Interaction Notes

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Use of Residue and Constant-Dyadic Information in Magnetic Singularity Identification

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Air Force Research Laboratory
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Abstract

Natural frequencies are an important target identifier due to their aspect independence. For the case of symmetrical targets using the low-frequency diffusion poles in magnetic singularity identification, the dyadic residues and constant dyadics group the poles and eigenvalues into scalar sets which have common angular dependence. This allows one to define ratios of scalar residues and eigenvalues as additional aspect-independent parameters which can be constrained in the target library for use in matching to experimental target responses.

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I. Introduction

In identification (discrimination) of buried metallic targets magnetic-singularity identification (MSI), using low-frequency diffusion poles in the singularity expansion method (SEM), is an appropriate technique. One can use the aspect-independent natural frequencies s_α (real and negative) of particular targets to attempt to fit the data and thereby decide which (if any) of the targets in a library is present in a measurement.

Besides natural frequencies there are the scalar M_α parts of the residues and the eigenvalues of the constant dyadics which are aspect independent. For symmetrical targets the natural frequencies and scalar residues and eigenvalues are grouped into sets, all with the same aspect and target-distance dependence. This paper then discusses how to use ratios of these within each set as aspect-independent target identifiers along with the natural frequencies.

2. SEM Representations of the Magnetic-Polarizability Dyadic

Summarizing from [1-4, 7] the magnetic-polarizability dyadic has the complex-frequency domain forms

$$\tilde{\overleftrightarrow{M}}(s) = \tilde{\overleftrightarrow{M}}(\infty) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha [s - s_\alpha]^{-1} \qquad \text{(delta-function response)}$$

$$\frac{1}{s}\tilde{\overleftrightarrow{M}}(s) = \frac{1}{s}\tilde{\overleftrightarrow{M}}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha [s - s_\alpha]^{-1} \qquad \text{(step-function response)} \qquad (2.1)$$

where the various terms have the properties

$\vec{M}_\alpha \cdot \vec{M}_\alpha = 1$, $\vec{M}_\alpha \equiv$ real unit vector for αth mode

M_α = real scalar

$s_\alpha < 0$ (all negative real natural frequencies)

$\tilde{\overleftrightarrow{M}}(\infty) = \sum_{\nu=1}^{3} M_\nu^{(\infty)} \vec{M}_\nu^{(\infty)} \vec{M}_\nu^{(\infty)}$ = entire function (a constant dyadic)

$\vec{M}_\nu^{(\infty)} \equiv$ real eigenvectors (three)

$\vec{M}_{\nu_1}^{(\infty)} \cdot \vec{M}_{\nu_2}^{(\infty)} = 1_{\nu_1,\nu_2}$ (orthonormal)

$\vec{M}_\nu^{(\infty)} \equiv$ real eigenvalues (non positive, not necessarily distinct)

$\tilde{\overleftrightarrow{M}}(0) = \sum_{\nu=1}^{3} M_\nu^{(0)} \vec{M}_\nu^{(0)} \vec{M}_\nu^{(0)}$ = static response (a constant dyadic)

$\vec{M}_\nu^{(0)} \equiv$ real eigenvalues (three)

$\vec{M}_{\nu_1}^{(0)} \cdot \vec{M}_{\nu_2}^{(0)} = 1_{\nu_1,\nu_2}$ (orthonormal) (2.2)

$M_\nu \equiv$ real eigenvalues (non negative, not necessarily distinct)

$s = \Omega + j\omega \equiv$ complex frequency or two-sided Laplace-transform variable

Multiplying and dividing by s in (2.1) we have the alternate representations

$$\tilde{\overleftrightarrow{M}}(s) = \tilde{\overleftrightarrow{M}}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \, s[s - s_\alpha]^{-1}$$

(step-associated delta-function response)

$$\frac{1}{s}\overset{\tilde{\leftrightarrow}}{M}(s) = \frac{1}{s}\overset{\tilde{\leftrightarrow}}{M}(\infty) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha s^{-1}[s - s_\alpha]^{-1} \tag{2.3}$$

(delta-associated step-function response)

The first of these forms is particularly convenient because, as one considers the lower natural frequencies (small $|s_\alpha|$) in extraction from real data, the constant term is easily isolated, and we know that

$$\overset{\tilde{\leftrightarrow}}{M}(0) \begin{cases} = (0_{n,m}) \Rightarrow \text{ non-ferrous target} \\ \neq (0_{n,m}) \Rightarrow \text{ ferrous target} \end{cases} \tag{2.4}$$

All the pole terms in this case take the form of modified poles giving zero response at zero frequency [5]. Comparing the two forms of the delta-function response we have

$$\overset{\tilde{\leftrightarrow}}{M}(\infty) = \overset{\tilde{\leftrightarrow}}{M}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \tag{2.5}$$

where we have assumed this series to be convergent. Combining this result with the properties of the high- and low-frequency dyadics gives us some idea of the properties of the M_α (particularly for small $|s_\alpha|$) noting the negative real property of the s_α.

