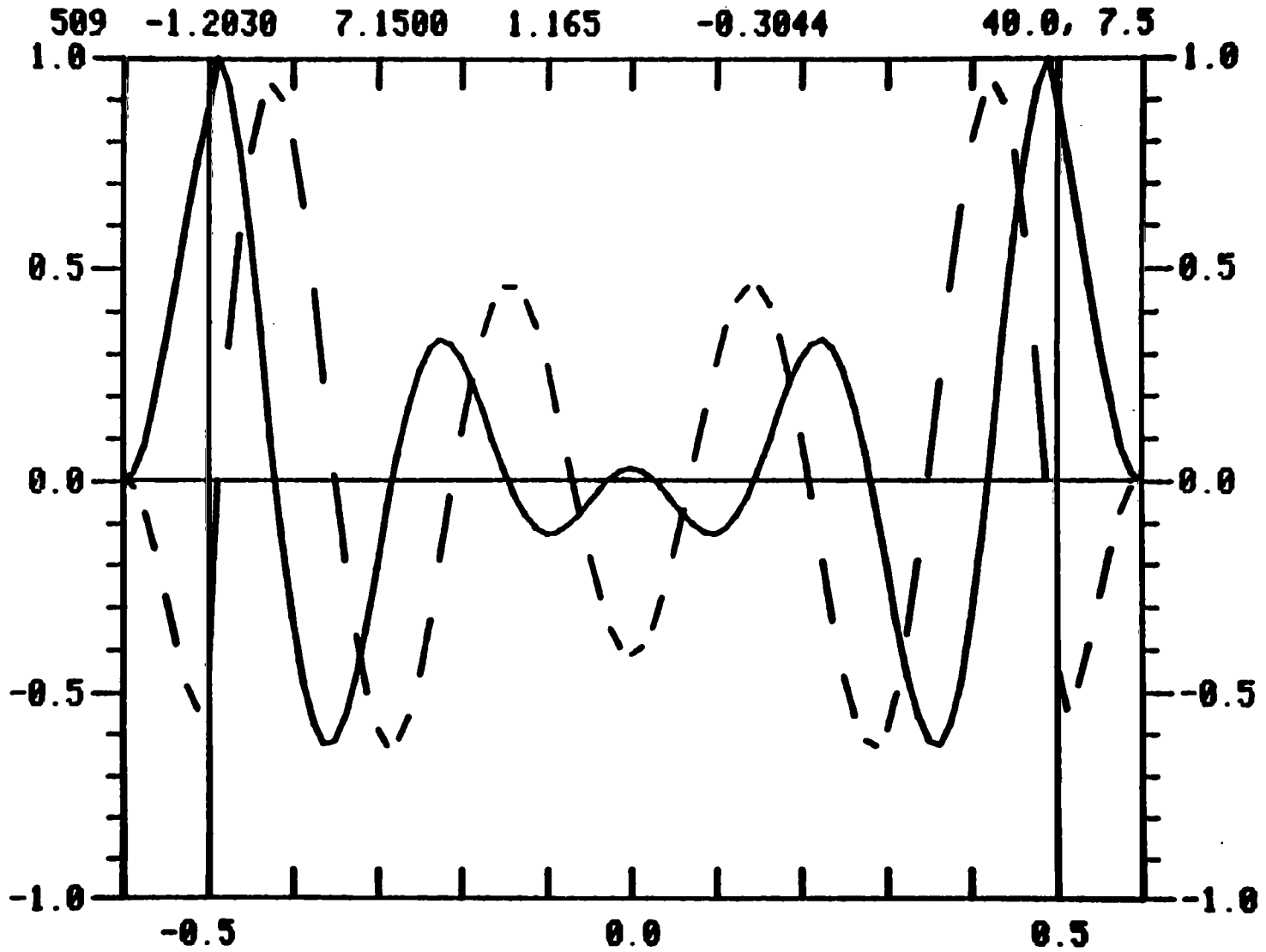


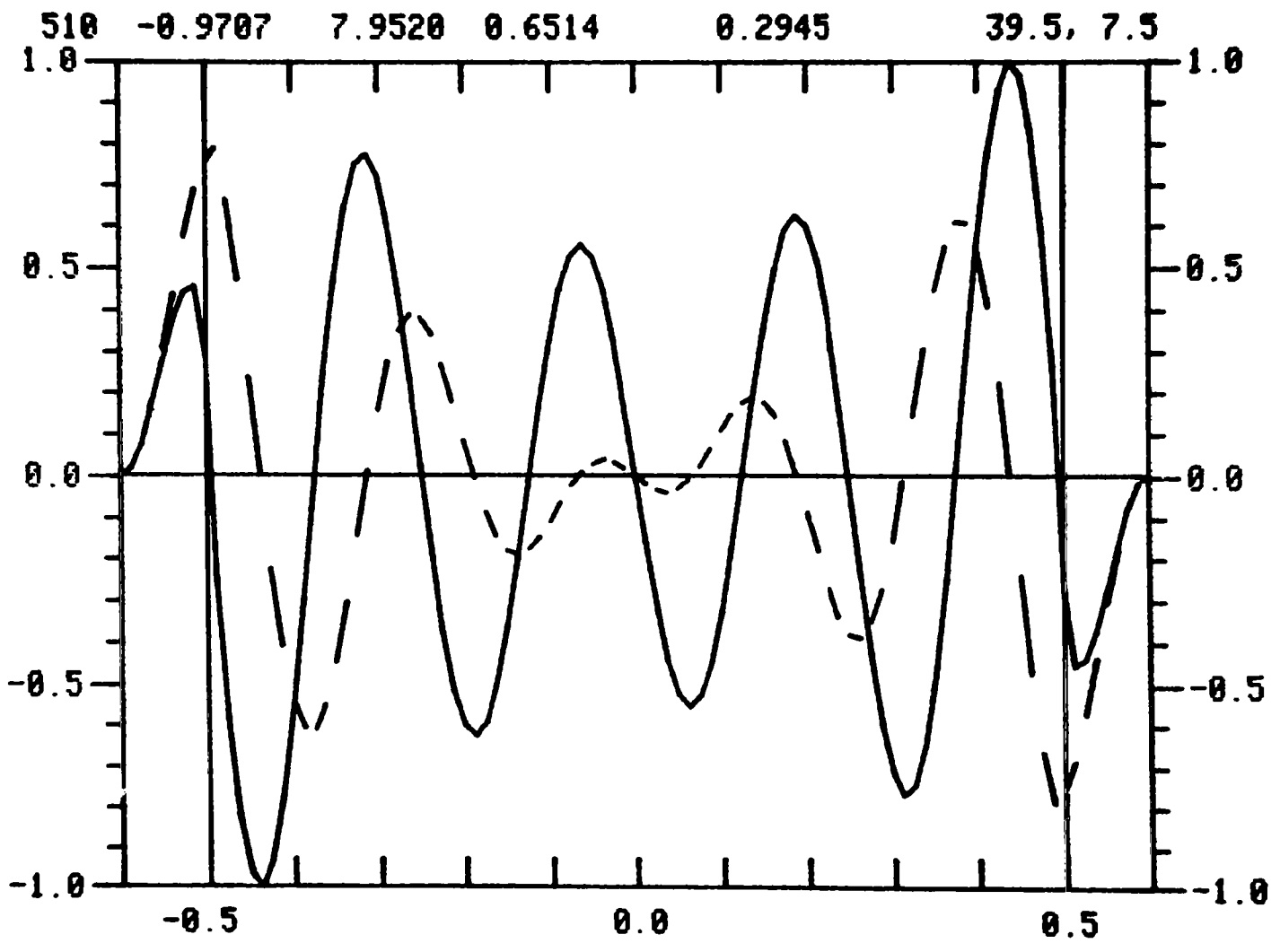


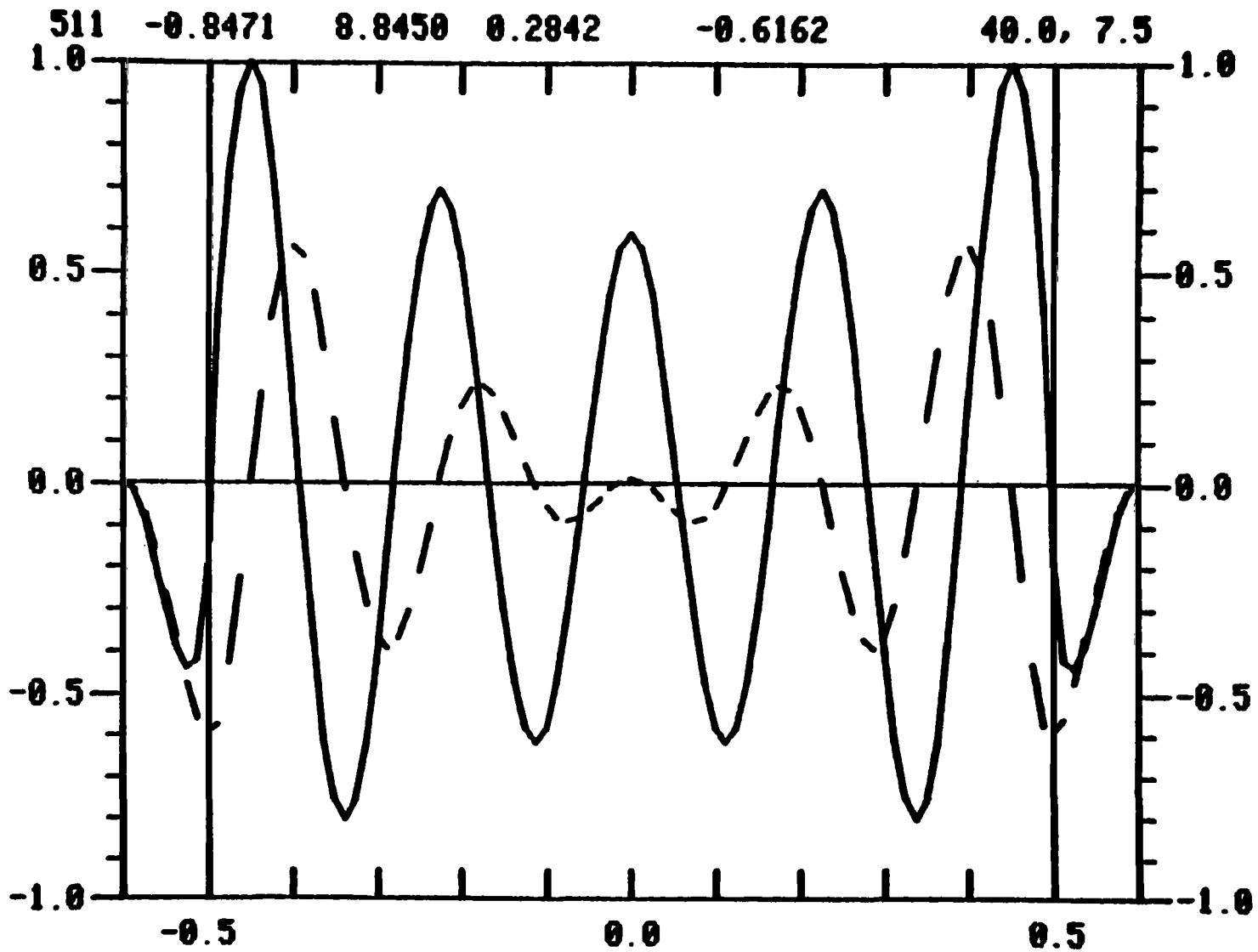


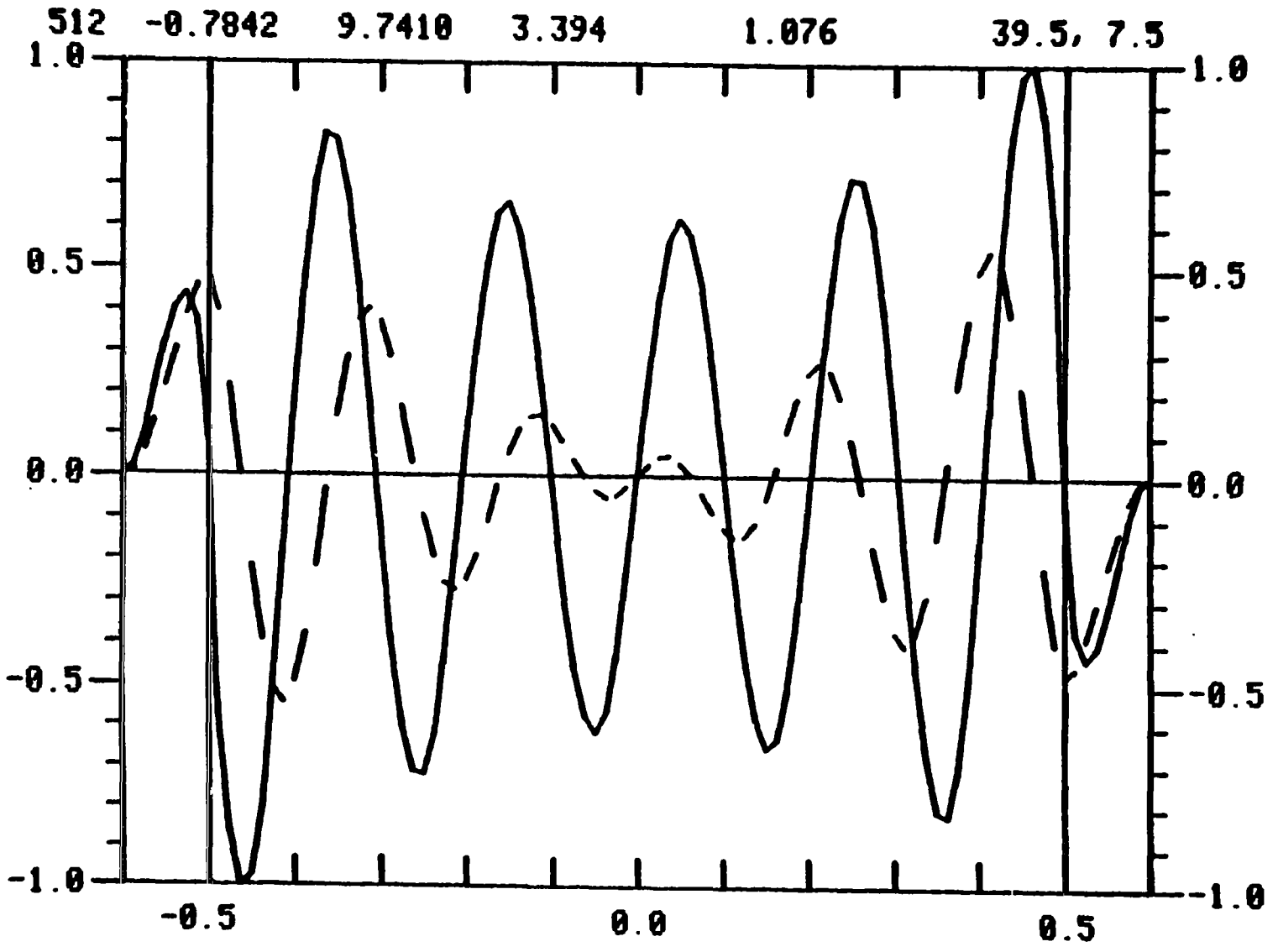


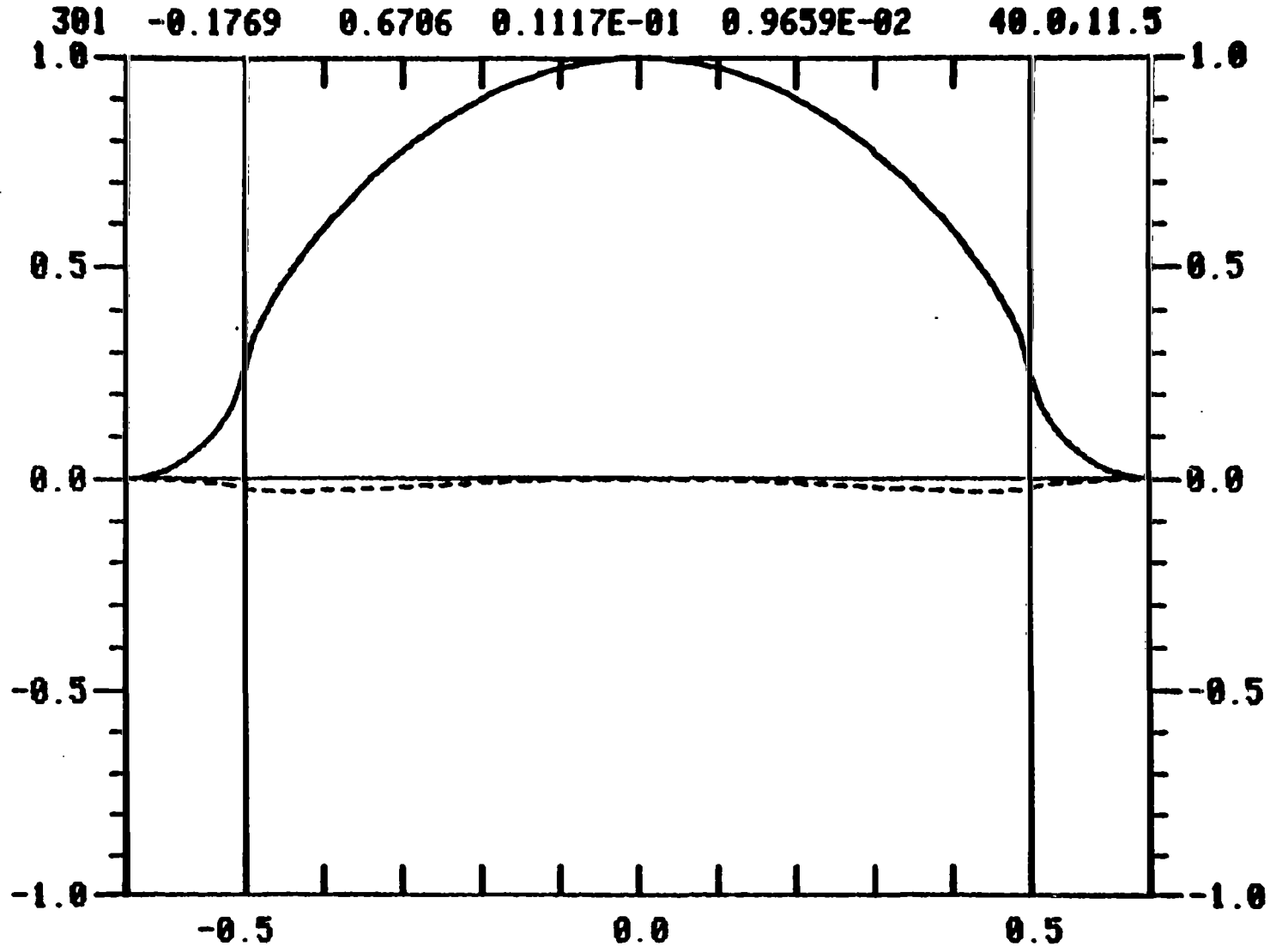




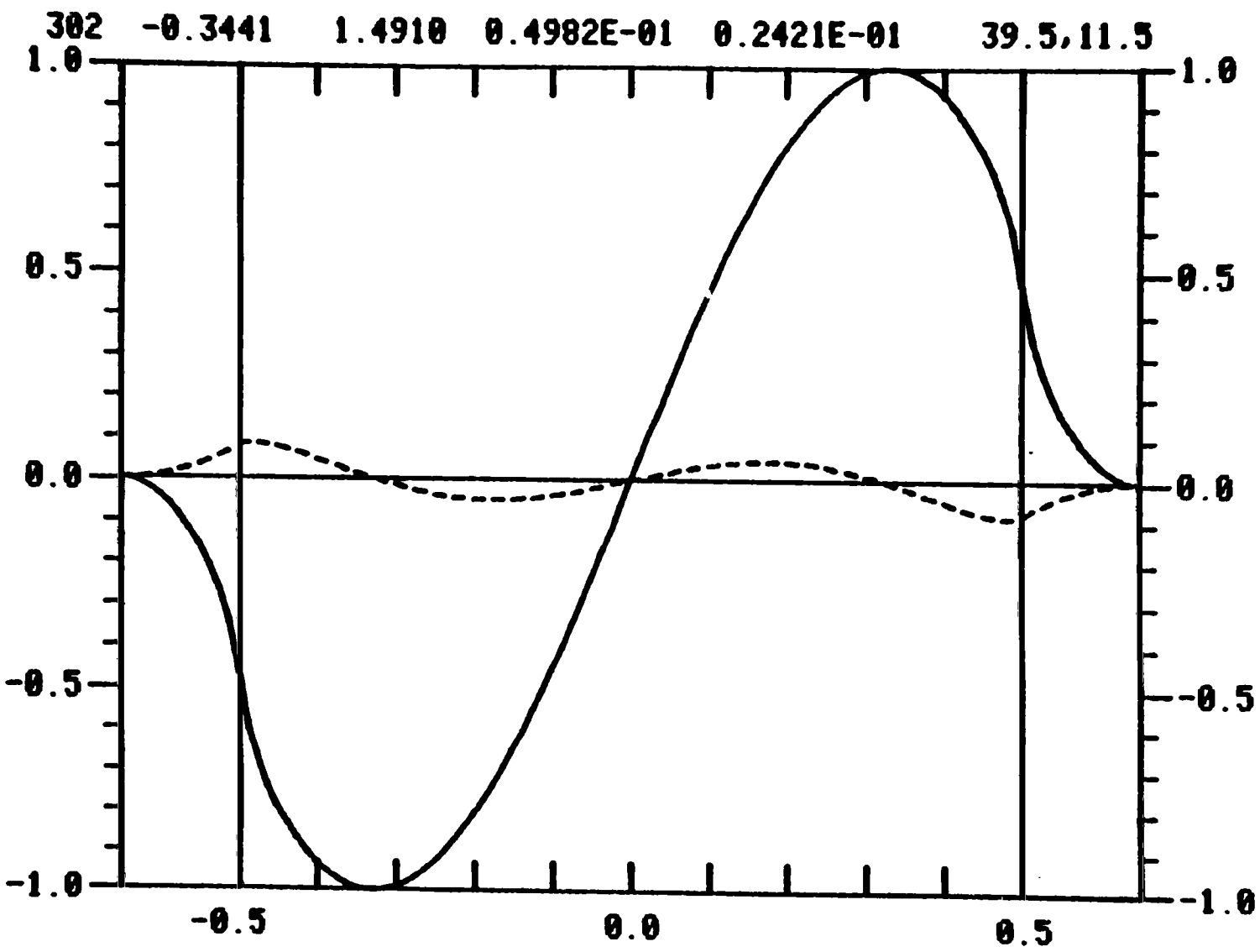


