Interaction Notes

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Discrimination of Buried Targets Via the Singularity Expansion

Carl E. Baum
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Abstract

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1. Introduction

A problem of significant current interest concerns the detection and identification of unexploded ordnance (UXO), whether mines or dud rounds [35]. This exists on many old battlefields around the world, and on many military bases (including those being converted to civilian use). While some UXOs may lie on the surface and be comparatively easy to identiffy (and remove or destroy), others may be burried out of sight under the ground or water surface at various depths. This latter case presents a much more difficult problem.

A commonly used technique for detecting such buried UXOs is the measurement of the distortion of the earth's magnetic field by sensitive magnetometers (including gradiometers). However, this applies only to ferrous targets. Inductive metal detectors (continuous wave (CW) at some frequency, or pulsed) are also used and are sensitive to metal targets in general. As one can see, this leaves out plastic mines in general. Furthermore, detection is not the limiting problem. The above (and other) techniques primarily only detect the presence (and to some degree, location) of some "targets" without indicating whether or not these are of significance for removal. In other words, the false alarm rate is high, and this has important economic consequences for site cleanup (mitigation). What is needed is techniques for discriminating UXOs from other "targets" such as shrapnel, pipes, rocks, tree roots, and other junk in general, before one digs up the object that has put some blip on our "radar screen".

One can consider this target-identification problem as a general problem in inverse scattering, but such a completely general approach is extremely difficult in the context of real-world soil and water. A related simpler approach relys on target signatures in which we have some finite library of such signatures for specific models of mortar shells, mines, etc., [11, 14, 31, 32]. By a signature is meant here a set of parameters (not too large in number) in a mathematical scattering model. There are various models of interest, for example, for identifying various types of aircraft via a transient (or multi-frequency) radar, different models being appropriate for various regimes of time and frequency.

For the buried UXO problem, we concentrate on the singularity expansion method (SEM) with its aspect-independent natural frequencies (pole locations) in the complex frequency plane. This is divided into three general types for present discussion. Electromagnetic singularity identification (EMSI) is concerned with metal and dielectric targets with wavelengths of the order of the target dimension in the surrounding medium (soil or water). This is appropriate for a special kind of ground-penetrating radar (GPR) operating in the general range of frequencies from roughly 100 MHz to 1 GHz (more or less). Magnetic singularity identification (MSI) is concerned with metal targets with diffusion depths in the metal of the order of the target dimensions (and is insensitive to a non-ferrous external medium). This is appropriate for a special kind of metal detector using coils for transmission and reception, but designed to analyze waveforms for natural frequencies corresponding to pure exponential decays with μs to ms time scales. Acoustic singularity identification (ASI) is like EMSI except that sound waves are employed with important frequencies now in the kHz range. Each of these has its own advantages and disadvantages, and are thus complementary. Theses are all discussed in detail in [17, 21–25].

2. The Singularity Expansion Method

As indicated in fig. 1, let there be an incident plane wave of the form

$$
\begin{aligned}
\tilde{\vec{E}}^{(inc)}(\vec{r},s) &= E_0\tilde{f}(s)\,\vec{1}_p\, e^{-\tilde{\gamma}(s)\vec{1}_i\cdot\vec{r}} \\
\tilde{\vec{H}}^{(inc)}(\vec{r},s) &= \frac{E_0}{\tilde{Z}(s)}\tilde{f}(s)\,\vec{1}_p\, e^{-\tilde{\gamma}(s)\vec{1}_i\cdot\vec{r}} \\
\vec{1}_i &\equiv \text{direction of incidence}, \quad \vec{1}_p \equiv \text{polarization} \\
\sim &\equiv \text{Laplace transform (two-sided) over time } t \\
s &\equiv \Omega + j\omega \equiv \textit{Laplace}-\text{transform variable or complex frequency} \\
\tilde{\gamma}(s) &\equiv \left[s\mu_0(\sigma + s\varepsilon)\right]^{1/2} \equiv \text{propagation constant} \\
\tilde{Z}(s) &\equiv \left[\frac{s\mu_0}{\sigma + s\varepsilon}\right]^{1/2} \equiv \text{wave impedance} \\
\mu_0 &\equiv \text{permeability (free space)} \\
\varepsilon &\equiv \text{permittivity} \\
\sigma &\equiv \text{conductivity} \\
f(t) &\equiv \text{incident waveform}
\end{aligned}
\tag{2.1}
$$

In free space, or in a lossless dispersionless dielectric, the incident wave takes the simple form $f(t - \vec{1}_i \cdot \vec{r}[\mu_0\varepsilon]^{1/2})$, but the above form allows for propagation in more general dispersive and lossy media where the constitutive parameters can even be allowed to be frequency dependent.

As a step in calculating the scattered field, one can calculate the current on a target via an integral equation of the general form

$$
\left\langle \tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}\,'_s; s)\, ;\, \tilde{\vec{J}}_s(\vec{r}\,'_s, s) \right\rangle = \tilde{\vec{E}}^{(inc)}(\vec{r}_s, s) \tag{2.2}
$$

The particular form here is that of the tangential field components (subscript t) on the perfectly conducting body with surface S (the domain of integration over $\vec{r}\,'_s$ in the symmetric product (no conjugation)). The kernel here is an impedance operator, simply related to the dyadic Green's function for the surrounding medium. The above form is generalizable to various other situations with volume current density (with integration over the volume V). Also note that, assuming reciprocity for the target, the kernel is symmetric (equals its transpose).