In time domain the magnetic-polarizability dyadic is a convolution operator. From (2.1) we have interesting time-domain functions for this purpose as

$$\overset{\leftrightarrow}{M}(t) = \overset{\tilde{\leftrightarrow}}{M}(\infty)\,\delta(t) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha e^{s_\alpha t} u(t)$$

(delta-function response)

$$\int_{-\infty}^{t} \overset{\leftrightarrow}{M}(t')\,dt' = \overset{\tilde{\leftrightarrow}}{M}(0)\,u(t) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha e^{s_\alpha t} u(t) \tag{2.6}$$

(step-function response)

The second of these forms is particularly convenient in that $\overset{\leftrightarrow}{M}(0)$ is exhibited as a clearly distinguishable term, the others decaying to zero in late time, and (2.4) applies.

For completeness we also have time-domain functions corresponding to (2.3) as

$$\overleftrightarrow{M}(t) = \tilde{\overleftrightarrow{M}}(0)\,\delta(t) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha \left[\frac{\delta(t)}{s_\alpha} + e^{s_\alpha t} u(t)\right]$$

(step-associated delta-function response)

$$\int_{-\infty}^{t} \overleftrightarrow{M}(t')\,dt' = \tilde{\overleftrightarrow{M}}(\infty)\,u(t) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \left[-1 + e^{s_\alpha t}\right] u(t) \tag{2.7}$$

(delta-associated step-function response)

These seem less convenient to use than the forms in (2.6).

3. Properties of Residue and Constant Dyadics

All the M_α (scalar parts of residues) and all the eigenvalues $M_\nu^{(\infty)}$ and $M_\nu^{(0)}$ of the corresponding constant dyadics are *aspect independent*, i.e., they are invariant to the relative orientation of the target with respect to the transmitter and receiver. The effect of the target orientation is contained in the $\overrightarrow{M}_\alpha$ and the eigenvectors $\overrightarrow{M}_\nu^{(\infty)}$ and $\overrightarrow{M}_\nu^{(0)}$ of the corresponding constant dyadics. These all rotate with the target.

The magnetic-polarizability dyadic appears in the scattered magnetic field as [1, 6]

$$\overleftrightarrow{\tilde{H}}^{(sc)}(\overrightarrow{r}_s, s) = \frac{1}{4\pi R^3}[3\ \overrightarrow{1}_R\ \overrightarrow{1}_R - \overleftrightarrow{1}] \cdot \overleftrightarrow{\tilde{M}}(s) \cdot \overleftrightarrow{\tilde{H}}^{(inc)}(\overrightarrow{r}_0, s)$$

$\overrightarrow{r}_0 \equiv$ target location

$\overrightarrow{r}_s \equiv$ receiver (sensor) location (3.1)

$$R \equiv |\overrightarrow{r}_s - \overrightarrow{r}_0| \ , \quad \overrightarrow{1}_R \equiv \frac{\overrightarrow{r}_s - \overrightarrow{r}_0}{R}$$

$$\overleftrightarrow{1} \equiv \overrightarrow{1}_x\ \overrightarrow{1}_x + \overrightarrow{1}_y\ \overrightarrow{1}_y + \overrightarrow{1}_z\ \overrightarrow{1}_z \equiv \text{identity}$$

where R is assumed large compared to the target dimensions so that the scattered field need include only the magnetic-dipole term. The incident magnetic field can be quite general (quasistatic). If the transmitter loop is small enough it can also be characterized by a magnetic dipole, giving

$$\overleftrightarrow{\tilde{H}}^{(inc)}(\overrightarrow{r}_0, s) = \frac{1}{4\pi|\overrightarrow{r}_0 - \overrightarrow{r}_t|}[3\ \overrightarrow{1}_0\ \overrightarrow{1}_0 - \overleftrightarrow{1}] \cdot \overrightarrow{\tilde{m}}^{(inc)}(s) \quad (3.2)$$

