3, 6-8, 13, 15, 16, 19-22
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Interaction Notes

Note 558

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Some Simple Formulae for Transient Scattering

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Abstract

In transient scattering measurements one needs an adequate received signal to successfully detect and perhaps identify the scatterer. This depends on the properties of the scattering dyadic and the transmit and receive antennas. For the case of backscattering (colocated transmit/receive antennas) received signals are estimated with choice of some kind of impulse radiating antenna with a large band ratio. The effect of propagation through an interface (e.g., air/soil) is also included.

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1. Introduction

Over the years much calculations (e.g., [17]) and measurements of electromagnetic scattering have been performed. While these are performed primarily in the frequency domain, they can be applied in time domain as well by inverse Laplace/Fourier transformation provided phase is retained (which is not the case for radar cross sections).

For transient remote sensing, one has some transmitting and receiving antennas which send out a pulse and receive the pulse scattered by some object of interest. Such antennas also need to be characterized and optimized for their transient performance. Various types of impulse radiating antennas (IRAs) are suitable for this purpose [22]. One needs to know what are the characteristics and amplitudes of the received voltage as a function of the transmitter voltage waveform and amplitude and the scattering dyadic operator for the object of interest. In the present paper, some simple canonical scatterers are considered for this purpose.

In addition to the antennas and scatterer, one may need to consider the properties of the intervening media. A common example concerns the interface between air and soil for the case of buried targets [10, 21].

2. Transient Scattering

Approximating the wave incident on the scatterer as a plane wave in free space, we have [18]

$$\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_0 \vec{1}_p \tilde{f}(s) e^{-\gamma \vec{1}_i \cdot \vec{r}}$$

$$\vec{E}^{(inc)}(\vec{r},t) = E_0 \vec{1}_p f\left(t - \frac{\vec{1}_i \cdot \vec{r}}{c}\right)$$

$\vec{1}_i \equiv$ direction of incidence , $\vec{1}_p \equiv$ polarization

$\vec{1}_i \cdot \vec{1}_p = 0$, $f(t) \equiv$ incident waveform

$c = [\mu_0 \varepsilon_0]^{-\frac{1}{2}} \equiv$ speed of light

$E_0 =$ amplitude factor (V/m)

$\gamma \equiv \frac{s}{c} \equiv$ propagation constant

$t \equiv$ time

$\sim \equiv$ Laplace transform (2-sided) over time (2.1)

$s \equiv \Omega + j\omega \equiv$ Laplace-transform variable or complex frequency

The far scattered field can then be written

$$\tilde{\vec{E}}_f^{(sc)}(\vec{r},s) = \frac{e^{-\gamma r}}{4\pi r} \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_0, \vec{1}_i; s) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s)$$

$\vec{1}_0 \equiv$ direction to observer at $\vec{r}$

$\tilde{\vec{E}}_f^{(sc)}(\vec{r},s) \cdot \vec{1}_0 = 0$, $r = |\vec{r}|$

$$\vec{E}_f^{(sc)}(\vec{r},t) = \frac{1}{4\pi r} \overleftrightarrow{\Lambda}(\vec{1}_0, \vec{1}_i; t) \overset{\circ}{\cdot} \vec{E}^{(inc)}\left(\vec{0}, t - \frac{r}{c}\right)$$

$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_0, \vec{1}_i; s) \equiv$ scattering dyadic (meters)

$\overleftrightarrow{\Lambda}(\vec{1}_0, \vec{1}_i; t) \circ \equiv$ scattering dyadic operator

$\vec{1}_0 \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_0, \vec{1}_i; s) = \vec{0} = \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_0, \vec{1}_i; s) \cdot \vec{1}_i$ (2.2)

$\tilde{\overleftrightarrow{\Lambda}}^T(\vec{1}_0, \vec{1}_i; s) = \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i, -\vec{1}_0; s)$ (reciprocity)

$\circ \equiv$ convolution with respect to time

f

Here, $\vec{r} = \vec{0}$ is located at some convenient position on or near the scatterer.