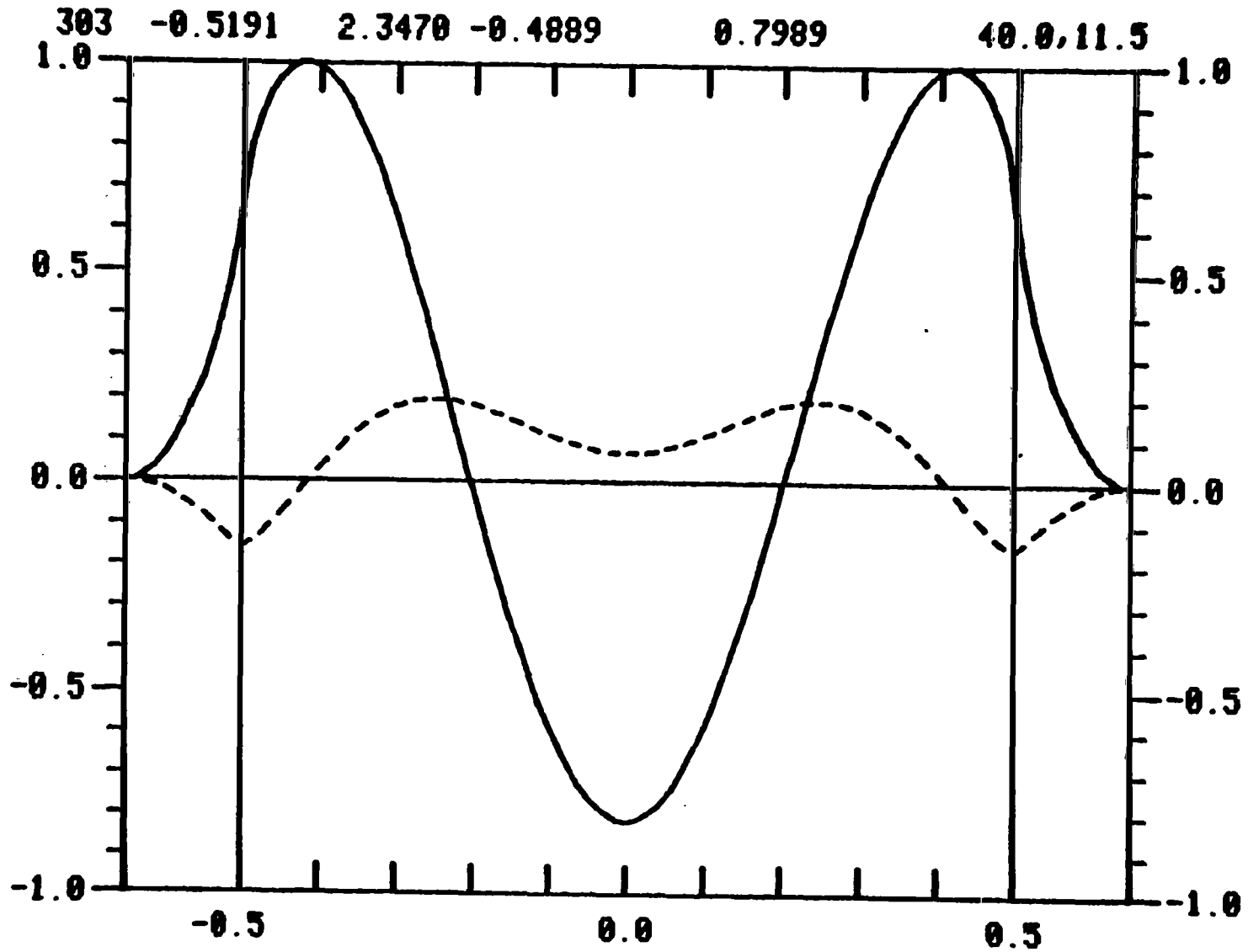


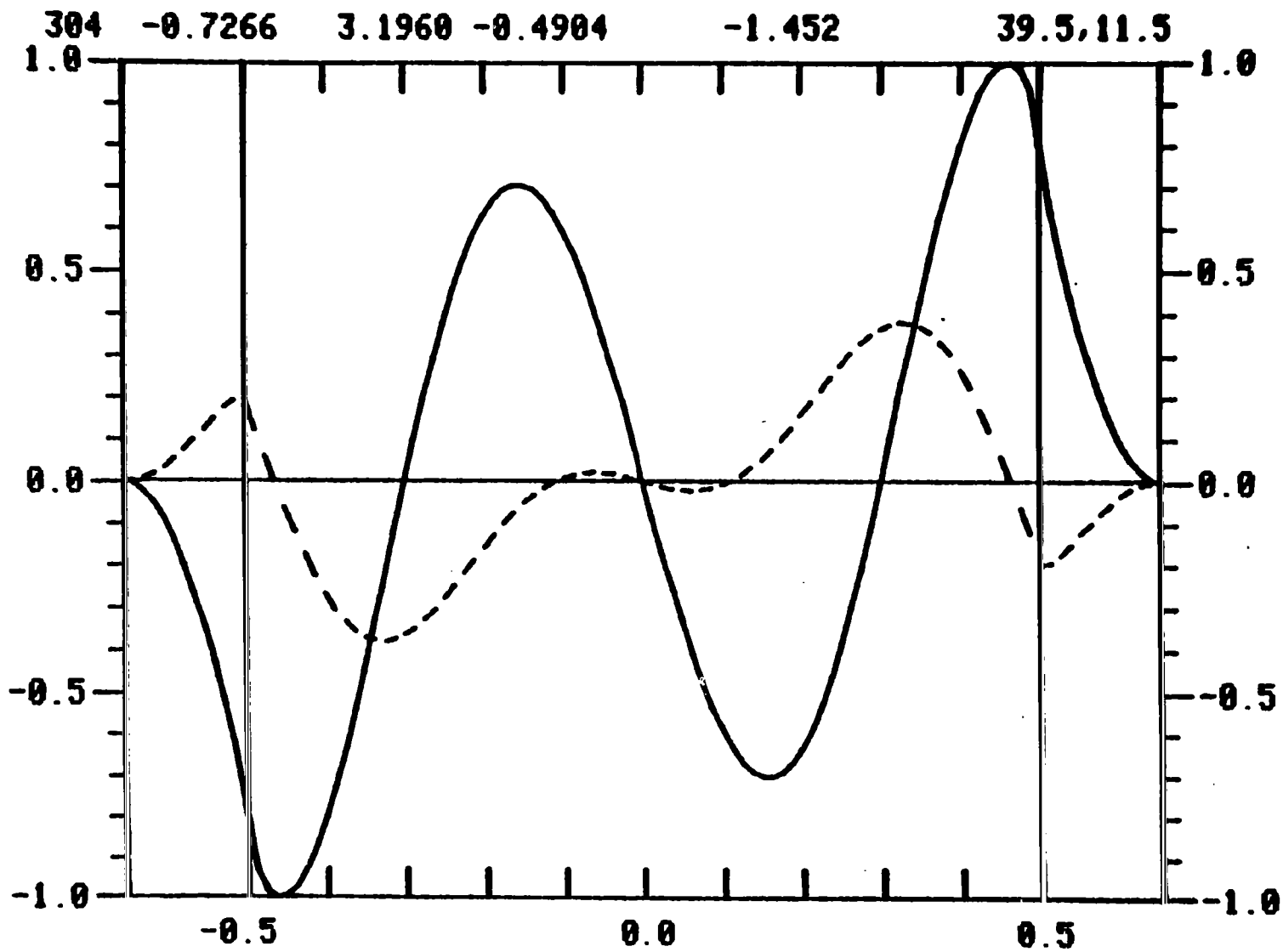


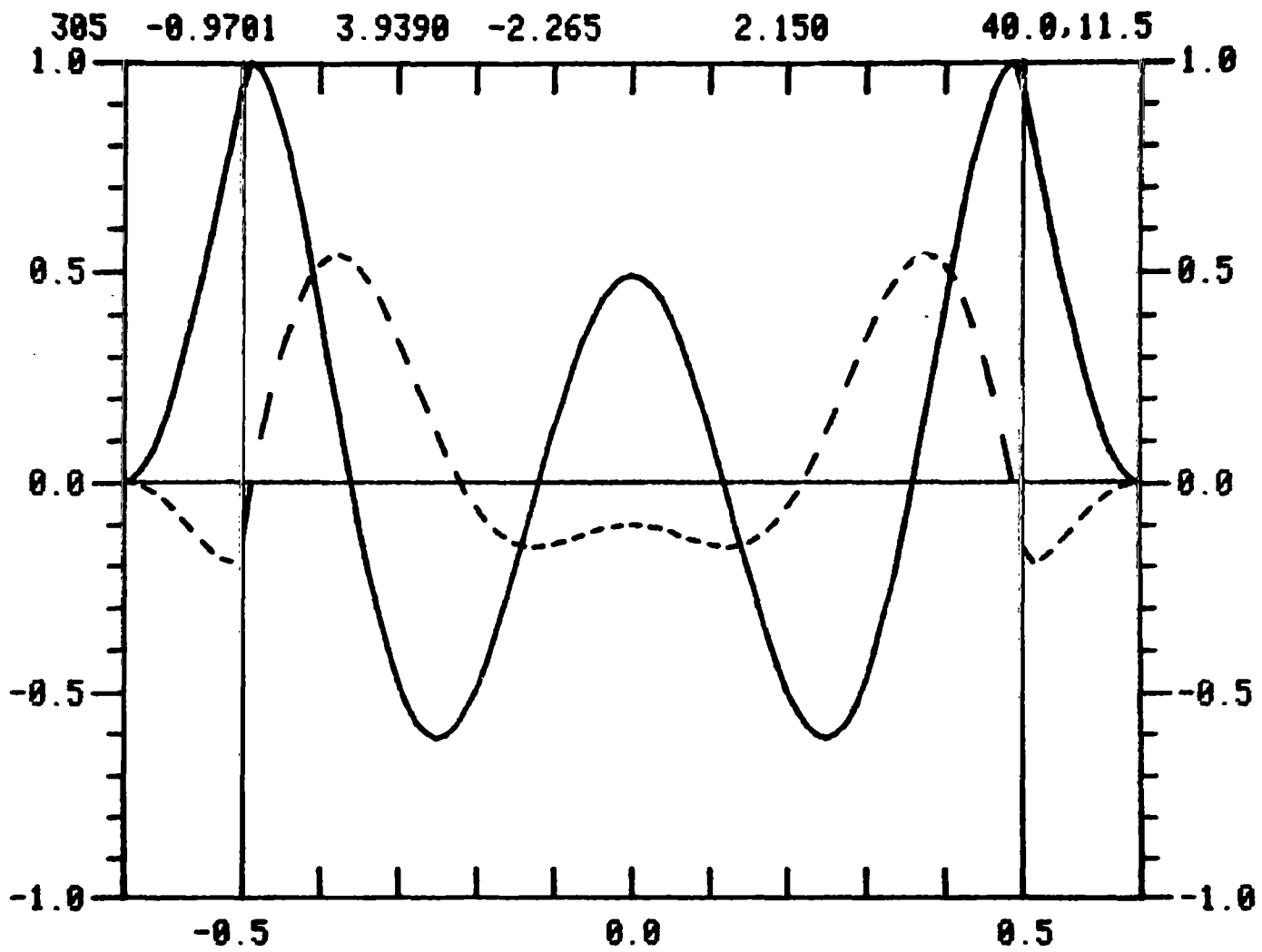


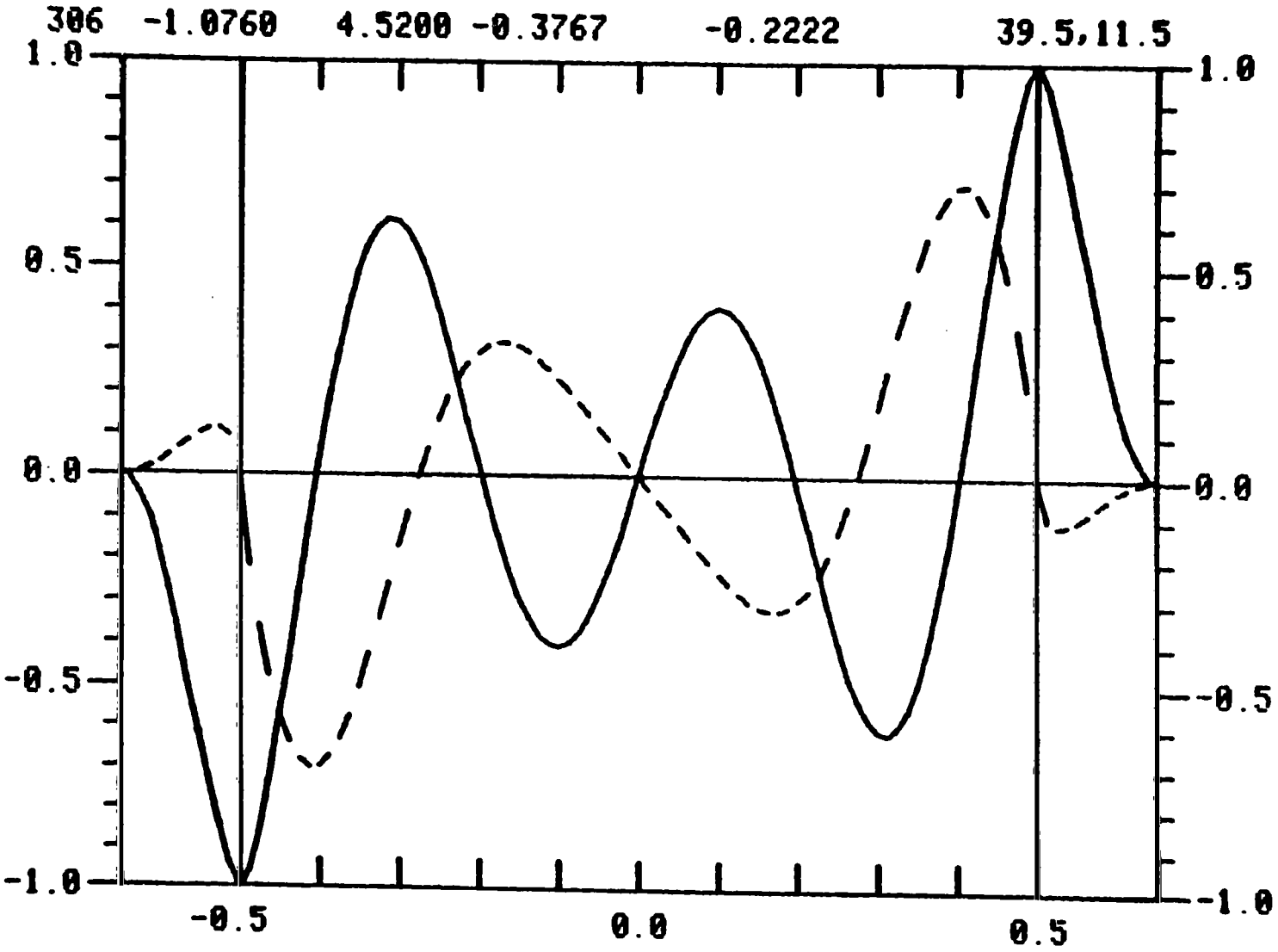
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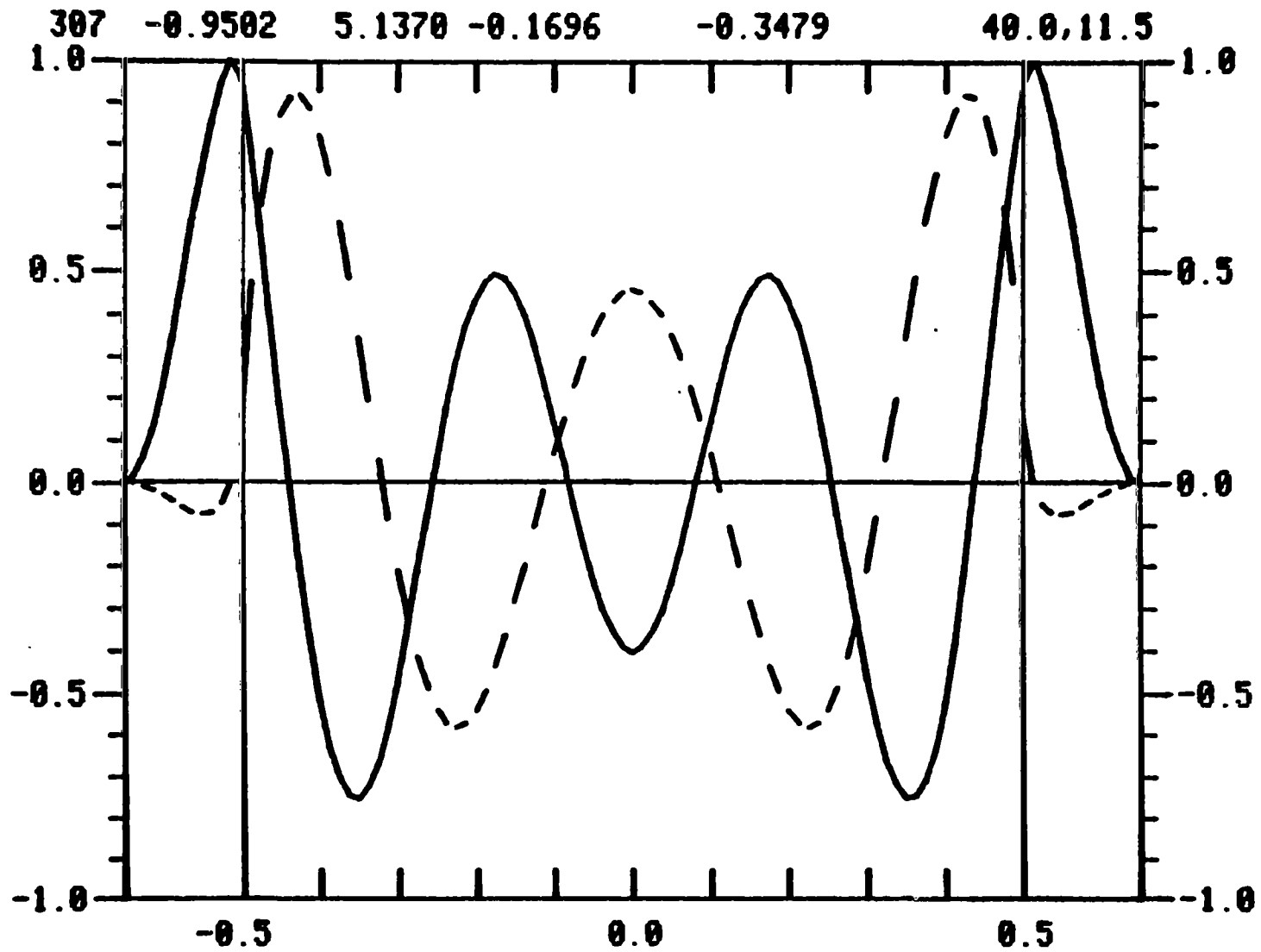