$\overrightarrow{r}_t \equiv$ transmitter location

$$\overrightarrow{1}_0 \equiv \frac{\overrightarrow{r}_t - \overrightarrow{r}_0}{|\overrightarrow{r}_t - \overrightarrow{r}_0|}$$

$$\overrightarrow{\tilde{m}}^{(inc)}(s) = \tilde{m}^{(inc)}(s)\ \overrightarrow{1}_c$$

$\overrightarrow{1}_c$ orientation of transmitter coil

One can combine (3.1) and (3.2) using $\overrightarrow{1}_s$ as the orientation of the receiver coil. If, in addition $\overrightarrow{r}_t = \overrightarrow{r}_s$ such an expression becomes quite symmetry.

So one way to identify the target is to have a stored set of the s_α, M_α, eigenvalues (aspect independent), and the corresponding $\vec{M}_\alpha$ and eigenvectors (aspect dependent). Then one attempts to rotate and translate the target in a computer to match the measured data. Note that if there is a problem with the absolute amplitude (say due to the sensitivity of the result to the target distance R), then one can scale the M_α to one particular one of these (a dominant one) which we might call M_1. The relative values M_α / M_1 will be insensitive to R, but are still weighted by the target orientation via the $\vec{M}_\alpha$. This gives more constraints on the fitting than the s_α alone. Of course, the s_α can be used without rotating and translating the target, but this does not use the aspect-independent M_α and eigenvalue information.

4. Influence of Symmetry

As discussed in [2, 6], various symmetries of the target (point symmetry groups) simplify the form of $\overleftrightarrow{\tilde{M}}(s)$. Table 4.1 summarizes these results. Here we see the various $\vec{M}_\alpha$ lining up according to symmetry axes and planes. Thus we see in symmetry category 1 the scalar function $\tilde{M}_2(s)$. The vectors associated with poles and constants in this function are all the same, i.e., $\vec{1}_z$. Since these all have the same dot product with the incident field and vectors pointed toward the receiver, all the residues and constants *scale the same way with aspect and distance.* As such the ratios of such terms appearing in the experiment are independent of the presumed unknown target aspect and location.

Generalizing, consider some scalar magnetic-polarizability function appearing in Table 4.1 which we write as

$$\begin{aligned} \tilde{M}_q(s) &= \tilde{M}_q(\infty) + \sum_\alpha M_{q_\alpha}[s - s_{q_\alpha}]^{-1} \\ \frac{1}{s}\tilde{M}_q(s) &= \frac{1}{s}\tilde{M}_q(0) + \sum_\alpha \frac{M_{q_\alpha}}{s_{q_\alpha}}[s - s_{q_\alpha}]^{-1} \end{aligned} \tag{4.1}$$

Table 4.1. Decomposition of Magnetic Polarizability Dyadic According to Target Point Symmetries.

Form of $\overleftrightarrow{\tilde{M}}(s)$	Symmetry Types (Groups)	Symmetry Category
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \overleftrightarrow{\tilde{M}}_t(s)$ $(\overleftrightarrow{\tilde{M}}_t(s)\cdot\vec{1}_z = \vec{0})$	R_z (single symmetry plane) C_2 (2-fold rotation axis)	1
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \tilde{M}_x(s)\,\vec{1}_x\,\vec{1}_x + \tilde{M}_y(s)\,\vec{1}_y\,\vec{1}_y$	$C_{2a} = R_x \otimes R_y$(two axial symmetry planes) D_2 (three 2–fold rotation axes)	2
$\tilde{M}_z(s)\,\vec{1}_z\,\vec{1}_z + \tilde{M}_t(s)\,\overleftrightarrow{1}_z$ $\left(\overleftrightarrow{1}_z = \overleftrightarrow{1} - \vec{1}_z\,\vec{1}_z \Rightarrow \text{double degeneracy}\right)$	C_N for $N \geq 3$ (N-fold rotation axis) S_N for N even and $N \geq 4$ (N-fold rotation-reflection axis) D_{2d} (three 2-fold rotation axes plus diagonal symmetry planes)	3
$\tilde{M}(s)\,\overleftrightarrow{1}$ $\left(\overleftrightarrow{1} \Rightarrow \text{triple degeneracy}\right)$	O_3 (generalized sphere) T, O, Y (regular polyhedra)	4

following (2.1), the alternate forms from (2.3) are

$$\tilde{M}_q(s) = \tilde{M}_q(0) + \sum_\alpha \frac{M_{q\alpha}}{s_{q\alpha}} s[s - s_{q\alpha}]^{-1}$$
$$\frac{1}{s}\tilde{M}_q(s) = \frac{1}{s}\tilde{M}_q(\infty) + \sum_\alpha M_\alpha s^{-1}[s - s_\alpha]^{-1} \tag{4.2}$$