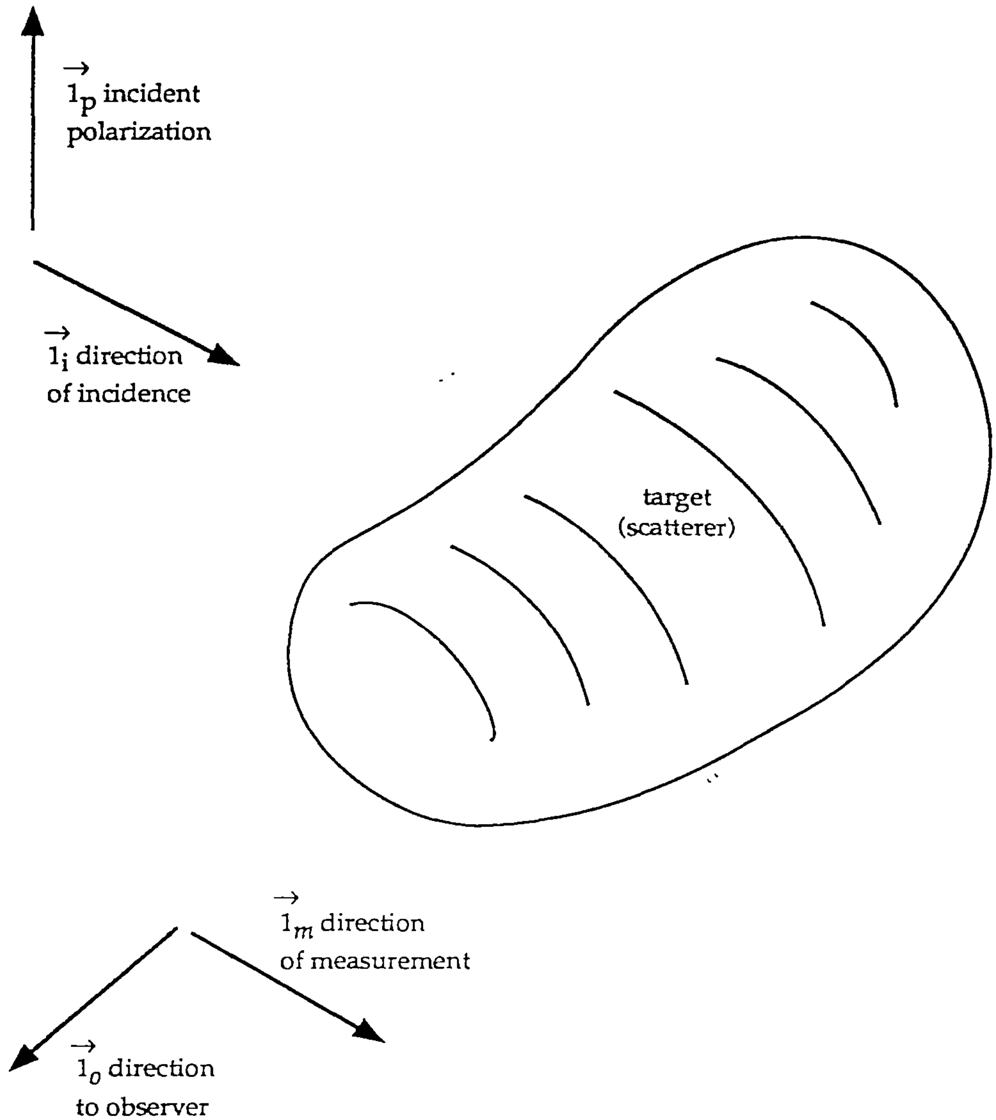

Fig. 1. Scattering of an Incident Wave By a Target

The details of SEM are found in many references [1–20]. Here we summarize. The natural frequencies satisfy

$$\left\langle \tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}\,'_s; s) \,;\, \tilde{\vec{j}}_{s_\alpha}(\vec{r}\,'_s) \right\rangle = \vec{0}$$

$$\left.\begin{array}{l} \left(\tilde{Z}_{t_{n,m}}(s_\alpha)\right)\cdot\left(j_{s_n}\right)_\alpha = (0_n) \\ \det\left(\left(\tilde{Z}_{t_{n,m}}(s_\alpha)\right)\right) = 0 \end{array}\right\} \text{numerical (matricized) form as in the moment method} \tag{2.3}$$

$$s_\alpha \equiv \text{natural frequency}\,, \quad \vec{j}_{s_\alpha} \equiv \text{natural mode}$$

In the numerical/matrix form we have a way of computing natural frequencies and modes. From the above we have immediately that s_α and $\vec{j}_{s_\alpha}$ have *no* dependence on incident-field parameters. This aspect independence of the s_α is of great advantage in identifying that the target is of a particular type. Constructing the complete response (current) gives

$$\begin{aligned} \tilde{\vec{J}}_s(\vec{r}_s, s) = E_0 \sum_\alpha \tilde{f}(s_\alpha)\, \eta_\alpha(\vec{1}_i, \vec{1}_p)\, \vec{j}_{s_\alpha}(\vec{r}_s)\left[s - s_\alpha\right]^{-1} e^{-(s-s_\alpha)t_0} \\ + \text{ singularities of } \tilde{f}(s) + \text{possible entire function} \\ + \text{ branch cuts (in dispersive external medium)} \end{aligned} \tag{2.4}$$

where only first-order poles have been included, but poles of higher order are possible in special circumstances. In time domain, the poles are replaced by $e^{s_\alpha t}u(t - t_0)$. Note the inclusion of a turn-on time t_0 which can be chosen for convenience (say the time the wave first touches the target). The coupling coefficient is

$$\eta_\alpha(\vec{1}_i, \vec{1}_p) = \frac{\vec{1}_i \cdot \left\langle e^{-\gamma_\alpha \vec{1}_i \cdot \vec{r}_s}\,,\, \vec{j}_{s_\alpha}(\vec{r}\,'_s) \right\rangle}{\left\langle \vec{j}_{s_\alpha}(\vec{r}_s); \frac{\partial}{\partial s} \tilde{\overleftrightarrow{Z}}_t(\vec{r}_s, \vec{r}\,'_s; s)\Big|_{s=s_\alpha}; \vec{j}_s(\vec{r}\,'_s) \right\rangle} \tag{2.5}$$

$$\gamma_\alpha \equiv \tilde{\gamma}(s_\alpha)$$

and here is where information concerning the incident wave is contained. The entire function (singularity at $s = \infty$) is basically an early-time contribution in time domain. It is the late-time portions of the scattered fields in which the natural frequencies are most clearly visible. An important result for finite size targets in free space (or similar dispersionless media) is the absence of branch-cut singularities. For lossy earth, however, such a term is present. Note that all the above terms occur in conjugate symmetric pairs (exept for singularities on the real s axis where they are real) corresponding to the Laplace transform of real-valued temporal quantities.