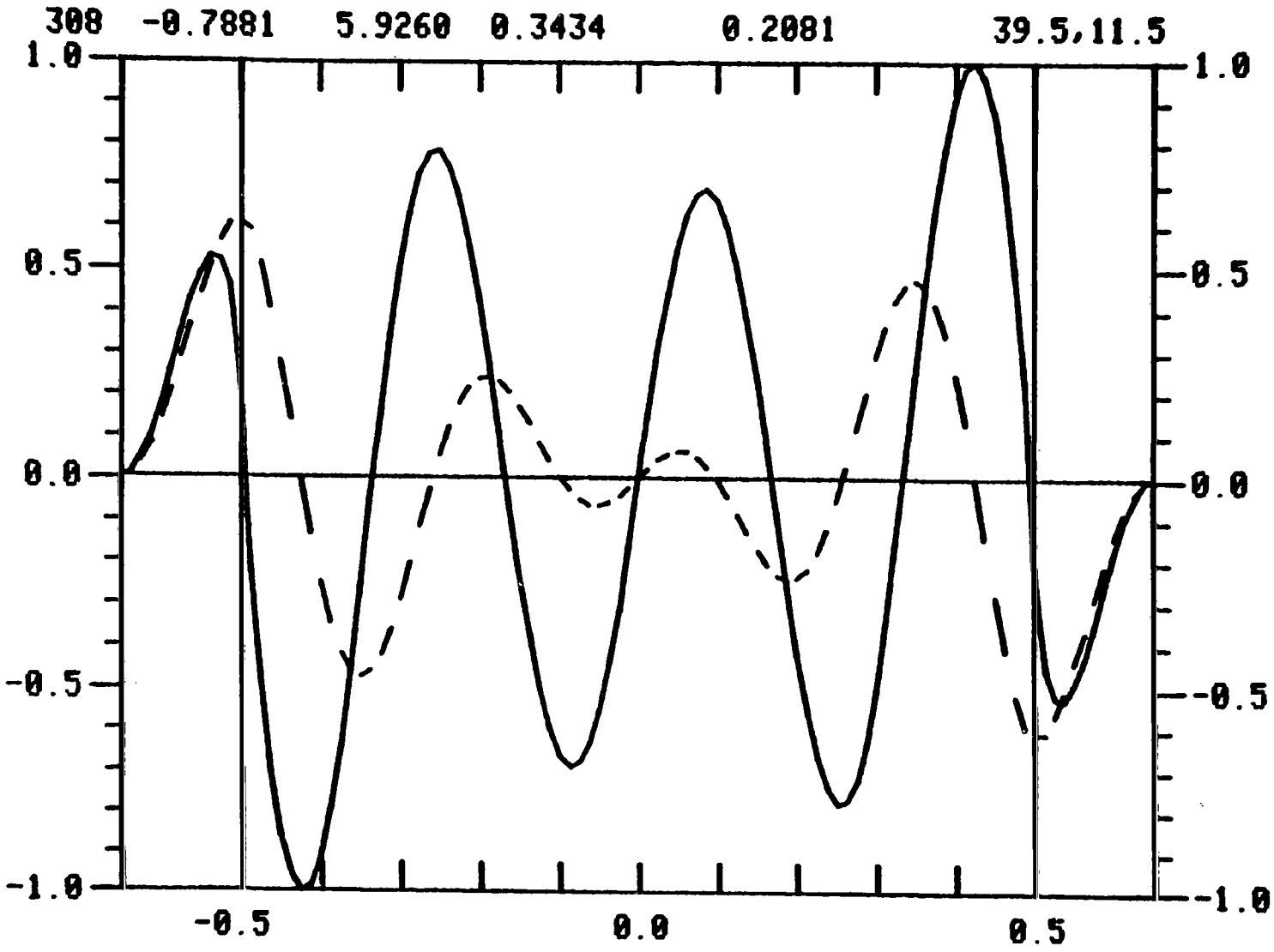




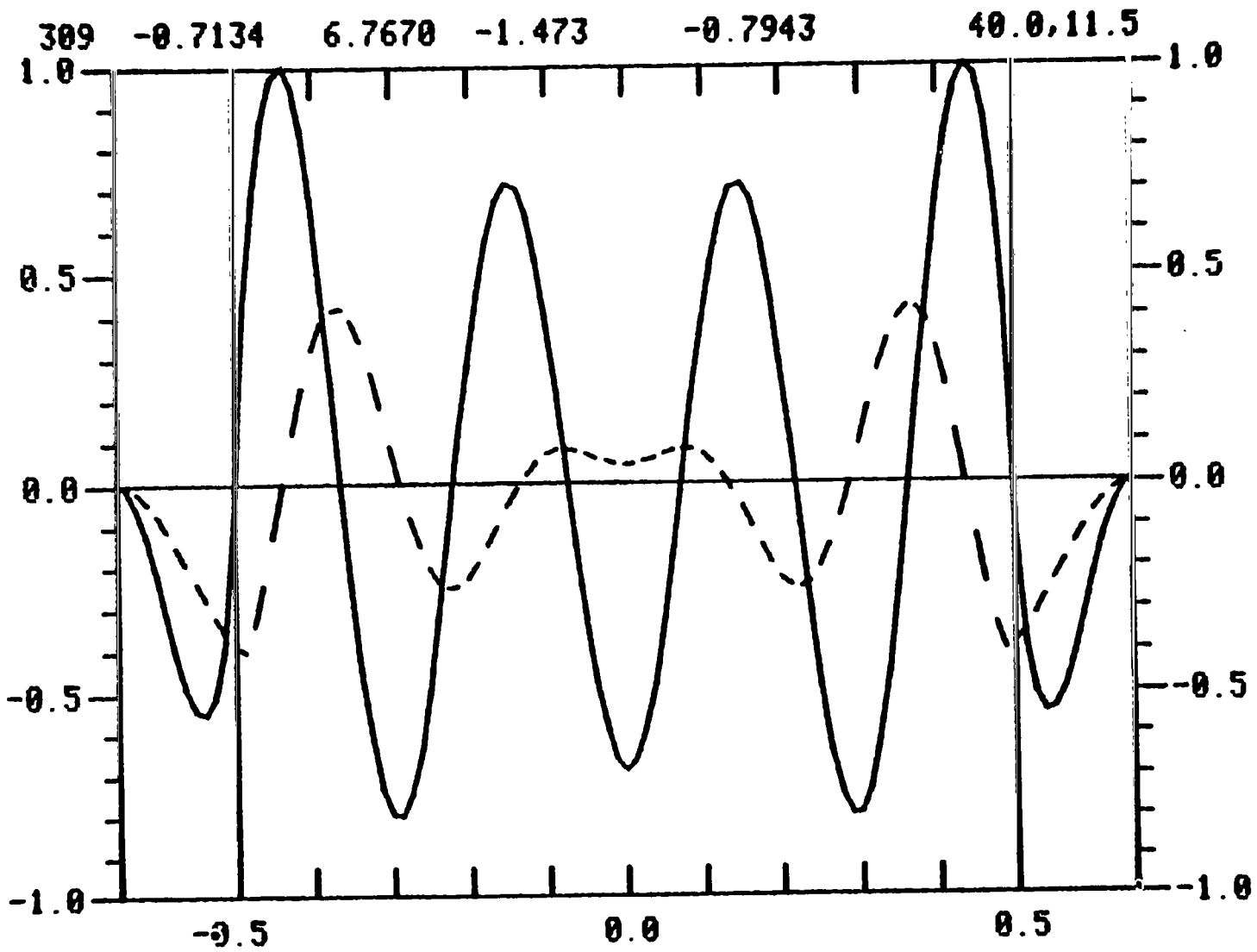


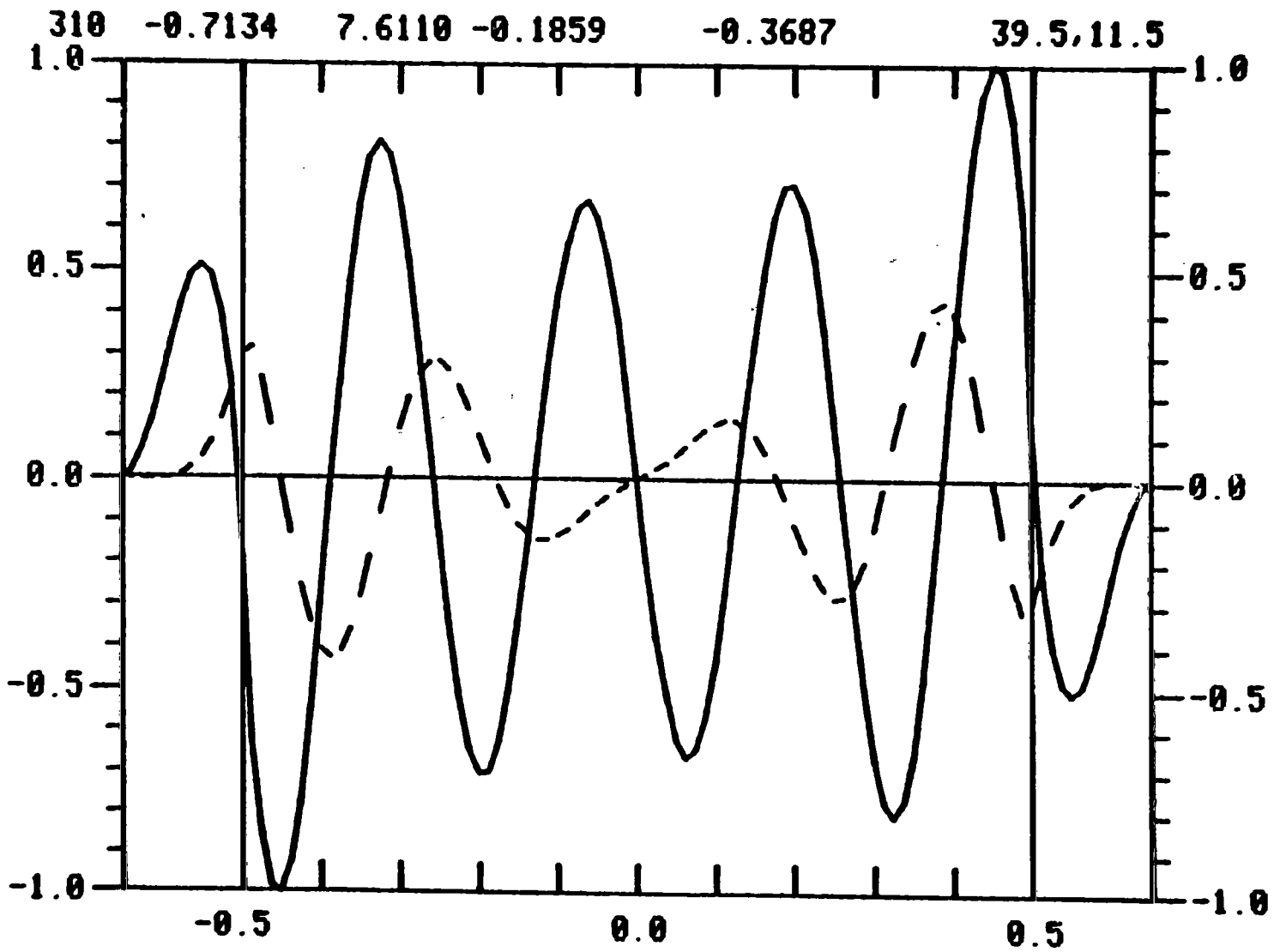
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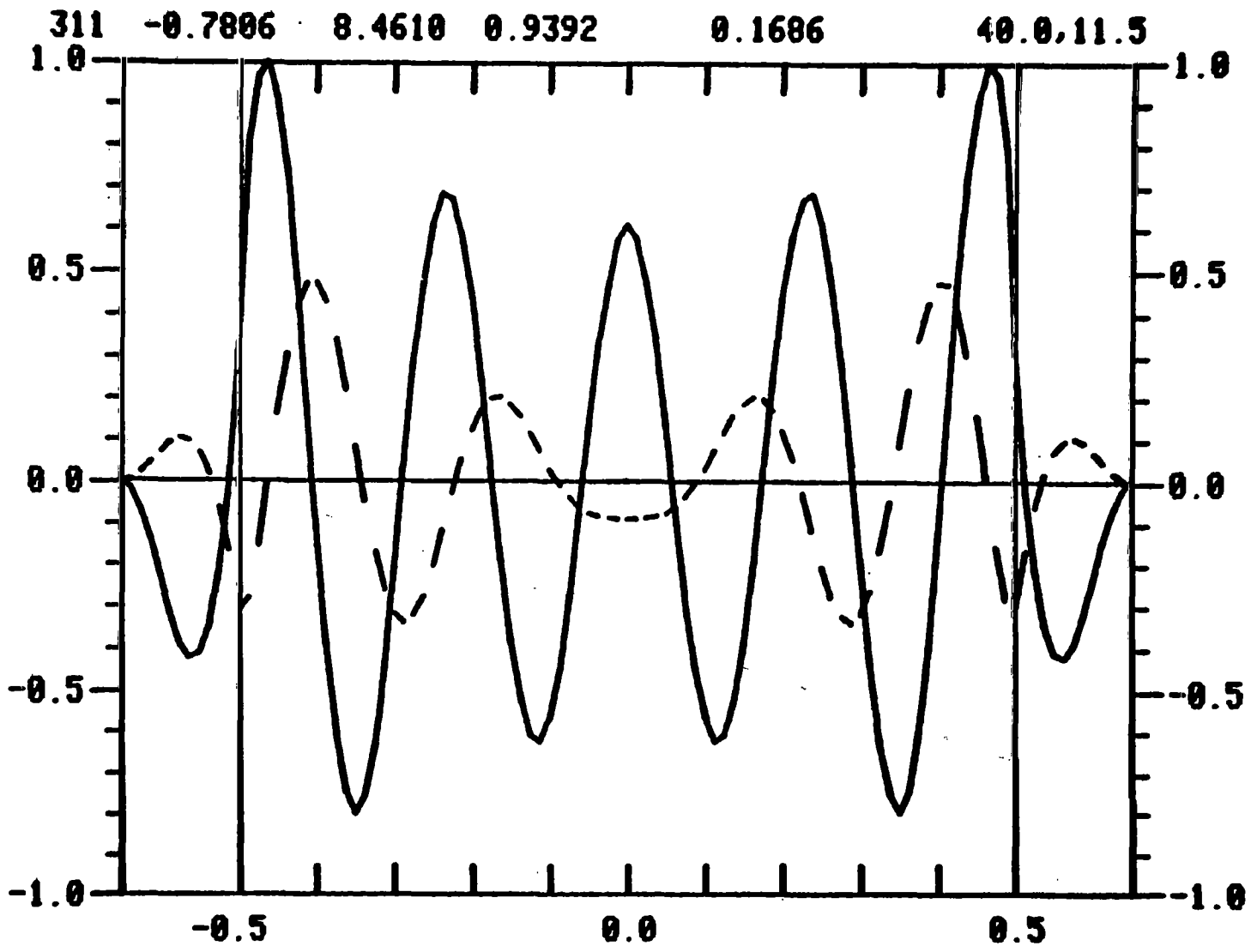




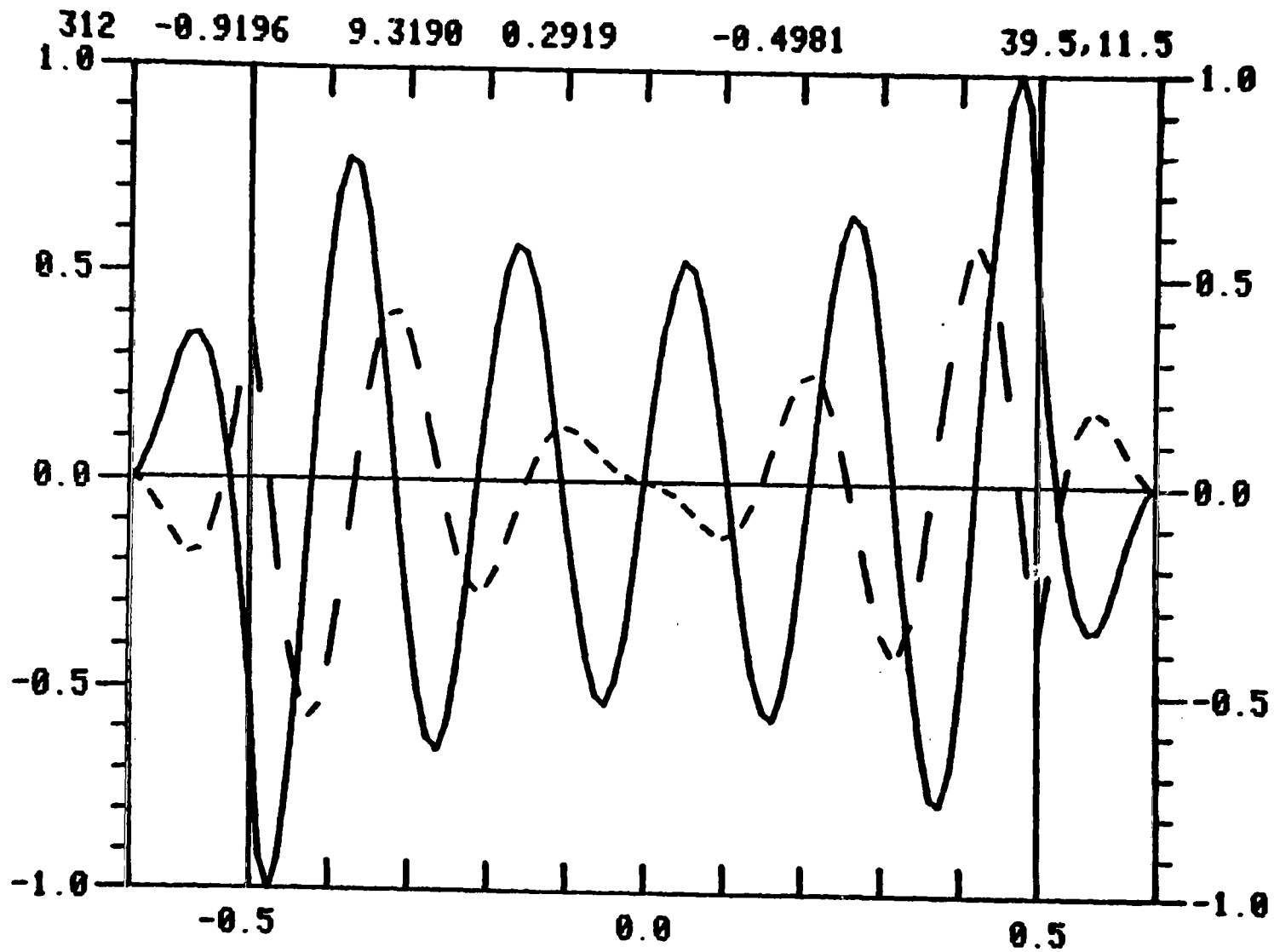
72







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III. SEM QUANTITIES FOR FIRST ORDER AZIMUTHAL VARIATION

First order current density, natural modes are presented in this section. Poles, natural modes, and normalization constants are given for the first four dominant resonances of a cylinder, whose aspect ratio (length/width) is each of the following: 100,50, 20,10,5, and 3.3333.

The real and imaginary parts of the natural modes are graphed. Each mode is represented by a pair of graphs. One graph shows the real and imaginary parts of the t-directed current density; the other graph is of the real and imaginary parts of the θ -directed current density. The pole (s), normalization constant (B), the pole index (M), the number of zones in the z-direction (NZ) on half of the structure, the number of zones in the p-direction (NR) on half of the structure, and the argument of the natural mode normalization (NORM ANG), are displayed with each pair of graphs.

The pole (s) and the normalization constant (B) are written in their real and imaginary parts. The pole index (M) and the number of zones (NZ) and (NR) have the same meanings as in the zeroth-order variation case: the last digits of the pole index specify pole number and the preceding digits correspond to aspect ratio, the number of

zones in the z-direction on half of the structure and the number of zones in the p-direction on half of the structure indicate the number of natural mode points. If the pole index is odd there are $2*(NZ+NR)-1$ points in the t-direction and $2*(NZ+NR)$ points in the θ -direction; if the index is even there are $2*(NZ+NR)+1$ points in the t-direction and $2*(NZ+NR)$ points in the θ -direction. For a given aspect ratio there are the same number of natural mode points for each pole. The poles are normalized to L/c , as in the zeroth order case. The natural modes are, however, normalized with respect to their symmetric product as indicated in [3] rather than with respect to their peak magnitudes as in the zeroth-order case. The quantity labeled "NORM ANG" indicates the argument of the H_{mi} of [2] for each natural mode. The normalization constants B_{mi} were computed based on the natural modes after they had been normalized. Therefore, $\text{ang}\{H_{mi}\}$ does not bear on expansions in terms of the data presented here. It is included in the data record solely for the purpose of interpretations of the numerical process used in computing the data which might conceivably be desirable in the future.