The time-domain forms in (2.6) and (2.7) can be similarly written by inspection. Here the subscript q is coordinate related as

$$q = \begin{cases} z \text{ in symmetry category 1} \\ x, y, \text{ and } z \text{ in symmetry category 2} \\ z \text{ and } t \text{ (for transverse) in symmetry category 3} \\ 0 \text{ (or whatever, for } O_3 \text{ and related symmetries in symmetry category 4} \end{cases} \tag{4.3}$$

So take some dominant pole in $\tilde{M}_q(s)$, say the lowest order one designated by s_{q_1} with residue M_{q_1}, and form the ratios

$$R_{q\alpha} \equiv \frac{M_{q\alpha}}{M_{q_1}} \quad , \quad R_{q_1} = 1$$
$$R_q^{(0)} \equiv \frac{\tilde{M}_q(0)}{M_{q_1}} \quad (= 0 \text{ for nonferrous target}) \tag{4.4}$$
$$R_q^{(\infty)} \equiv \frac{\tilde{M}_q(\infty)}{\tilde{M}_{q_1}}$$

Then in processing the data for, say the delta-function response by dividing out the transmitter and receiver responses, the $R_{q\alpha}$ will be an *invariant of the measurement.* If one of the residues is increased by moving and/or orienting the transmitter and receiver, all of the residues in this set will be increased by the same factor. The ratios in (4.4) can then be stored in a computer as an invariant part (aspect and location independent) of the target signature, just like the s_α.

It should be emphasized that the invariance of these ratios applies within each scalar $\tilde{M}_q(s)$ separately. As we have seen, there may be zero through three such functions depending on the kind of symmetry (or lack thereof) applicable to a particular target. In general our target response as seen by the sensor will involve a linear combination of the $\tilde{M}_q(s)$, together with perhaps other terms. This means that the target response can be written as

$$\tilde{H}^{(sc)}(s) = \tilde{H}^{(inc)}(s)\sum_{q} a_q \tilde{M}_q(s) \;\; +\text{– possible other terms}$$

$$= \tilde{H}^{(inc)}(s)\sum_{q} a_q \;\; M_{q_1}\left[R_q^{(0)} + \sum_{z} R_{q\alpha} \frac{s}{s_{q\alpha}} [s - s_{q\alpha}]^{-1} \right] + \text{ possible other terms}$$

$$\vec{H}^{(inc)}(\vec{r}_0, s) \equiv \tilde{H}^{(inc)}(s) \;\; \vec{1}_{inc} \tag{4.5}$$

$\vec{1}_{inc} \equiv$ orientation of incident magnetic field at target (frequency independent)

$$\tilde{H}^{(sc)}(s) \equiv \vec{H}^{(sc)}(\vec{r}_s, s) \cdot \vec{1}_s$$

$\vec{1}_s \equiv$ orientation of sensor coil

where the form in the first of (2.3) has been used but other forms can be used as well. Here the significance of the ratio invariance within each $\tilde{M}_q(s)$ is apparent. Only the coefficient a_q or equivalently $a_q \;\; M_{q_1}$ depends on target location and orientation. The natural frequencies and ratios are invariant for a particular target and can presumably be determined beforehand for storage in the target library. Of course, M_{q_1} can also be determined beforehand, but it is masked by the variability of a_q.

5. Concluding Remarks

Symmetrical targets, having various alignments of the residue dyadics giving sets with common alignments, give a tighter target discriminant due to ratios within each such set. So besides having to match some number N_q of natural frequencies in the set, there are $N_q - 1$ residue ratios plus one ratio involving the DC response, or N_q ratios to be matched as well, doubling the number of aspect independent parameters within that set to be matched. With more than one such set in a target signature a linear combination of the responses for the various sets is required to match the data.

In the case of C_N symmetry for $N \geq 3$ we have a common type of target symmetry, including the case of a body of revolution with axial symmetry planes ($C_{\infty a} = O_2$ symmetry). As discussed in [6], this symmetry can be used to locate and orient such a buried target. As a part of this the $\tilde{M}_z(s)$ and parts of the response need to be separated. The present discussion has shown that one can use the residue and eigenvalue ratios in each of these sets to better identify such a target and more sensitively characterize these two parts of the response.

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