Our concern is with the scattered field for which we have the far field

$$\vec{\tilde{E}}_f(\vec{r},s) = \frac{e^{-\tilde{\gamma}(s)r}}{4\pi r}\,\overleftrightarrow{\tilde{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\vec{\tilde{E}}^{(inc)}(\vec{0},s)$$

$$r \equiv |\vec{r}|$$

$$\overleftrightarrow{\tilde{\Lambda}}(\vec{1}_o,\vec{1}_i;s) = \overleftrightarrow{\tilde{\Lambda}}^T(-\vec{1}_i,-\vec{1}_o;s)\ \text{(reciprocity)} \equiv \text{scattering dyadic} \qquad (2.6)$$

$$\vec{1}_o \equiv \text{direction to observer}$$

For backscattering (monostatic) this reduces to

$$\vec{1}_o = -\vec{1}_i\ ,\quad \overleftrightarrow{\tilde{\Lambda}}_b(\vec{1}_i,s) \equiv \overleftrightarrow{\tilde{\Lambda}}(-\vec{1}_i,\vec{1}_i;s) = \overleftrightarrow{\tilde{\Lambda}}^T(\vec{1}_i;s)\ \text{(symmetric)} \qquad (2.7)$$

Applying this to the SEM form gives

$$\overleftrightarrow{\tilde{\Lambda}}(\vec{1}_o,\vec{1}_i;s) = \sum_\alpha \vec{c}_\alpha(-\vec{1}_o)\,\vec{c}_\alpha(\vec{1}_i) \ + \text{entire function} + \text{possible branch cuts}$$

$$\vec{c}_\alpha(\vec{1}_i) = w_\alpha\left\langle \overleftrightarrow{1}_i\, e^{-\gamma_\alpha \vec{1}_i\cdot\vec{r}_s'}\,;\vec{j}_{s_\alpha}(\vec{r}_s')\right\rangle \equiv \text{normalized coupling vector} \qquad (2.8)$$

$$w_\alpha^2 = -s_\alpha\,\mu_0\left\langle \vec{j}_{s_\alpha}(\vec{r}_s);\frac{\partial}{\partial s}\overleftrightarrow{\tilde{Z}}_t(\vec{r}_s,\vec{r}_s';s)\Big|_{s=s_\alpha};\vec{j}_{s_\alpha}(\vec{r}_s')\right\rangle^{-1}$$

$$\overleftrightarrow{1}_i \equiv \overleftrightarrow{1} - \vec{1}_i\vec{1}_i\ ,\quad \overleftrightarrow{1} \equiv \vec{1}_x\vec{1}_x \equiv \vec{1}_y\vec{1}_y \equiv \vec{1}_z\vec{1}_z \equiv \text{identity}$$

So, in addition to the natural frequencies, these coupling vectors can give some additional information about the target, specifically about its orientation (polarization). Note that there can be degeneracies in the case of symmetry, in which there are more than one $\vec{c}_\alpha$ with the same s_α.

The above gives the basic form for SEM as appropriate to EMSI (used with a GPR) discussed in the next two sections. Later, when discussing acoustics/elastodynamics, a very similar form is obtained for ASI. For MSI, however, near fields and the magnetic-polarizability dyadic will give a different form, but one with natural frequencies (purely real) and vectors coming from the decomposition of the dyadic.

3. Perfectly Conducting Target in Lossy Dielectric Medium

Consider now the (relatively) simple case of a perfectly conducting target. Here, we observe from the form of the kernel

$$
\begin{aligned}
\overleftrightarrow{\tilde{Z}}_t(\vec{r}_s,\vec{r}_s\,';s) &= -s\mu\,\overleftrightarrow{1}_S(\vec{r}_s)\cdot\overleftrightarrow{\tilde{G}}_0(\vec{r}_s,\vec{r}_s\,';s)\cdot\overleftrightarrow{1}_S(\vec{r}_s\,') \\
&= -s\mu_0\,\overleftrightarrow{1}_S(\vec{r}_s)\cdot\left\{\left[-2\zeta^{-3}-2\zeta^{-2}\right]e^{-\zeta}\,\vec{1}_R\,\vec{1}_R\right. \\
&\quad\left.+\left[\zeta^{-3}+\zeta^{-2}+\zeta^{-1}\right]e^{-\zeta}\left[\overleftrightarrow{1}-\vec{1}_R\,\vec{1}_R\right]\right\}\cdot\overleftrightarrow{1}_S(\vec{r}_s\,') \\
R &= |\vec{r}_s-\vec{r}_s\,'|\ ,\quad \vec{1}_R = \frac{\vec{r}_s-\vec{r}_s\,'}{R}\ \text{ for }\ \vec{r}_s \neq \vec{r}_s\,' \\
\zeta &= \tilde{\gamma}(s)R\ ,\quad \overleftrightarrow{1}_S(\vec{r}_s) = \overleftrightarrow{1}-\vec{1}_S(\vec{r}_s)\,\vec{1}_S(\vec{r}_s) \equiv \text{transverse dyadic at } \vec{r}_s \\
\vec{1}_S(\vec{r}_s) &\equiv \text{unit surface normal at } \vec{r}_s
\end{aligned}
\tag{3.1}
$$

involving the dyadic Green's function of the external uniform isotropic medium, that there is a special scaling relationship.

Summarizing from [17, 21], let

$$
\tilde{\gamma}^{(0)}(s) = \frac{s^{(0)}}{c} = s^{(0)}\left[\mu_0\varepsilon_0\right]^{-1/2} \equiv \text{propagation constant of free space}
\tag{3.2}
$$

and equate this to the propagation constant in the external medium of interest (e.g., soil). Applying this to the natural frequencies we have

$$
\begin{aligned}
\gamma_\alpha^{(0)} &= \frac{s_\alpha^{(0)}}{c} = \left[s_\alpha\mu_0\left(\sigma+s_\alpha\varepsilon\right)\right]^{1/2} \equiv \gamma_\alpha \\
s_\alpha &= -\frac{\sigma}{2\varepsilon} + \left[\left[\frac{\sigma}{2\varepsilon}\right]^2 + \frac{\varepsilon}{\varepsilon_0}\,s_\alpha^{(0)^2}\right]^{1/2}
\end{aligned}
\tag{3.3}
$$

This result comes from observing that the natural frequencies come from (2.3) in the form of particular γ_α, as in (3.1). Changing the form $\tilde{\gamma}(s)$ takes changes the s_α, not the γ_α. Furthermore, note from (2.3) and (2.8) that the natural modes and coupling vectors are unchanged (since γ_α does not change) which we indicate symbolically as

$$\vec{j}_{s_\alpha}(\vec{r}_s) = \vec{j}^{(0)}_{s_\alpha}(\vec{r}_s) \quad , \quad \vec{c}_\alpha(\vec{1}_i) = \vec{c}^{(0)}_\alpha(\vec{1}_i) \tag{3.4}$$

noting, of course, that these are applied to the new (shifted) natural frequencies. With these scaling relationships it is not necessary to calculate the natural frequencies, say from a discretized form of (2.3). One can measure these parameters, say in an anechoic chamber, and then use these scaling equations to scale to a lossy dielectric medium of interest.