A complete listing of each pole, and natural mode is included here. The companion Tektronix 4050-Series tape is formatted in the following form:

File 1	Table of Contents
File 2	Aspect Ratio 100
File 3	Aspect Ratio 50

File 4	Aspect Ratio 20
File 5	Aspect Ratio 10
File 6	Aspect Ratio 5
File 7	Aspect Ratio 3.3333

Each file contains all poles, normalization constants, and natural modes calculated for that given aspect ratio. Each record for a given pole is presented in the following form: Pole Index, Pole Location, Normalization Constant, Number of Zones on half of the sidewall (NZ), Number of Zones on a cap (NR), NORM Ang, and the Tabulated Natural Mode, where the number of natural mode points equals $4*(NZ+NR)-1$. The Fortran format statements to read the data in transferring it from tape via the Tektronix communications interface are for the Pole Index, Pole, Beta, NZ, NR, NORM ANG.:

```
FORMAT(1X,I5,1X,4G12.4,6X,F4.1,1X,F4.1,6X,F6.2)
```

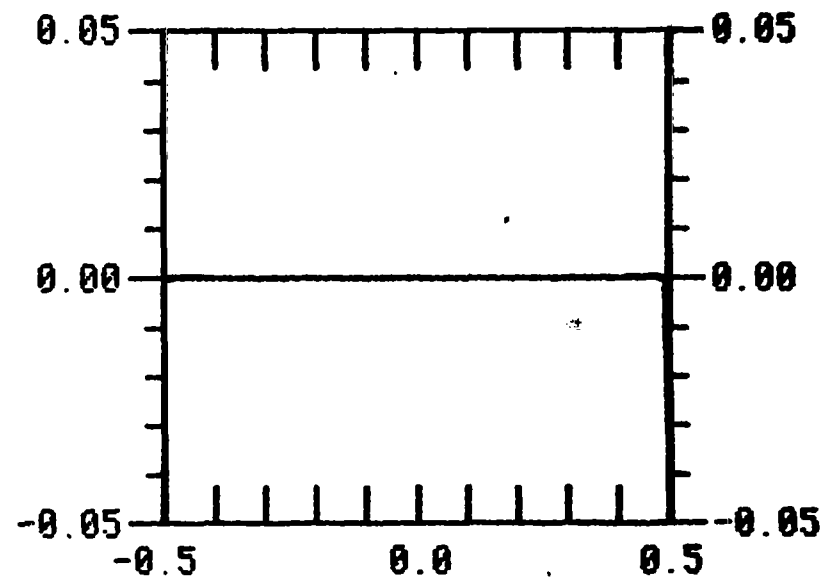
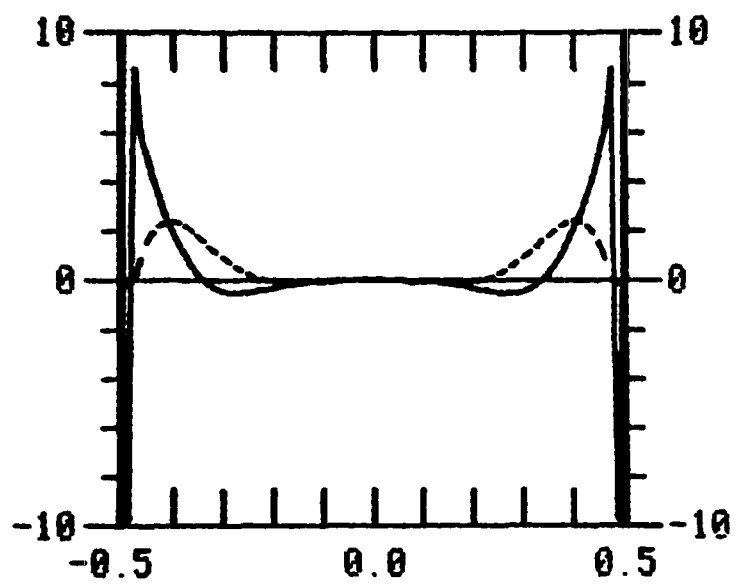
For the Natural Modes:

```
FORMAT(4(1X,2E12.6))
```

INDEX	POLE	NORMALIZATION	NZ	NR
10001	-2.864	.9007	.5619E-05 .9635E-05	39.5 1.5 88.24
10002	-2.828	1.812	-.3661E-08-.8122E-08	40.0 1.5 87.09
10003	-2.783	2.740	.0000E+00 .0000E+00	39.5 1.5 1.51
10004	-2.732	3.686	.1375E-08-.4295E-08	40.0 1.5 86.52
5001	-2.462	.8850	.1171E-05 .1913E-05	39.5 1.5 87.09
5002	-2.418	1.782	-.6747E-08-.2232E-07	40.0 1.5 89.95
5003	-2.366	2.707	-.2807E-04 .1227E-05	39.5 1.5 89.95
5004	-2.310	3.651	.3080E-08-.6295E-08	40.0 1.5 85.94
2001	-1.933	.8478	.9248E-04 .1965E-03	39.5 1.5-89.95
2002	-1.883	1.729	-.5806E-08-.7511E-07	40.0 1.5-89.38
2003	-1.816	2.646	-.4464E-03-.3176E-04	39.5 1.5-89.38
2004	-1.749	3.589	.1969E-07-.2411E-07	40.0 1.5-88.81
1001	-1.549	.8126	-.2718E-05-.1470E-05	39.5 3.5 89.38
1002	-1.477	1.684	.6301E-08-.4590E-07	40.0 3.5 89.38
1003	-1.381	2.592	-.3465E-06-.9263E-06	39.5 3.5 88.81
1004	-1.288	3.516	.1275E-07-.1058E-07	40.0 3.5 88.81
501	-1.179	.7820	-.6480E-05-.3648E-05	39.5 7.5 89.38
502	-1.035	1.617	.1342E-07-.2872E-07	40.0 7.5 88.81
503	-.8936	2.447	-.1852E-06 .5245E-06	39.5 7.5 89.38
504	-.8050	3.283	.8464E-08-.4380E-08	40.0 7.5-89.38
301	-.9492	.7693	.1445E-07-.1892E-07	39.5 11.5 88.24
302	-.7486	1.507	.1412E-07-.1849E-07	40.0 11.5 88.81
303	-.6430	2.247	-.1577E-04 .1068E-04	39.5 11.5-89.38
304	-.6195	3.024	.3819E-08-.1651E-08	40.0 11.5-79.64