For large $s_\alpha^{(0)}$ compared to σ / ε, in what is called for high-frequency window [33], we have

$$s_\alpha = \left[\frac{\varepsilon_0}{\varepsilon}\right]^{1/2} s_\alpha^{(0)} - \frac{\sigma}{2\varepsilon} + 0\left(s_\alpha^{(0)^{-1}}\right) \text{ as } s_\alpha^{(0)} \to \infty \tag{3.5}$$

For σ and ε assumed independent of s, this is a simple affine transformation (dilation and translation) in the s plane. As such, the "pattern" of the natural frequencies for a particular target is an aid in identification. As we can see from the dilation term, the natural frequencies will be reduced by a significant factor from their free-space values, and shifted slightly to the left (more damping) for typical $\varepsilon/\varepsilon_0$ (say 10 or so) and s (10^{-3} to 10^{-2} S/m).

As part of the design of a GPR to exploit the above results, one needs to propagate a signal from the ground surface to the target, and from the target (scattered signal) back to an antenna on or above the ground surface. Their are various ways to design appropriate antennas [26–30] to transmit and receive appropriate pulses, perhaps using the Brewster angle for better transmission through the ground surface. Letting d be some distance of interest for propagation in this medium, we have the propagation factor

$$\begin{aligned} e^{-\tilde{\gamma}(s)d} &= e^{-\frac{s}{v}d} e^{-\frac{\sigma Z_\infty d}{2}}\left[1+O\left(s^{-1}\right)\right] \text{ as } s \to \infty \\ v &= [\mu_0 \varepsilon]^{-1/2} \quad , \quad Z_\infty = \left[\frac{\mu_0}{\varepsilon}\right]^{1/2} \\ e^{-\frac{s}{v}d} &\equiv \text{delay (by time } d/v) \\ e^{-\frac{\sigma Z_\infty d}{2}} &\equiv \text{attenuation factor} \end{aligned} \tag{3.6}$$

If σ and ε can be approximated as frequency independent, then in this high-frequency window pulses are propagated with attenuation, but without dispersion. Propagation distances of a few meters are typically useful with acceptable attenuation. Expressions (3.6) and related for the electric and magnetic fields can be readily expressed in time domain (for delta- or step-function response) to give exact responses, including the low-frequency dispersion [17, 21].

There are various errors to contend with, such as associated with inhomogeneities (rocks, etc.) in the external medium as well as the ground/air interface. If these are close to the target (in units of target size) they can affect the s_α. Furthermore, σ and ε can vary with frequency (in the range of interest). This can be partly compensated by measuring these constitutive parameters in situ, say by measuring the reflection of the incident wave (from the GPR, including appropriate variations of the direction of incidence and polarization) from the ground/air interface.

4. Dielectric Target in Lossy Dielectric Medium

Let now the target be characterized as a simple dielectric with constant permittivity ε_2 and permeability μ_0 with

$$\varepsilon_r \equiv \frac{\varepsilon_2}{\varepsilon_1} \tag{4.1}$$

where now a subscript 1 will be used to designate parameters of the external medium. For present we consider the case that ε_r is small compared to 1. With ε_2 as about $2\varepsilon_0$ or $3\varepsilon_0$ (corresponding to typical plastic explosive) and typical soil permittivities (in the 100 MHz range) as about $10\,\varepsilon_0$ (or $81\,\varepsilon_0$ in water), one can justify such an approximation. For later use we have

$$\begin{aligned}
\tilde{Z}_1 &= \sqrt{\frac{\mu_0}{\varepsilon_1}}\left[1+\frac{\sigma_1}{s\varepsilon_1}\right]^{-\frac{1}{2}} \equiv \text{wave impedance in external medium} \\
Z_2 &= \sqrt{\frac{\mu_0}{\varepsilon_2}} \equiv \text{wave impedance in target} \\
\tilde{\xi} &\equiv \frac{Z_2}{\tilde{Z}_1} = \frac{\tilde{\gamma}_1}{\tilde{\gamma}_2} = \varepsilon_r^{-\frac{1}{2}}\left[1+\frac{\sigma_1}{s\varepsilon_1}\right]^{\frac{1}{2}}
\end{aligned} \tag{4.2}$$

In the limit of small ε_r (or large $\tilde{\xi}$) one can look at the properties of the external and internal resonances [17, 24]. For the external resonances we can think of the target surface S as a perfect magnetic conductor (infinite surface impedance) in the limit as $\tilde{\xi} \to \infty$. This lets one consider the dual problem in which S is a perfect conductor and the electric and magnetic fields are interchanged [13]. The s_α then are the same as for a metal target as in the previous section, and the scaling relationships there can be applied from free space measurements of a metal coated dielectric target to obtain the s_α in soil (or water). Of course, this procedure only gives the unperturbed external resonances, but this can be used as a starting point.

Leaving behind the external resonances, go on to the internal resonances which are, in general, more important due to their smaller damping (higher Q). In this case the lower external wave impedance (large $\tilde{\xi}$) allows one to consider the internal resonances as the usual cavity resonances with S considered as a perfect conductor in the first approximation. From an experimental point of view one can cover the dielectric target with metal and couple electromagnetic fields into the cavity through a small penetration (small wire coupling loop, etc.) to determine the interior s_α (on the $j\omega$ axis for a lossless cavity). Then one considers the perturbation away from these s_α into the left-half s plane due to the external medium. Write

$$s_\alpha = s_\alpha^{(0)} + \Delta s_\alpha$$

$$s_\alpha^{(0)} \equiv \text{natural frequency (unperturbed) for target as lossless cavity with perfectly conducting boundary} \tag{4.3}$$

$$\Delta s_\alpha \equiv \text{perturbation of natural frequency}$$

For the perturbation of the natural frequencies we can consider canonical problems to get some idea of what the perturbation from the cavity problem is. A simple canonical problem is a dielectric slab of thickness ℓ. As an infinite slab, we constrain the direction of incidence to be perpendicular to the slab to make the natural frequencies unique and give a simple transmission-line problem. The reflection or backscattering coefficient is

$$\tilde{R}_0 = \frac{\left[\tilde{\xi}^2 - 1\right]\sinh\left(\tilde{\gamma}_2 \ell\right)}{2\tilde{\xi}\cosh\left(\tilde{\gamma}_2 \ell\right) + \left[\tilde{\xi}^2 + 1\right]\sinh\left(\tilde{\gamma}_2 \ell\right)} \tag{4.4}$$

The poles are found from the zeros of the denominator as

$$0 = \frac{\tilde{\xi}^2(s_\alpha) + 1}{2\tilde{\xi}(s_\alpha)} + \coth\left(\tilde{\gamma}_2(s_\alpha)\ell\right) \tag{4.5}$$

Large $\tilde{\xi}$ gives the limiting (unperturbed) form as

$$\begin{aligned} &T_\ell \equiv \sqrt{\mu_0 \varepsilon_2}\,\ell\,,\; s_\alpha^{(0)} = j\omega_\alpha^{(0)} \\ &0 = \sin\left(\omega_\alpha^{(0)} T_\ell\right),\; \omega_\alpha^{(0)} T_\ell = n\pi \;\text{ for } n = 1, 2, \ldots \end{aligned} \tag{4.6}$$

Continuing the expansion as in (4.3) we have

$$\begin{aligned} \Delta s_\alpha T_\ell &= -\operatorname{arctanh}\left[\frac{2\tilde{\xi}(s_\alpha)}{\tilde{\xi}^2(s_\alpha) + 1}\right] \\ &= -2\tilde{\xi}^{-1}(s_\alpha) + O(\tilde{\xi}^{-3}) \;\text{ as } \tilde{\xi} \to \infty \\ \tilde{\xi}(s_\alpha) &= \tilde{\xi}(s_\alpha^{(0)})\left[1 + O(\Delta s_\alpha)\right] \;\text{ as } \Delta s_\alpha \to 0 \\ \Delta s_\alpha T_\ell &= -2\tilde{\xi}^{-1}(s_\alpha) + O(\tilde{\xi}^{-3}) \;\text{ as } \tilde{\xi} \to \infty \end{aligned} \tag{4.7}$$

For a pure dielectric external medium ($\sigma_1 = 0$) we have

$$\Delta s_\alpha T_\ell = -2\varepsilon_r^{\frac{1}{2}} + O(\varepsilon_r) \;\text{ as } \varepsilon_{r \to 0} \tag{4.8}$$

which is a particularly simple result. Including a non-zero σ_1 we have

$$\Delta s_\alpha T_\ell = -2\varepsilon_r^{\frac{1}{2}}\left[1+\frac{j}{2}\frac{\sigma_1}{\omega_\alpha^{(0)}\varepsilon_1}+O\left(\left(\frac{\sigma_1}{\omega_\alpha^{(0)}\varepsilon_1}\right)^2\right)\right]+O\left(\tilde{\xi}^{-3}\right)$$

$$\text{as } \frac{\sigma_1}{s_\alpha^{(0)}\varepsilon_1}\to 0 \text{ and } \tilde{\xi}\to\infty \tag{4.9}$$

showing the leading effect in the high-frequency limit as a shift to the left in the s plane with a smaller shift of the imaginary part (toward the origin).

A second canonical problem is a dielectric sphere of radius a. Leaving behind the details of the spherical vector wave functions [17, 24], the exterior natural frequencies for $\tilde{\xi}\to\infty$ are given as

$$\begin{aligned} \Gamma^{(1)} &\equiv \tilde{\gamma}_1 a = \left[s\mu_0(\sigma_1+s\varepsilon_1)\right]^{\frac{1}{2}} a \\ \left[\Gamma_\alpha^{(1,H)}k_n\left(\Gamma_\alpha^{(1,H)}\right)\right]' &= 0 \quad , \quad k_n\left(\Gamma_\alpha^{(1,E)}\right)=0 \\ \Gamma_\alpha^{(1,H)} &\equiv \text{ roots for external H (or TE) modes} \\ \Gamma_\alpha^{(1,E)} &\equiv \text{ roots for external E (or TM) modes} \end{aligned} \tag{4.10}$$

where an H mode has a radial (normal to the sphere surface) component of the magnetic field, and similarly for an E mode. The prime indicates differentiation with respect to the argument of the spherical Bessel function.

For the internal resonances we have for large $\tilde{\xi}$

$$\begin{aligned} \Gamma^{(2)} &\equiv \tilde{\gamma}_2 a = s\sqrt{\mu_0\varepsilon_2}\,a \\ i_n\left(\Gamma_\alpha^{(2,H)}\right)&=0 \quad , \quad \left[\Gamma_\alpha^{(2,E)}i_n\left(\Gamma_\alpha^{(2,E)}\right)\right]' = 0 \\ \Gamma_\alpha^{(2,H)} &\equiv \text{ roots for internal H (or TE) modes} \\ \Gamma_\alpha^{(2,H)} &\equiv \text{ roots for internal E (or TM) modes} \end{aligned} \tag{4.11}$$

these roots being purely imaginary. For the interior H modes we then have

$$\begin{aligned} T_\alpha &\equiv \sqrt{\mu_0\varepsilon_2}\,a \\ s_\alpha^{(2,H,0)}T_a &\equiv \Gamma_\alpha^{(2,H)} \\ s_\alpha^{(2,H)} &\equiv s_\alpha^{(2,H,0)} + \Delta s_\alpha^{(2,H)} \end{aligned} \tag{4.12}$$

Performing the appropriate expansions leads to

$$\Delta s_{\alpha}^{(2,H)} T_a = -\tilde{\xi}^{-1} + O(\tilde{\xi}^{-2}) \text{ as } \tilde{\xi} \to \infty$$
$$= -\varepsilon_r^{\frac{1}{2}} + O(\varepsilon_r) \text{ as } \varepsilon_r \to 0 \text{ for } \sigma_1 = 0 \tag{4.13}$$

which is very similar to the slab results. For the interior E modes we have

$$s_{\alpha}^{(2,E,0)} T_a \equiv \Gamma_{\alpha}^{(2,E)} a \;,\quad s_{\alpha}^{(2,E)} \equiv s_{\alpha}^{(2,E,0)} + \Delta s_{\alpha}^{(2,E)} \tag{4.14}$$

which are used to obtain

$$\Delta s_{\alpha}^{(2,E)} T_a = -\tilde{\xi}^{-1} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} + O(\tilde{\xi}^{-2}) \text{ as } \tilde{\xi} \to \infty$$
$$= -\varepsilon_r^{\frac{1}{2}} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} + O(\varepsilon_r) \text{ as } \varepsilon_r \to 0 \text{ for } \sigma_1 = 0 \tag{4.15}$$

This is also similar to the slab results, but slightly more complicated.

These results have the general form for the interior resonances

$$s_{\alpha}^{(2)} T = -C\tilde{\xi}^{-1} + O(\tilde{\xi}^{-2}) \text{ as } \tilde{\xi} \to \infty$$

$C > 0$, $C \equiv$ dimensionless parameter depending on mode and target shape (4.16)

$T \equiv$ some characteristic time associated with propagation through the target (in medium 2)

This is primarily a shift to the left in the s plane associated with $\varepsilon_r^{\frac{1}{2}}$, but there is a small imaginary part associated with σ_1.