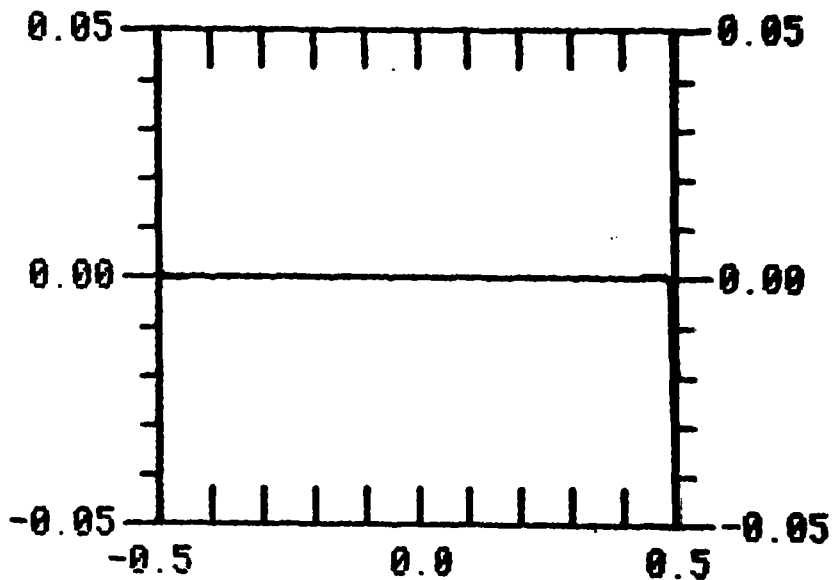
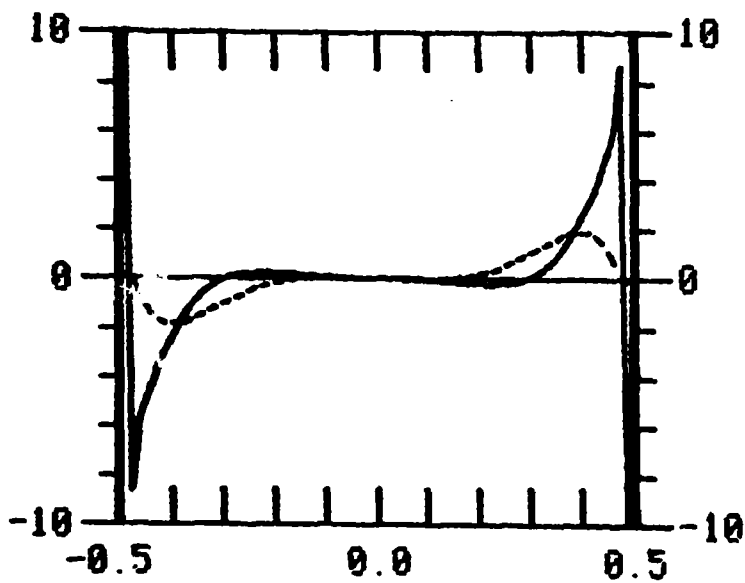
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81



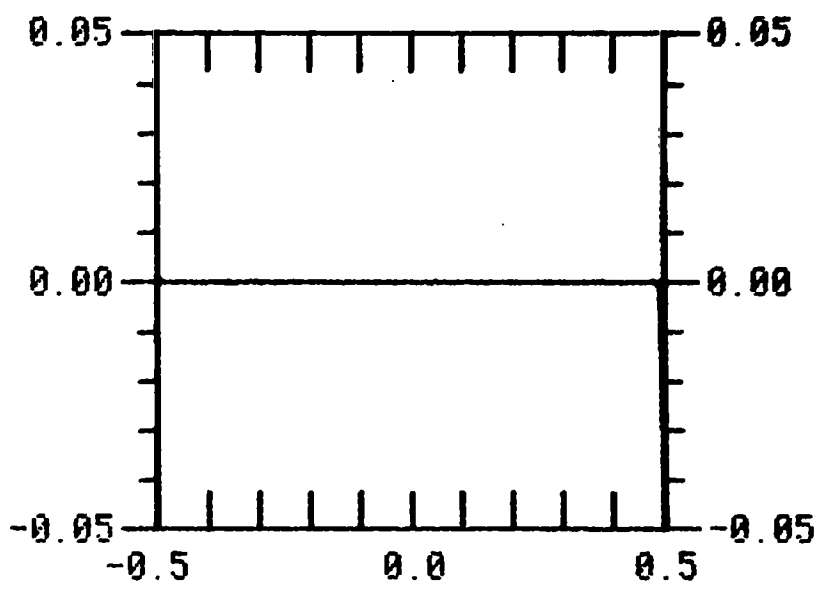
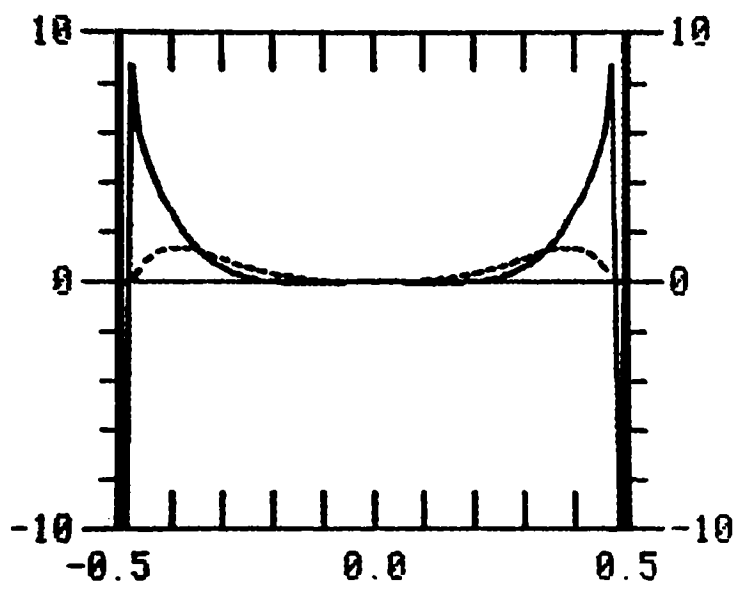
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82

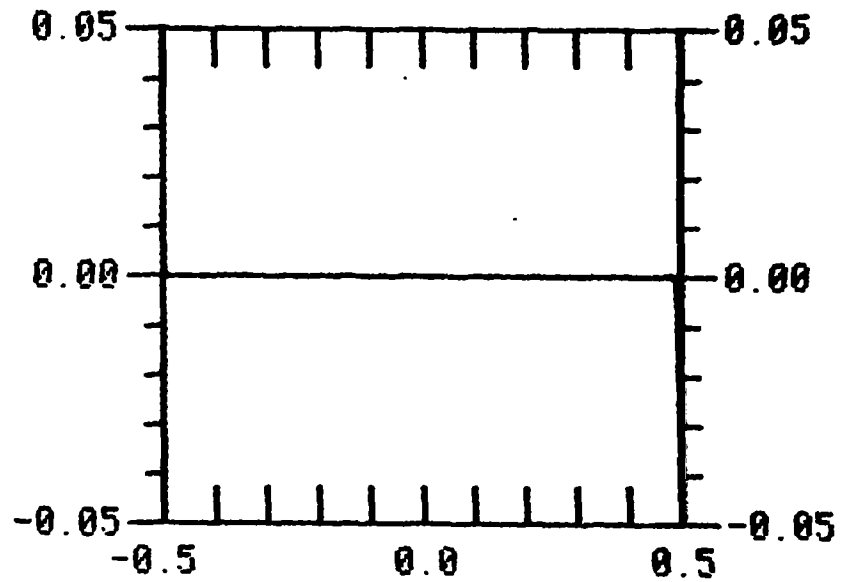
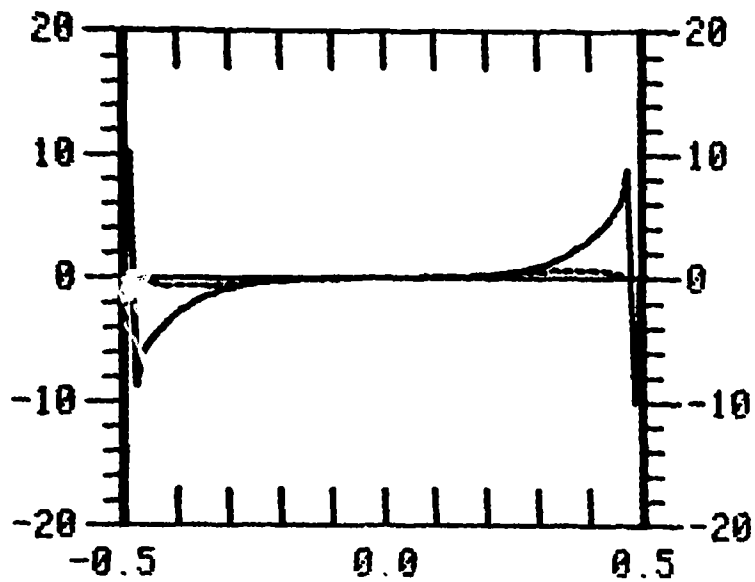


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83

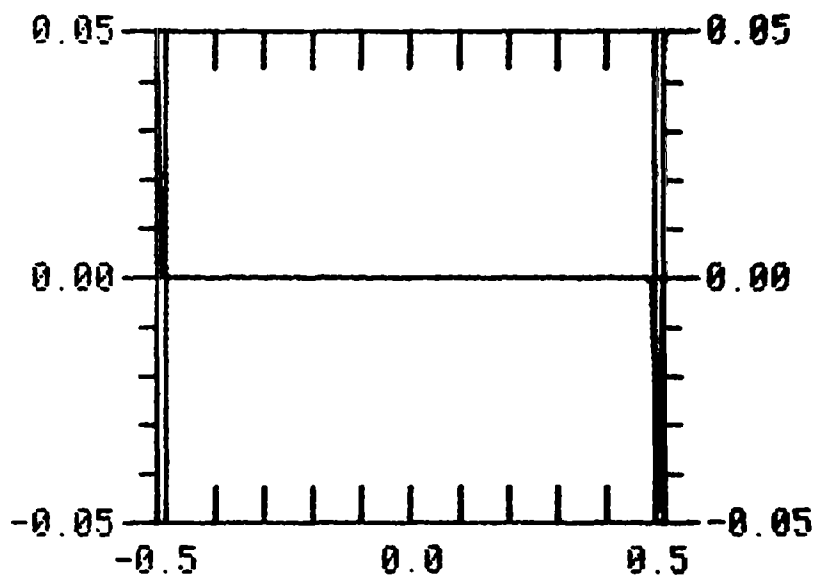
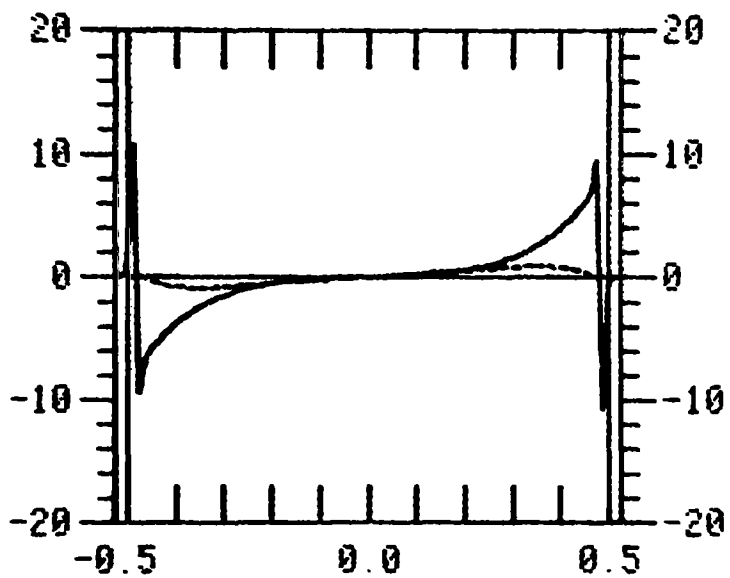


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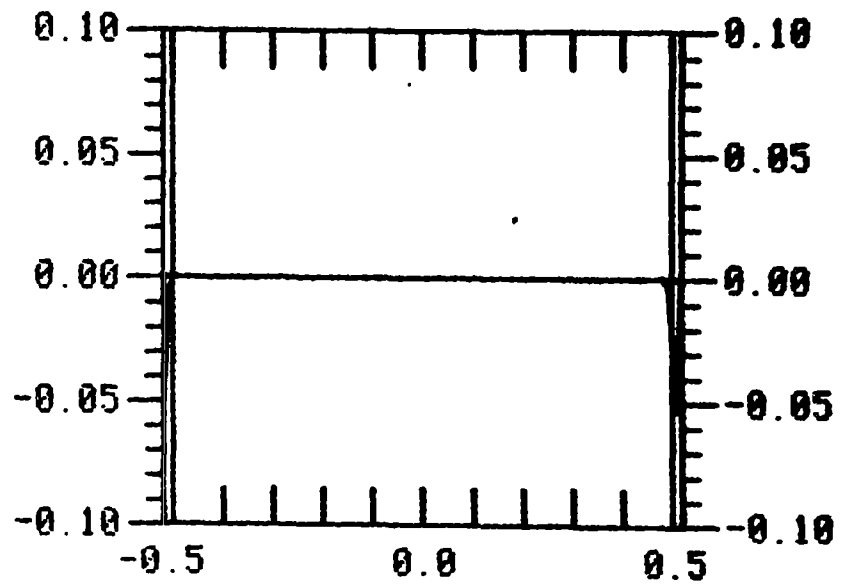
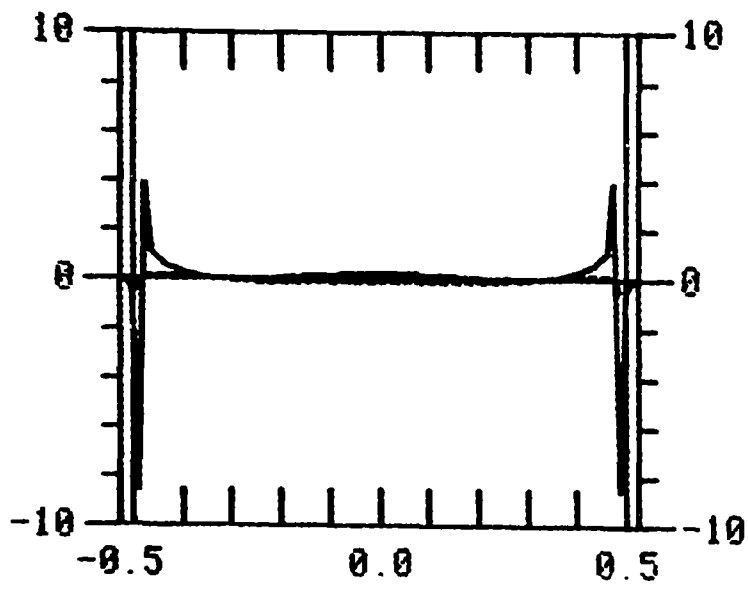
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85