5. Highly, But Not Perfectly, Conducting Targets

A technique of recent vintage [17, 22, 23] concerns the diffusive natural frequencies in metal targets (MSI). For this case one uses loops above the ground surface as transmitters and receivers of quasi-static magnetic fields, with wavelengths (or skin depths) in the external medium large compared to distances between loops and target. In this case, one is interested in the magnetic polarizability dyadic which gives the target an induced magnetic dipole moment as

$$
\begin{aligned}
\tilde{\vec{m}}(s) &= \tilde{\overleftrightarrow{M}}(s) \cdot \tilde{\vec{H}}^{(inc)}(s) \\
\tilde{\vec{H}}^{(inc)}(s) &= \text{incident magnetic field at target} \\
\tilde{\overleftrightarrow{M}}(s) &= \text{magnetic polarizability dyadic} \\
\tilde{\vec{m}}(s) &= \text{induced magnetic dipole moment}
\end{aligned}
\tag{5.1}
$$

The scattered magnetic field is then

$$
\tilde{\vec{H}}^{(sc)}(\vec{r},s) = \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1}\right] \cdot \tilde{\overleftrightarrow{M}}(s) \cdot \tilde{\vec{H}}^{(inc)}(s) \tag{5.2}
$$

with the usual spherical coordinates centered on the target. The distance r to the observer is assumed large compared to target dimensions so that higher order magnetic moments are not significant.

Summarizing, we have

$$
\begin{aligned}
\tilde{\overleftrightarrow{M}}(s) &= \tilde{\overleftrightarrow{M}}(\infty) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha [s - s_\alpha]^{-1} \\
\frac{1}{s}\tilde{\overleftrightarrow{M}}(s) &= \frac{1}{s}\tilde{\overleftrightarrow{M}}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha [s - s_\alpha]^{-1} \\
\vec{M}_\alpha \cdot \vec{M}_\alpha &= 1 \quad , \quad \vec{M}_\alpha \equiv \text{real unit vector for } \alpha\text{th mode} \\
M_\alpha &= \text{real scalar} \quad , \quad s_\alpha < 0 \text{ (all negative real natural frequencies)} \\
\tilde{\overleftrightarrow{M}}(\infty) &= \sum_{\nu=1}^{3} M_\nu^{(\infty)} \vec{M}_\nu^{(\infty)} \vec{M}_\nu^{(\infty)} \\
\vec{M}_\nu^{(\infty)} &\equiv \text{real eigenvectors (three)} \\
\vec{M}_\nu^{(\infty)} \cdot \vec{M}_\nu^{(\infty)} &= 1_{\nu_1,\nu_2} \quad \text{(orthonormal)}
\end{aligned}
$$

$$
\begin{aligned}
M_\nu^{(\infty)} &\equiv \text{real eigenvalues (nonpositive, not necessarily distinct)} \\
\overset{\leftrightarrow}{\tilde{M}}(0) &= \sum_{\nu=1}^{3} M_\nu^{(0)} \vec{M}_\nu^{(0)} \vec{M}_\nu^{(0)} \\
\vec{M}_\nu^{(0)} &\equiv \text{real eigenvectors (three)} \\
\vec{M}_{\nu_1}^{(0)} \cdot \vec{M}_{\nu_2}^{(0)} &= 1_{\nu_1,\nu_2} \quad \text{(orthonormal)} \\
M_\nu^{(0)} &\equiv \text{real eigenvalues (non negative, not necessarily distinct)} \\
1_{\nu_1,\nu_2} &= \begin{cases} 1 & \text{for } \nu_1 = \nu_2 \\ 0 & \text{for } \nu_1 \neq \nu_2 \end{cases}
\end{aligned}
\tag{5.3}
$$

Note the presence of only first order poles. The entire function is a constant (delta function in time domain). Transit times across the target in the external medium are so short on the present time scale as to be neglected. Fundamental to the above results is the neglect of $\omega\varepsilon$ compared to σ in the target. (Note that in general in the target we can have dyadic (anisotropic) constitutive parameters $\overset{\leftrightarrow}{\sigma}(\vec{r})$ and $\overset{\leftrightarrow}{\mu}(\vec{r})$.) From a discretized point of view the target can be viewed as a circuit comprised of inductors and resistors (LR) and appropriate circuit theorems applied, leading to only first order poles with s_α real and negative, corresponding to simple exponential decays in time domain. Inclusion of point symmetry (rotation and/or reflection) in the target further simplifies (5.3) by making the various unit vectors line up according to the planes and axes of symmetry.

This general form of the response is consistent with the well-known examples of a metal sphere and a simple loop. However, one need not calculate the s_α (and perhaps associated vectors) for each target type of interest. These can be measured to give the target-library entries. While my original thoughts concerning this technique were directed towards targets in the ground, it has become apparent that the same technique can be applied to security systems such as at airports [34]. One can think of this as a "smart" metal detector.

6. Acoustic/Elastodynamic Target Discrimination

There is an analog to the SEM representation for electromagnetic scattering in the case of acoustic/elastodynamic scattering, this being important for the case of UXO in water or water-saturated soil. Going by the name of acoustic resonance scattering, various canonical problems having analytic solutions (based on circular-cylindrical and spherical shells) have been treated [12]. One thing needed here is a more general form of the scattering, applicable to general shapes of bodies that are passive, linear, and reciprocal. Recent results [17, 25] have found the general form of the scattering poles directly analogous to the form in Section 2. For scalar p-wave scattering this consists in replacing the vector $\vec{c}_\alpha$ by scalar c_α , and similarly dyadics by scalars. Including s-waves in the incident and scattered waves complicates the problem significantly.

Summarizing from [17, 25], in terms of the displacement vector $\vec{u}(\vec{r},t)$ we have

$$
\begin{aligned}
\tilde{\overleftrightarrow{\tau}}(\vec{r},s) &= \left(\tilde{C}_{n,m,\ell,k}(s)\right) : \nabla \tilde{\vec{u}}(\vec{r},s) \equiv \text{stress tensor} \\
\left(\tilde{C}_{n,m,\ell,k}(s)\right) &\equiv \text{stiffness tensor} \\
C_{n,m,\ell,k} &= \tilde{\ell}_1(s)\, 1_{n,m}\ 1_{\ell,k} + \tilde{\ell}_2(s)\left[1_{n,\ell}\, 1_{m,k} + 1_{m,k}\, 1_{m,\ell}\right] \\
\tilde{\ell}_1(s),\ \tilde{\ell}_2(s) &\equiv \text{Lame constants} \\
\tilde{\overleftrightarrow{\tau}}(\vec{r},s) &= \tilde{\ell}_1(s)\left[\nabla \cdot \tilde{\vec{u}}(\vec{r},s)\right]\overleftrightarrow{1} + \tilde{\ell}_2(s)\left[\nabla \tilde{\vec{u}}(\vec{r},s) + \left[\nabla \tilde{\vec{u}}(\vec{r},s)\right]^T\right] \\
&= \tilde{\overleftrightarrow{\tau}}^T(\vec{r},s) \\
\tilde{v}_p(s) &= \left[\frac{\tilde{\ell}_1(s) + 2\,\tilde{\ell}_2(s)}{\rho_a}\right]^{-1/2} \equiv \text{p-wave speed (longitudinal, pressure)} \\
\tilde{v}_s(s) &= \left[\frac{\tilde{\ell}_2(s)}{\rho_a}\right]^{-1/2} \equiv \text{s-wave speed (transverse, shear)} \\
\rho_a &\equiv \text{mass density}
\end{aligned}
\tag{6.1}
$$