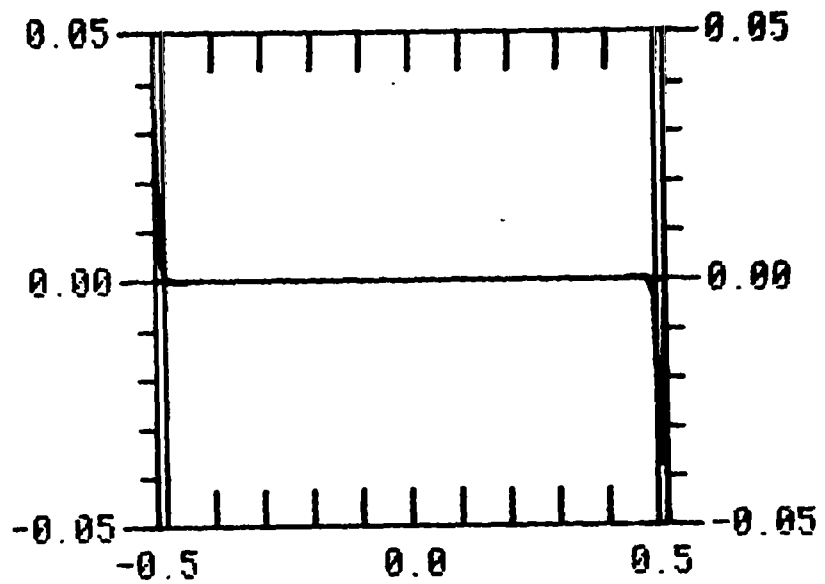
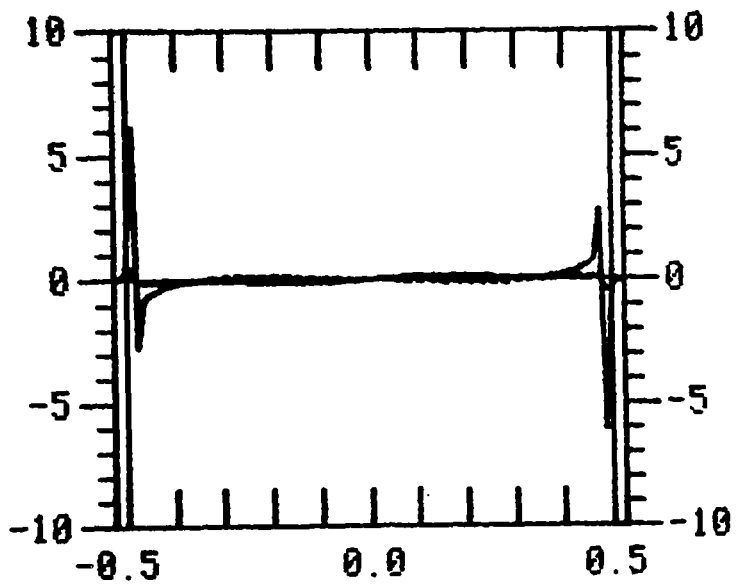
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98



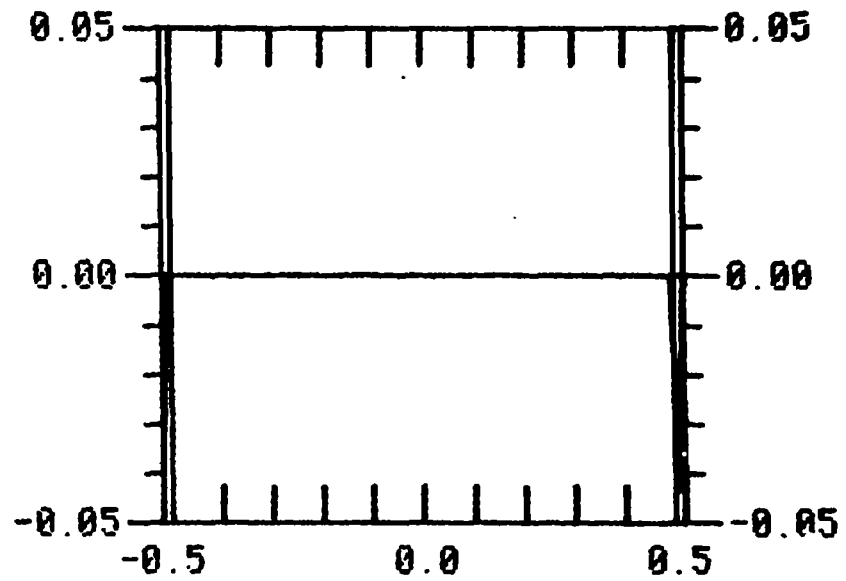
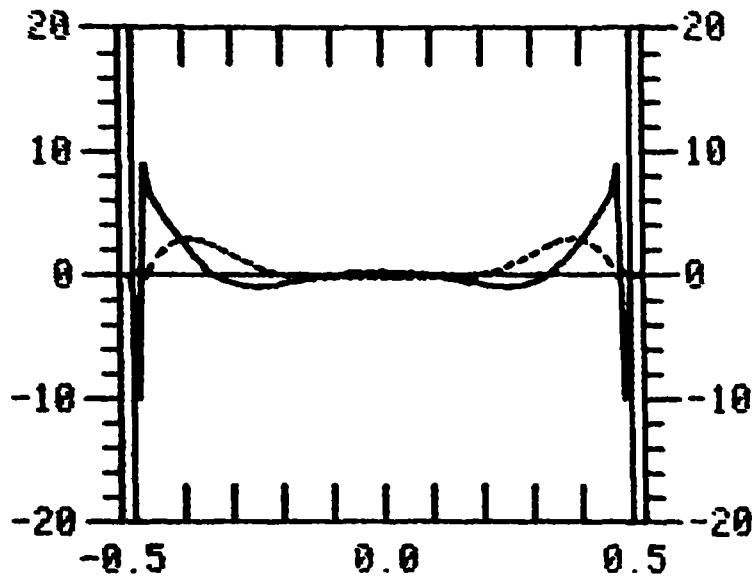
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87



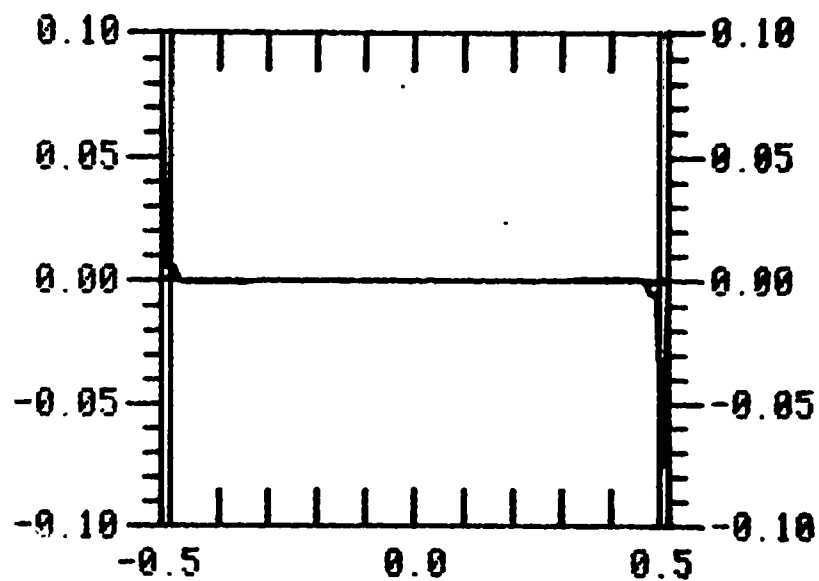
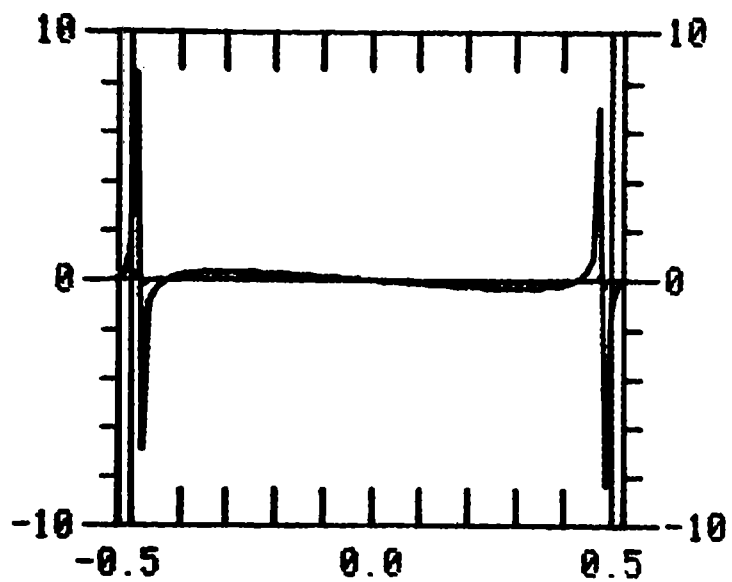
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88



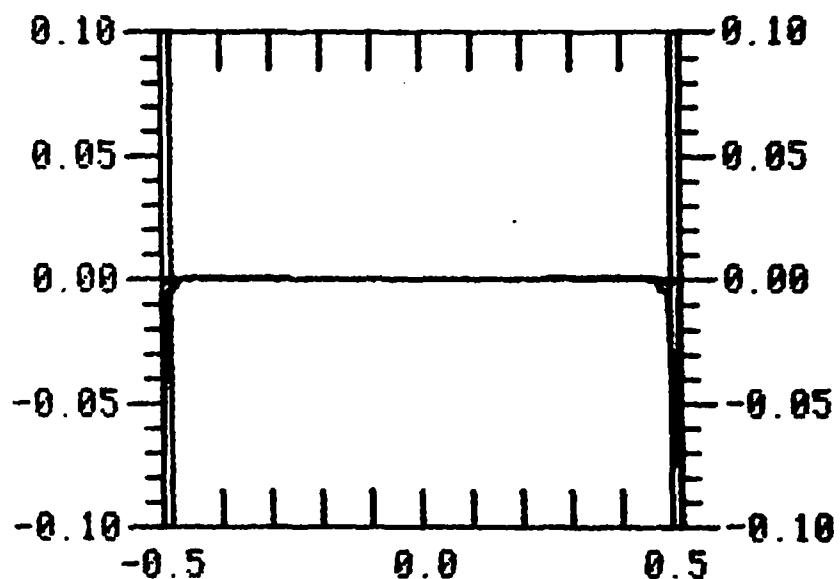
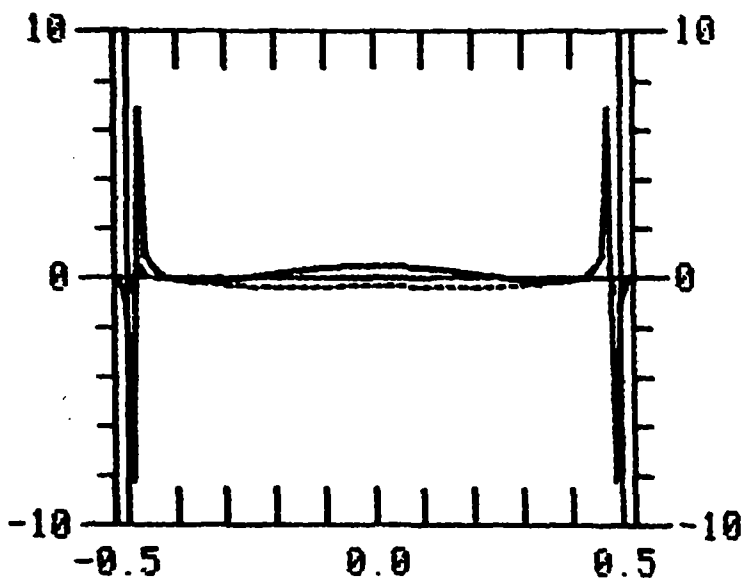
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68



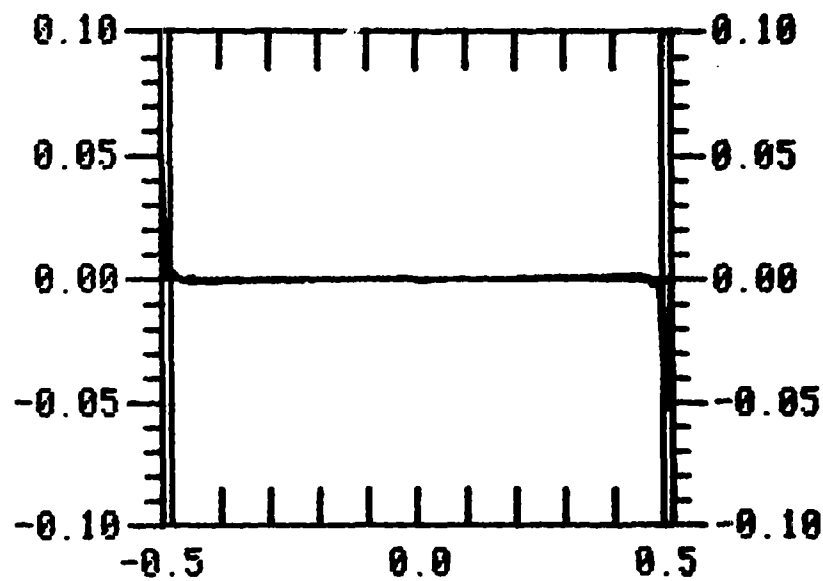
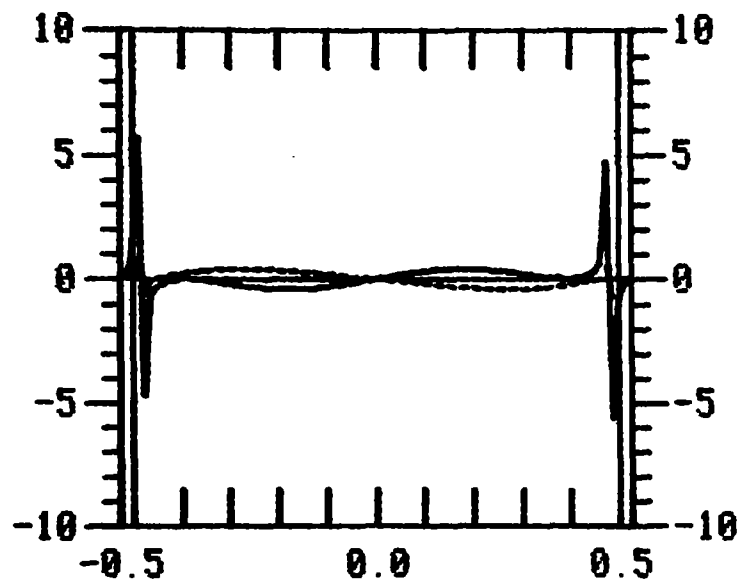
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06

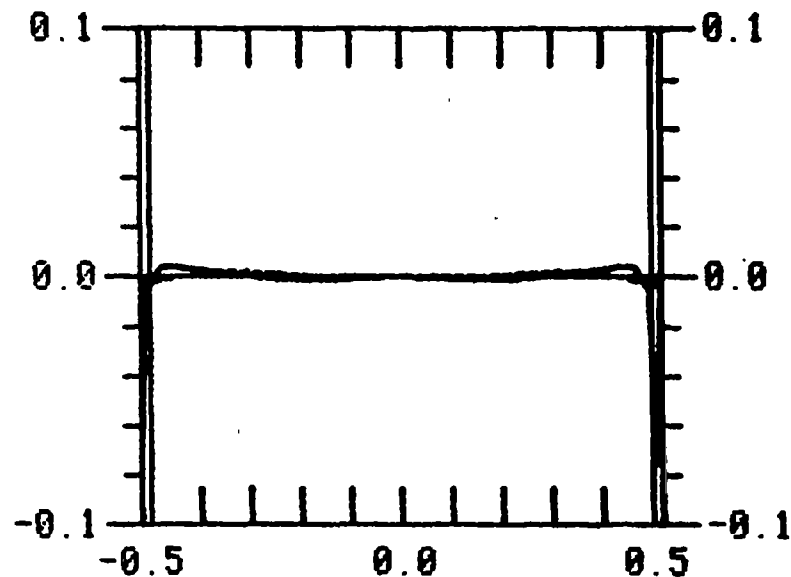
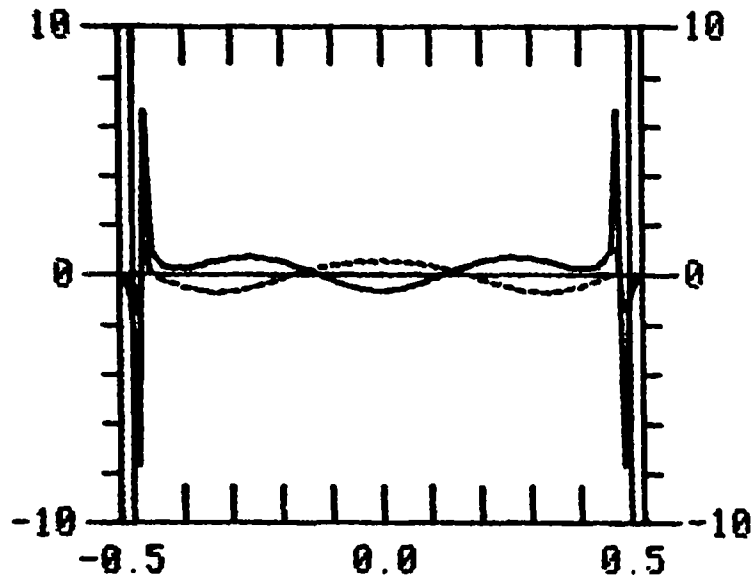


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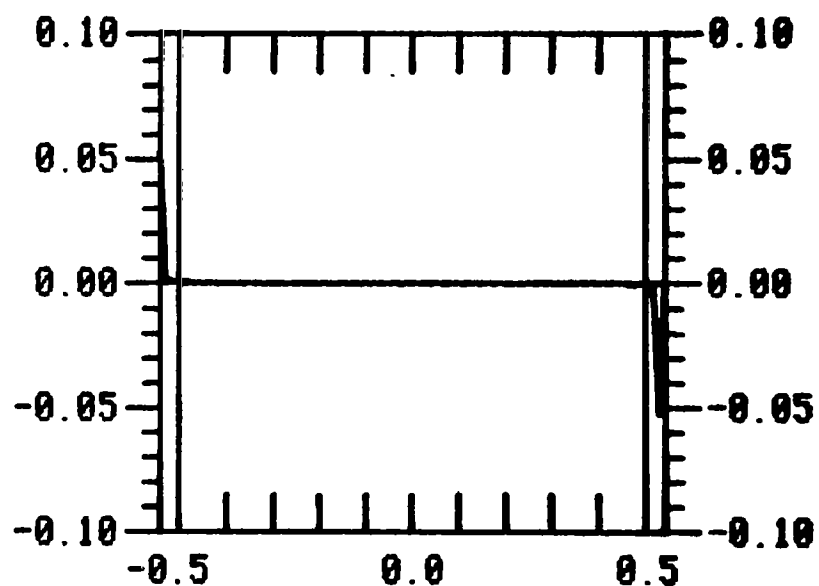
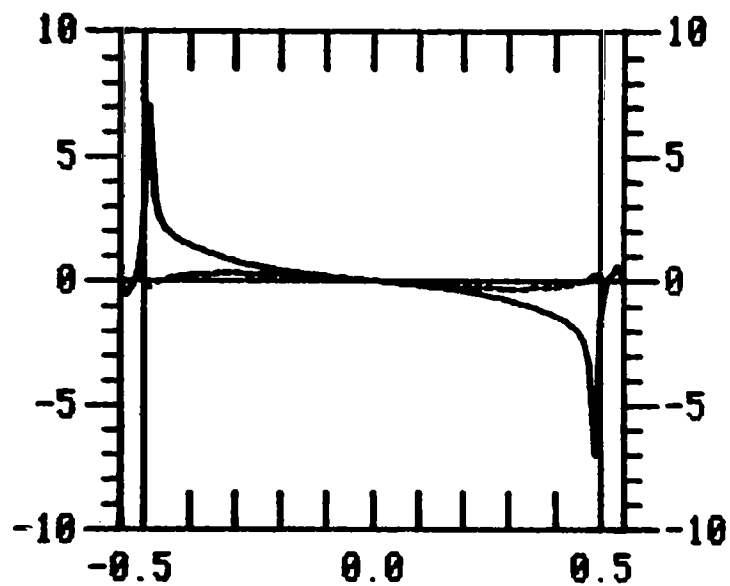
16



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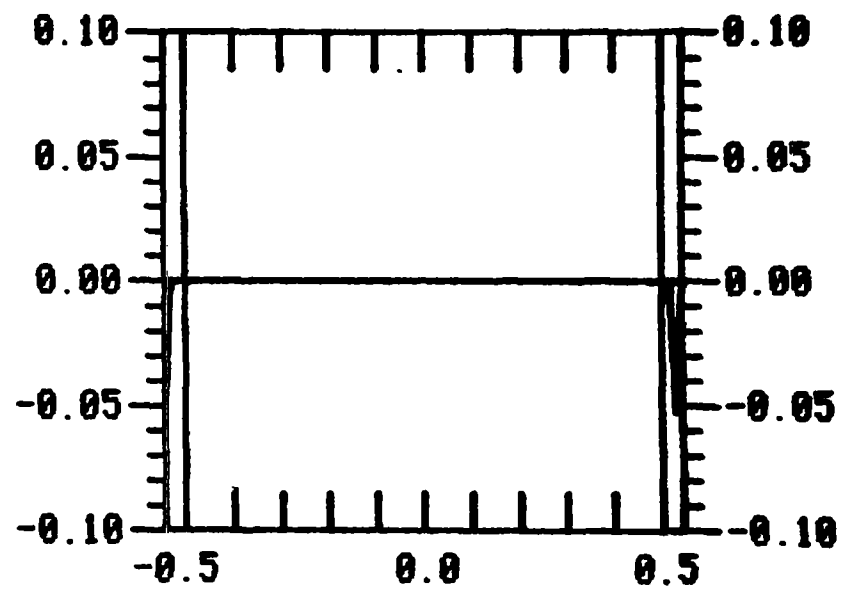
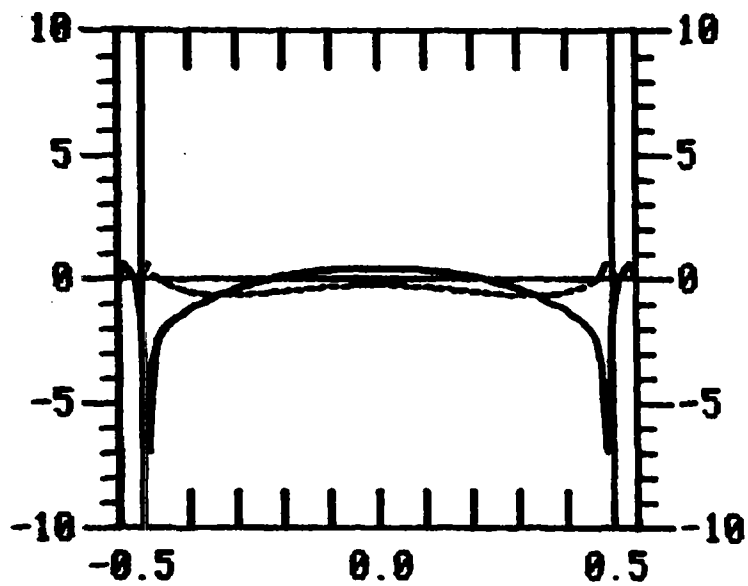


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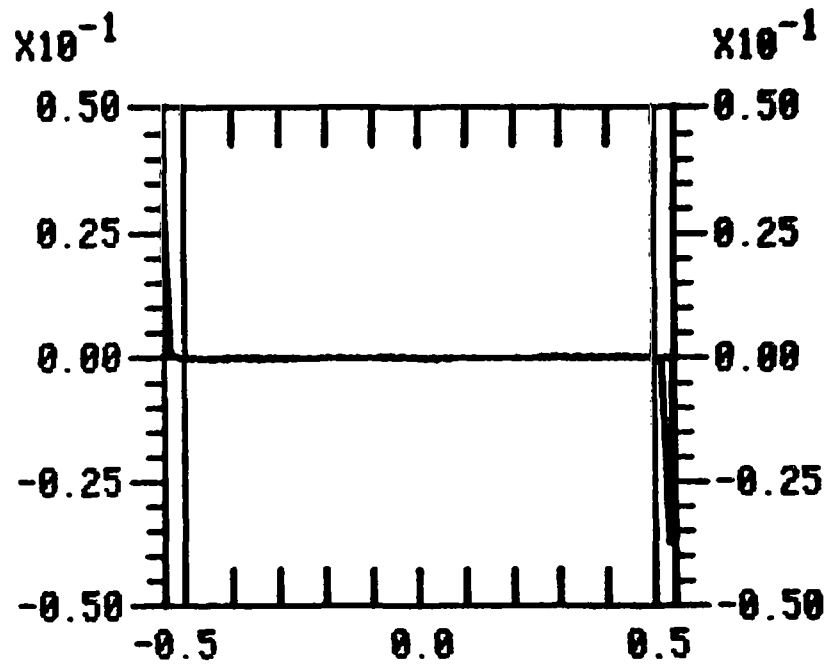
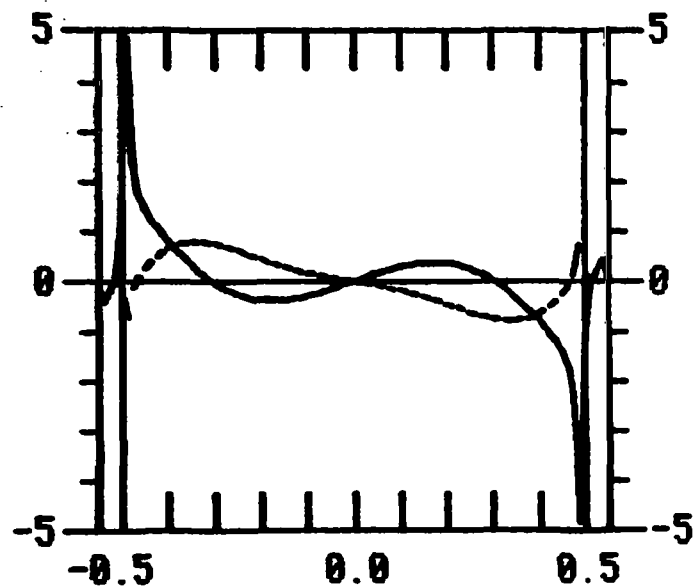
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94



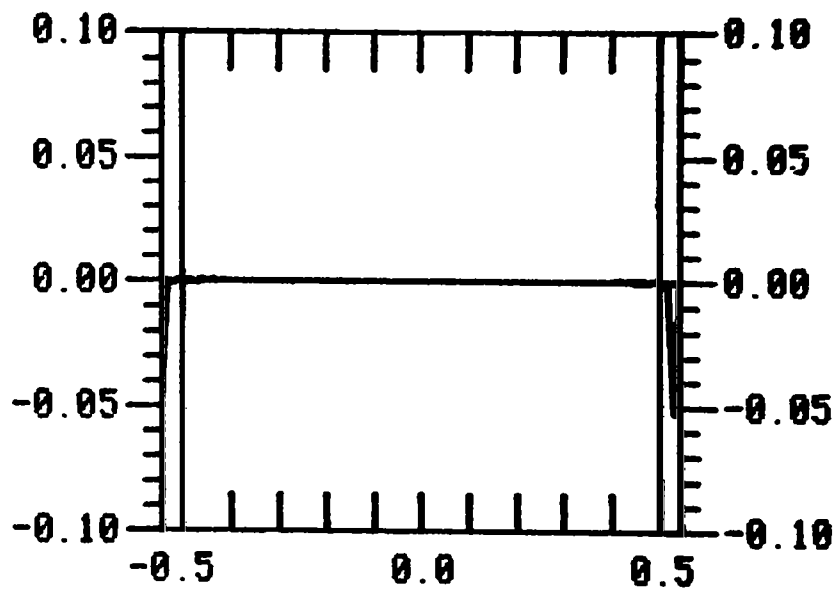
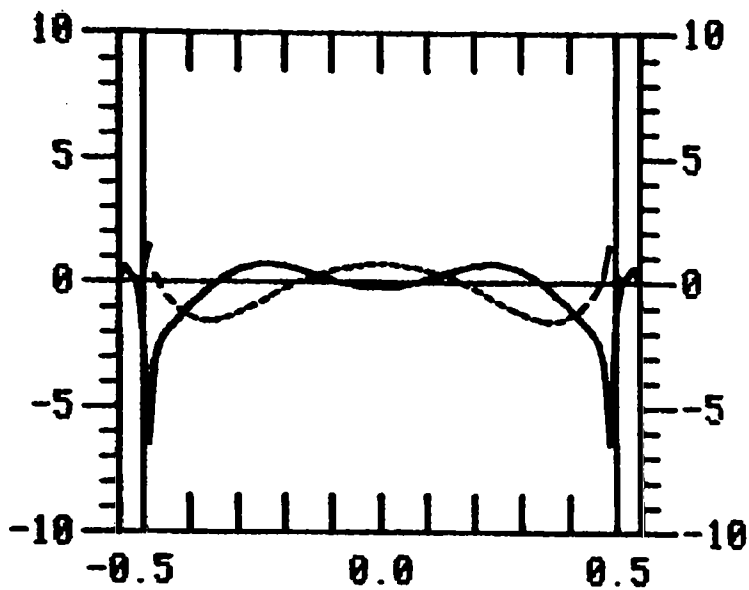
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95

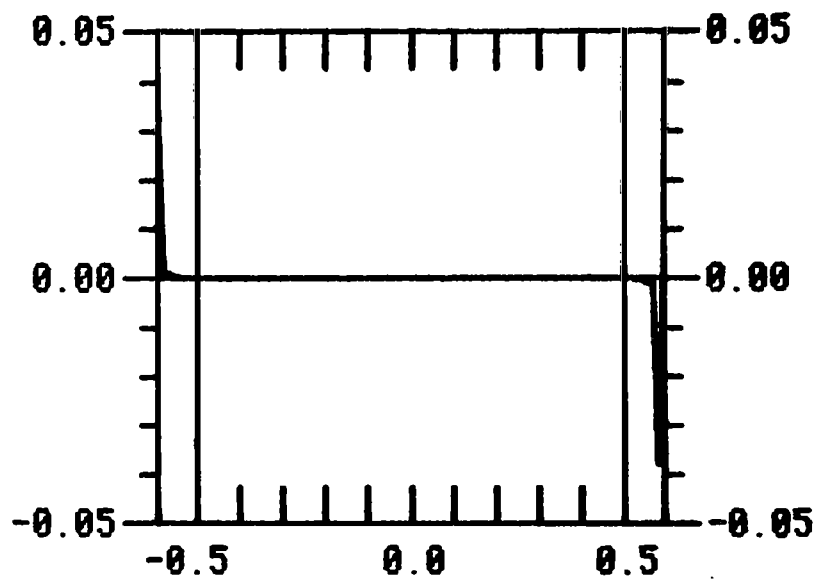
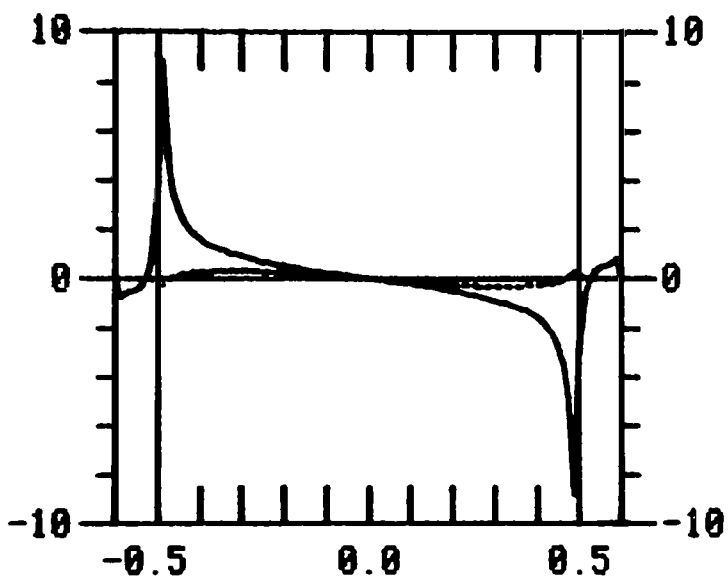


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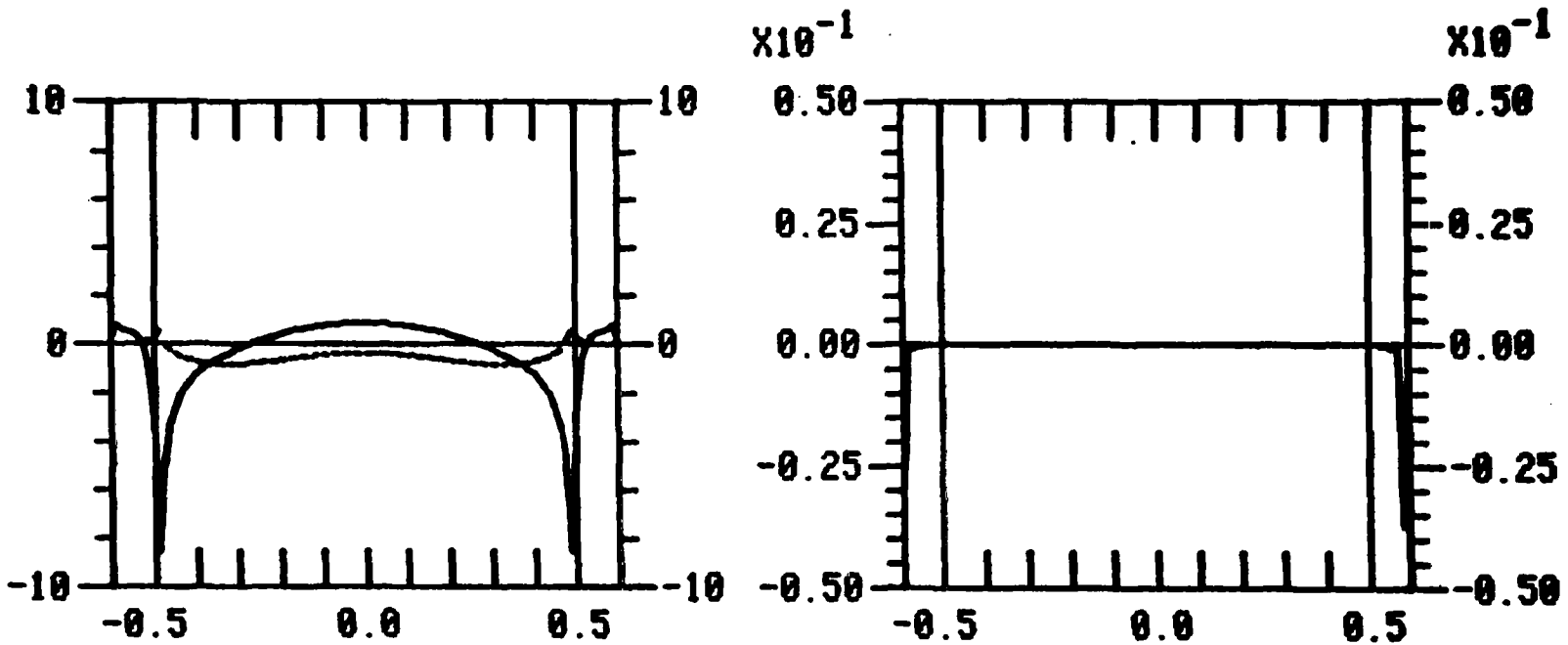
96