For frequency-independent (and thereby real and positive) Lame constants we have

$$
v_p > v_s > 0 \tag{6.2}
$$

The incident and scattered (far-field) waves have both p and s components in general with the forms

$$
\begin{aligned}
&\vec{\tilde{u}}^{(inc)}(\vec{r},s) = e^{-\tilde{\gamma}_p(s)\vec{1}_i\cdot\vec{r}}\,\vec{\tilde{u}}_p^{(inc)}(\vec{0},s) + e^{-\tilde{\gamma}_s(s)\vec{1}_i\cdot\vec{r}}\,\vec{\tilde{u}}_s^{(inc)}(\vec{0},s) \\
&\vec{\tilde{u}}_p^{(inc)}(\vec{0},s) = \tilde{u}_p^{(inc)}(\vec{0},s)\,\vec{1}_i \quad , \quad \vec{\tilde{u}}_s^{(inc)}(\vec{0},s)\cdot\vec{1}_i = 0 \\
&\vec{\tilde{u}}_f^{(sc)}(\vec{r},s) = \vec{\tilde{u}}_{f_p}^{(sc)}(\vec{r},s) + \vec{\tilde{u}}_{f_s}^{(sc)}(\vec{r},s) \quad , \quad \vec{r} = r\,\vec{1}_o \\
&\vec{\tilde{u}}_{f_p}^{(sc)}(\vec{r},s) = \tilde{u}_{f_p}^{(sc)}(\vec{r},s)\,\vec{1}_o \quad , \quad \vec{\tilde{u}}_{f_p}^{(sc)}(\vec{r},s)\cdot\vec{1}_o = 0 \\
&\qquad \tilde{\gamma}_p(s) \equiv \frac{s}{\tilde{v}_p} \quad , \quad \tilde{\gamma}_p(s) \equiv \frac{s}{\tilde{v}_s}
\end{aligned}
\tag{6.3}
$$

The incident and scattered fields are related by linearity as

$$
\begin{aligned}
\vec{\tilde{u}}_{f_p}^{(sc)}(\vec{r},s) &= \vec{1}_o\,\frac{e^{-\tilde{\gamma}_p(s)r}}{4\pi r}\Big[\tilde{\Lambda}_{p,p}(\vec{1}_o,\vec{1}_i;s)\,\vec{1}_i\cdot\vec{\tilde{u}}_p^{(inc)}(\vec{0},s) \\
&\qquad + \vec{\tilde{\Lambda}}_{p,s}(\vec{1}_o,\vec{1}_i;s)\cdot\vec{\tilde{u}}_s^{(inc)}(\vec{0},s)\Big] \\
\vec{\tilde{u}}_{f_s}^{(sc)}(\vec{r},s) &= \frac{e^{-\tilde{\gamma}_s(s)r}}{4\pi r}\Big[\vec{\tilde{\Lambda}}_{s,p}(\vec{1}_o,\vec{1}_i;s)\,\vec{1}_i\cdot\vec{\tilde{u}}_p^{(inc)}(\vec{0},s) \\
&\qquad + \overleftrightarrow{\tilde{\Lambda}}_{s,s}(\vec{1}_o,\vec{1}_i;s)\cdot\vec{\tilde{u}}_s^{(inc)}(\vec{0},s)\Big]
\end{aligned}
\tag{6.4}
$$

giving four scattering coefficients (one scalar, two vector, one dyadic).

Expand these four scattering coefficients in terms of poles (here assumed first order) as

$$
\begin{aligned}
\tilde{\Lambda}_{p,p}(\vec{1}_o,\vec{1}_i;s) &= \sum_\alpha \Lambda_\alpha^{(p,p)}(\vec{1}_o,\vec{1}_i)\,[s-s_\alpha]^{-1} + \text{other singularity terms} \\
\vec{\tilde{\Lambda}}_{p,s}(\vec{1}_o,\vec{1}_i;s) &= \sum_\alpha \vec{\Lambda}_\alpha^{(p,s)}(\vec{1}_o,\vec{1}_i)\,[s-s_\alpha]^{-1} + \text{other singularity terms} \\
\vec{\tilde{\Lambda}}_{s,p}(\vec{1}_o,\vec{1}_i;s) &= \sum_\alpha \vec{\Lambda}_\alpha^{(s,p)}(\vec{1}_o,\vec{1}_i)\,[s-s_\alpha]^{-1} + \text{other singularity terms} \\
\overleftrightarrow{\tilde{\Lambda}}_{s,s}(\vec{1}_o,\vec{1}_i;s) &= \sum_\alpha \overleftrightarrow{\Lambda}_\alpha^{(s,s)}(\vec{1}_o,\vec{1}_i)\,[s-s_\alpha]^{-1} + \text{other singularity terms}
\end{aligned}
\tag{6.5}
$$

The elastodynamic reciprocity theorem, when applied to the pole residues, gives

$$\begin{aligned}
\Lambda_{\alpha}^{(p,p)}(\vec{1}_o, \vec{1}_i) &= \Lambda_{\alpha}^{(p,p)}(-\vec{1}_i, -\vec{1}_o) \\
\vec{\Lambda}_{\alpha}^{(p,s)}(\vec{1}_o, \vec{1}_i) &= -\frac{\tilde{v}_s^2(s_\alpha)}{\tilde{v}_p^2(s_\alpha)} \vec{\Lambda}_{\alpha}^{(s,p)}(-\vec{1}_i, -\vec{1}_o) \\
\overleftrightarrow{\Lambda}_{\alpha}^{(s,s)}(\vec{1}_o, \vec{1}_i) &= \overleftrightarrow{\Lambda}^{(s,s)^T}(-\vec{1}_i, -\vec{1}_o)
\end{aligned} \tag{6.6}$$

Omitting the details, this is then used for non-degenerate natural modes to give a factored form for these scattering coefficients as

$$\begin{aligned}
\Lambda_{\alpha}^{(p,p)}(\vec{1}_o, \vec{1}_i) &= c_\alpha(-\vec{1}_o)\, c_\alpha(\vec{1}_i) \\
\overleftrightarrow{\Lambda}_{\alpha}^{(s,s)}(\vec{1}_o, \vec{1}_i) &= \vec{c}_\alpha(-\vec{1}_o)\, \vec{c}_\alpha(\vec{1}_i) \\
\vec{\Lambda}_{\alpha}^{(p,s)}(\vec{1}_o, \vec{1}_i) &= a_\alpha\, c_\alpha(-\vec{1}_o)\, \vec{c}_\alpha(\vec{1}_i) \\
\vec{\Lambda}_{\alpha}^{(s,p)}(\vec{1}_o, \vec{1}_i) &= a_\alpha^{-1}\, \vec{c}_\alpha(-\vec{1}_o)\, c_\alpha(\vec{1}_i) \\
a_\alpha = \pm j\, \frac{\tilde{v}_s(s_\alpha)}{\tilde{v}_p(s_\alpha)} \;&,\quad \vec{c}_a(\vec{1}_i) \cdot \vec{1}_i = 0 \text{ (transverse)}
\end{aligned} \tag{6.7}$$

These four coefficients are then representable by products of one scalar c_α and one vector $\vec{c}_a$ as functions of incoming and outgoing directions. These can be regarded as experimental observables which the factorization in (6.7) makes functions of only one angle (variable over 4π steraidians).

Comparing to the electromagnetic form in (2.8) we can see that (6.7) is a generaliation to include both transverse and longitudinal waves. While (2.8) has been derived from an integral-equation representation of the scattering, (6.7) has been derived by the imposition of reciprocity on the general form of the scattering coefficients (a procedure which can be applied just as easily to electromagnetic scattering). While (6.5) and (6.7) give the basic form for first order elastodynamic poles, there is still much to be done to bring the knowledge of the SEM terms to the same level of sophistication as in the electromagnetic case. While the present results apply to the elastodynamic case, there are important cases in which the external medium only supports significant p-waves (e.g., air, water). Even though the target may support both p- and s-waves, only p-waves are present in the far-field scattering. In such cases only the scalar c_α are needed to form the single scattering coefficient $\Lambda_{\alpha}^{(p,p)}$.

7. Concluding Remarks

As this summary has discussed, there are two electromagnetic techniques (EMSI and MSI) and one acoustic/elastodynamic technique (ASI) for discriminating buried targets based on patterns of natural frequencies (and perhaps residue vectors as well) in some target library. Figure 2 gives a diagramatic way of looking at these collectively. This has properties of a matrix and a Venn diagram (used in Boolean algebra). Here we can see how the different techniques overlap when considering different types of targets (metal, dielectric) embedded in different types of media (soil, water). Of course, this is a simplified view, and one can in principle, further subdivide the various domains for a more detailed evaluation of the various target/technique combinations. Note multiple techniques applicable to metal targets in both kinds of media. One would like to have more techniques for dielectric targets to fill out the diagram better, such as chemical (sniffer) and nuclear techniques, but these are beyond the scope of this paper.

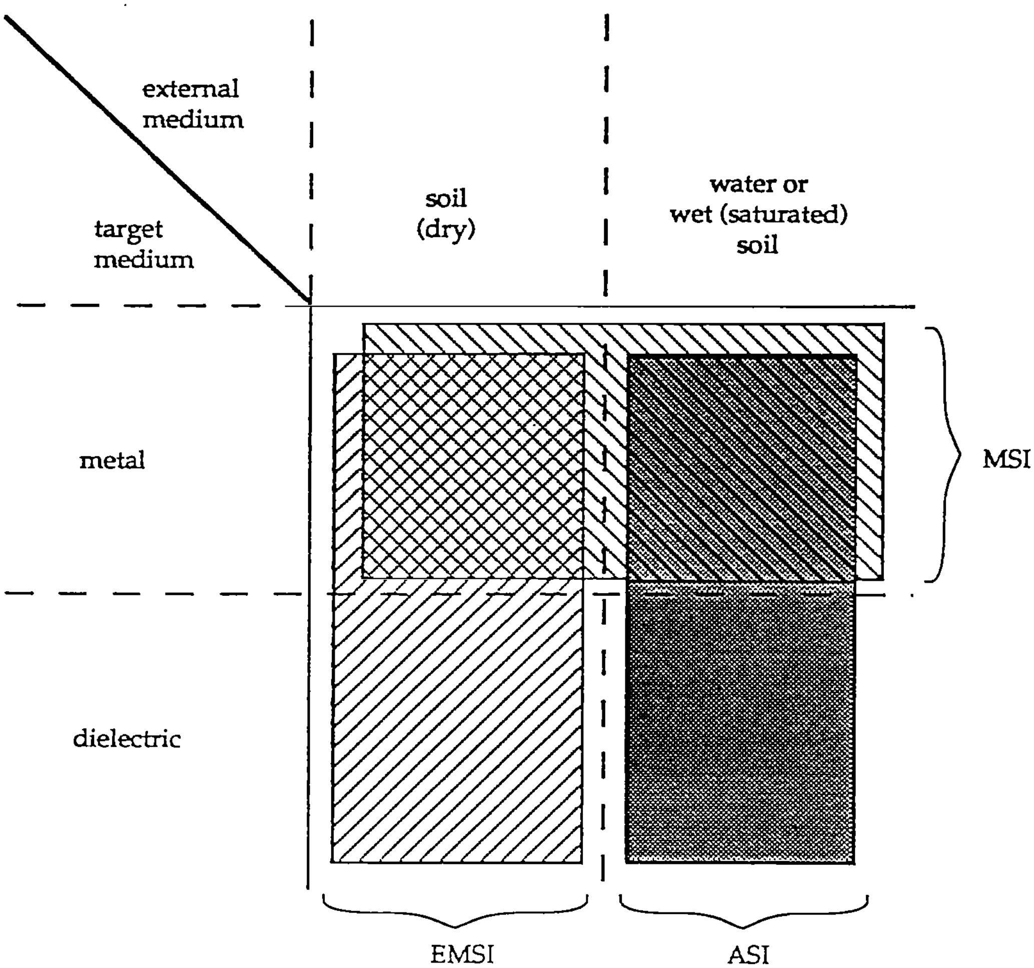

Fig. 2. SEM-Based Target Identification

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