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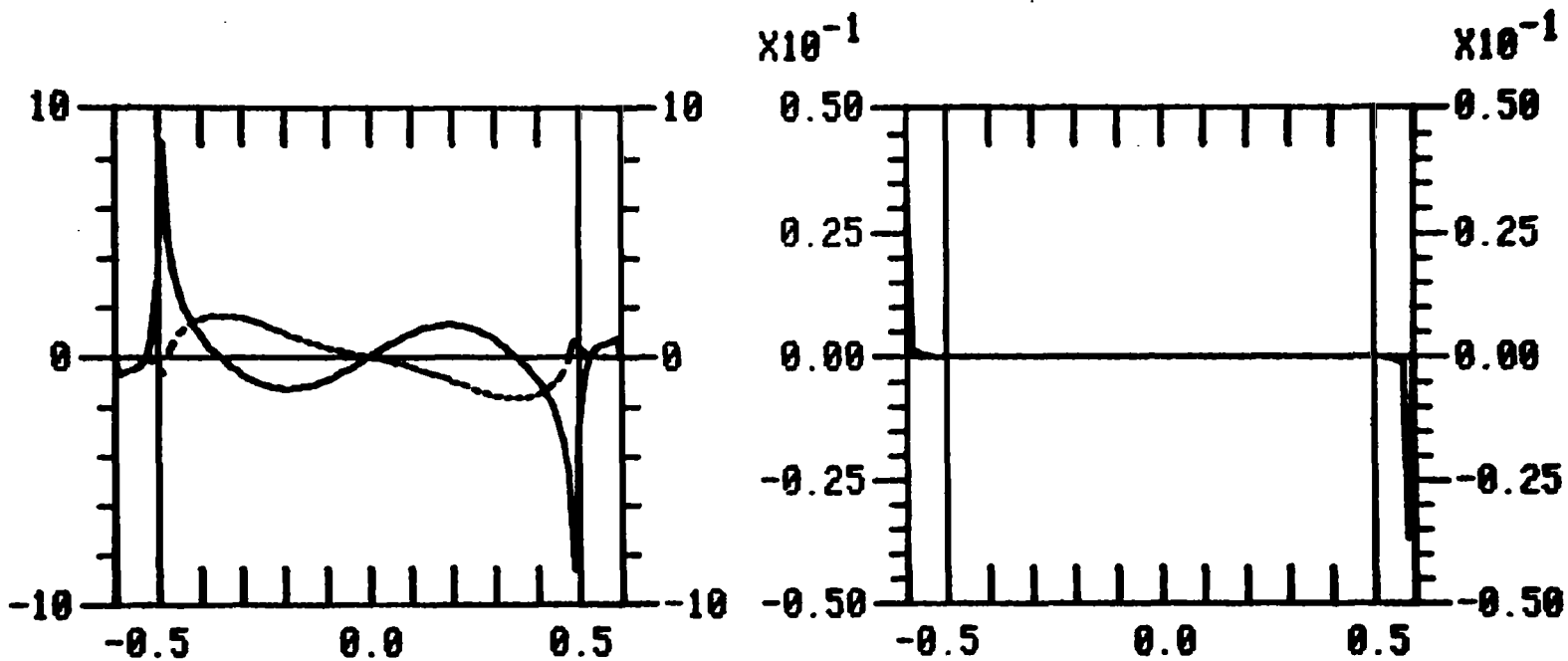


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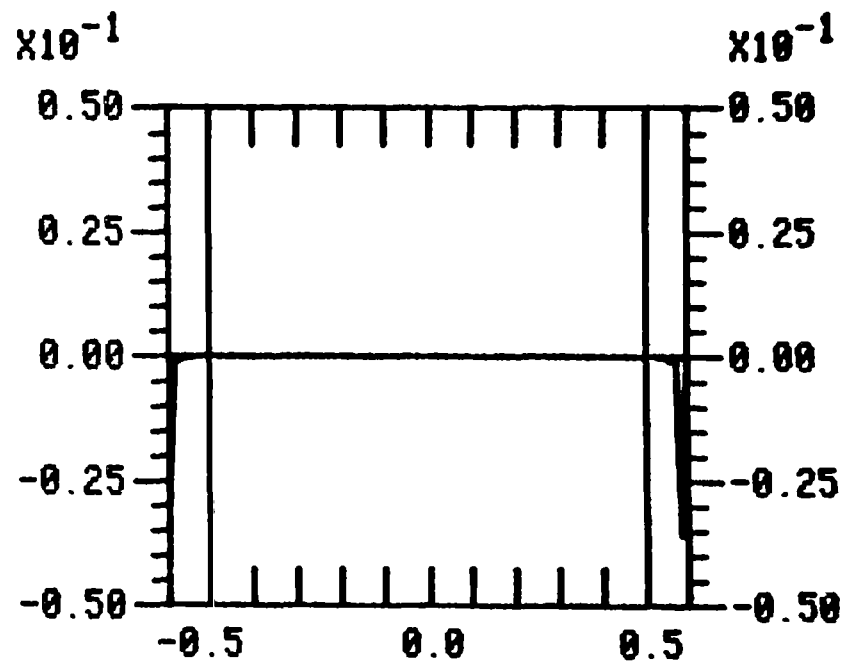
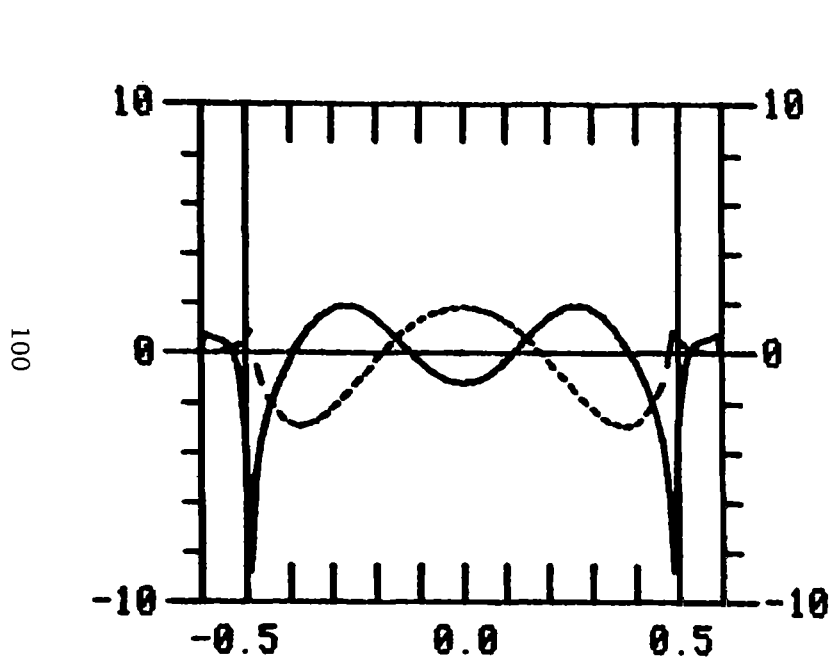


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66

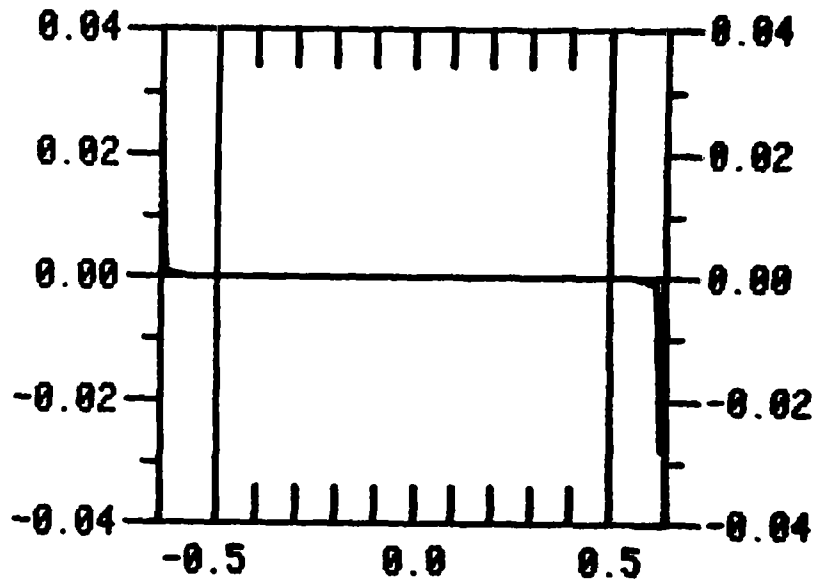
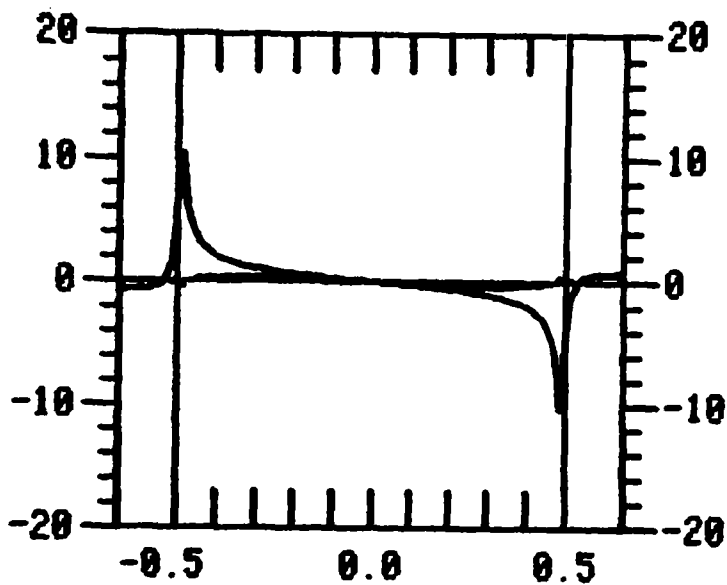


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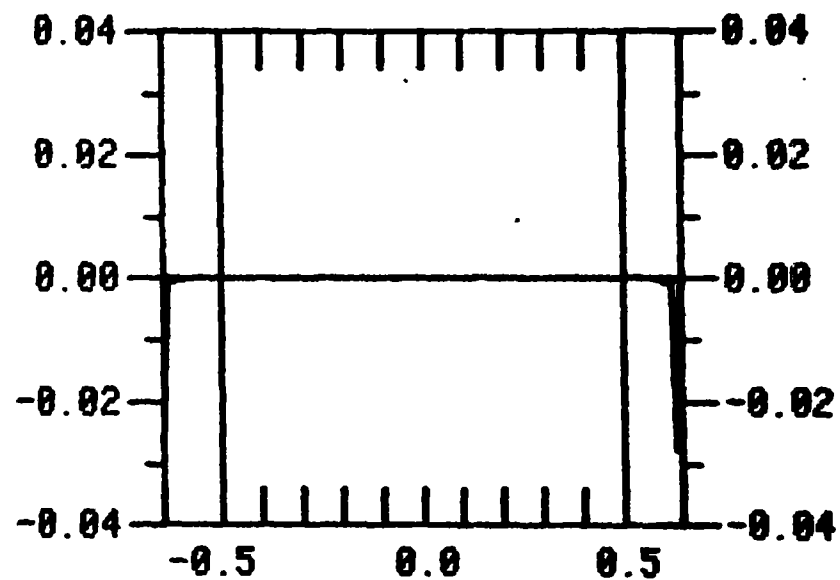
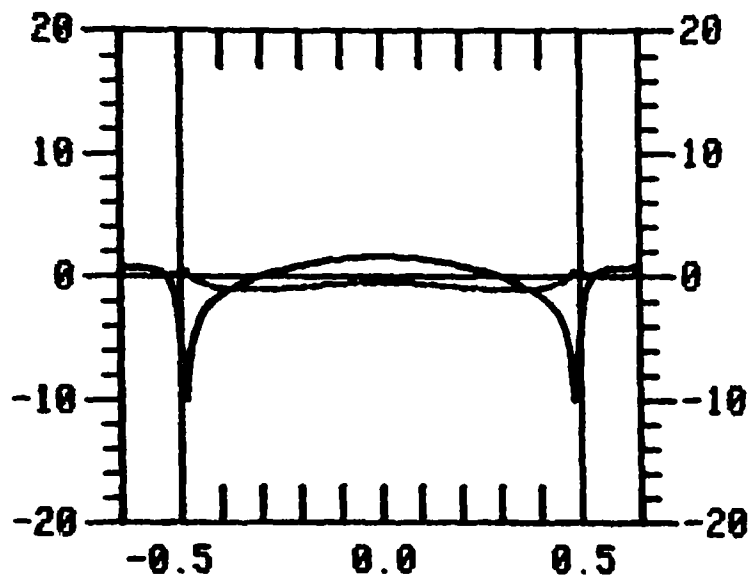
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101



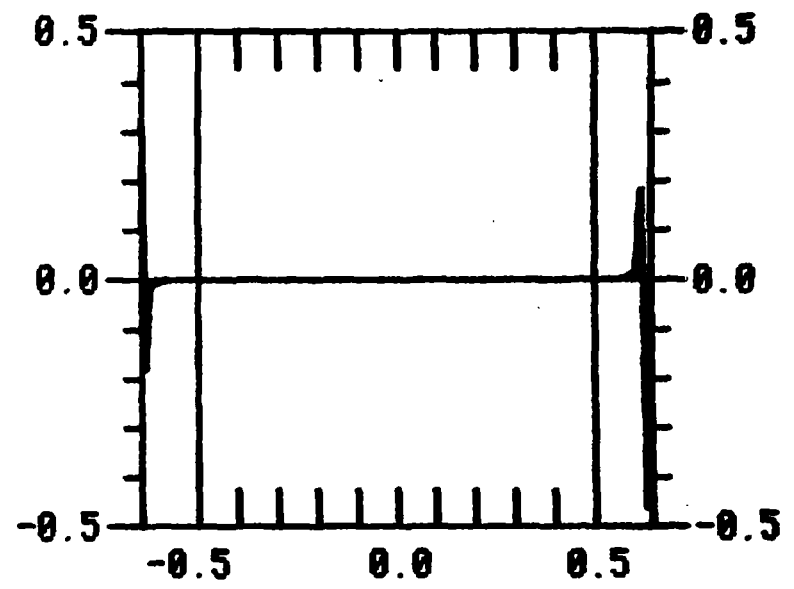
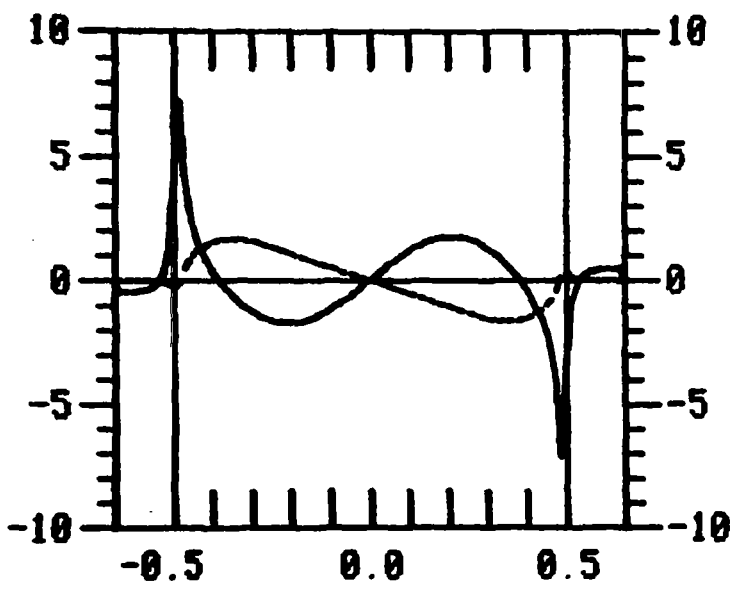
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102



303 -0.6430 2.2470 -0.1577E-04 0.1068E-04 39.5,11.5 -89.38

103



304 -0.6195 3.0240 0.3819E-08 -0.1651E-08 40.0,11.5 -79.64

104