For the important case of backscatter we have

$$\vec{1}_0 = -\vec{1}_i$$
$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i, s) \equiv \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i, \vec{1}_i, s) = \tilde{\overleftrightarrow{\Lambda}}_b^T(\vec{1}_i, s) \tag{2.3}$$

In this latter case we can have the usual h,v (horizontal-vertical) radar coordinates with a right handed system $(\vec{1}_h, \vec{1}_v, -\vec{1}_i)$ with

$$\vec{1}_h \times \vec{1}_v = -\vec{1}_i = \vec{1}_o \tag{2.4}$$

In this case, one envisions an observer at the radar looking in the direction $\vec{1}_i$ (toward the scatterer) with $\vec{1}_h$ to the right and $\vec{1}_v$ (approximately) up.

While the general form of the temporal form of the scattering dyadic is a convolution operator, there are special cases which simplify its form. As discussed in [18] one has cases in which the scattering dyadic takes the form

$$\begin{aligned} &\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) = \gamma^n \overleftrightarrow{K}(\vec{1}_o, \vec{1}_i) \to c^{-n} \overleftrightarrow{K}(\vec{1}_o, \vec{1}_i) \frac{d^n}{dt^n} \\ &\gamma^{-1} \overleftrightarrow{K}(\vec{1}_o, \vec{1}_i) \to c \overleftrightarrow{K}(\vec{1}_o, \vec{1}_i) \int_{-\infty}^{t} (\quad) dt' \\ &n = \text{integer} \end{aligned} \tag{2.5}$$

which are simply temporal derivatives or integrals. Our canonical examples include such cases.

A commonly used measure of far-field scattering is cross section (real) $A_c(j\omega)$ typically defined by

$$A_c(j\omega) = 4\pi r^2 \frac{|\tilde{\vec{E}}^{(sc)}(j\omega)|^2}{|\tilde{\vec{E}}^{(inc)}(j\omega)|^2} \quad (\text{meter}^2) \tag{2.6}$$

However, this loses phase information when considering individual frequencies, and loses dispersion information, making it unsuitable for transient scattering. It can be calculated from the scattering dyadic [6, 7]. Note that A_c is also a function of the polarization of the incident wave and whether one is considering some particular polarization of the scattered field. Using linear polarization (circular polarization being unsuitable for transient fields) then (2.2) becomes

$$\tilde{E}^{(sc)}_{f_n}(\vec{r}, j\omega) = \frac{e^{-j\omega/c}}{4\pi r} \tilde{\Lambda}_{n,m}(\vec{1}_o, \vec{1}_i; j\omega)\, \tilde{E}^{(inc)}_m(\vec{0}, j\omega) \tag{2.7}$$

and (2.6) gives

$$A_c(j\omega) = \frac{1}{4\pi}\left|\tilde{\Lambda}_{n,m}(\vec{1}_o, \vec{1}_i; j\omega)\right|^2 = \frac{1}{4\pi}\left|\tilde{\Lambda}_{b_{n,m}}(\vec{1}_i; j\omega)\right|^2 \quad \text{for backscattering} \tag{2.8}$$

The formulae in this paper can then be converted to cross-section for frequency-domain purposes.

3. Scattering Signals

Besides the properties of the scatterer one needs to radiate a pulse from some antenna and receive the pulse from the scatterer with an antenna (the same antenna or a different antenna). Beginning with the transmitting antenna located near $\vec{r}$ it has a far-field [1] given by [3]

$$\tilde{\vec{E}}_f^{(inc)}(\vec{0},s) = \frac{e^{-\gamma r}}{r}\tilde{\vec{F}}(\vec{1},s)\,\tilde{V}^{(trans)}(s) \;,\quad \tilde{\vec{F}}(\vec{1}_i,s)\cdot\vec{1}_i = 0 \tag{3.1}$$

where we can approximate this field as a plane wave incident on the scatterer, giving

$$\tilde{\vec{E}}_f^{(inc)}(\vec{0},s) = \tilde{\vec{E}}^{(inc)}(\vec{0},s) \tag{3.2}$$

In reception an antenna at $\vec{r}$ receives the scattered field as

$$\tilde{V}^{(rec)}(s) = \tilde{\vec{h}}_a(\vec{1}_o,s)\cdot\tilde{\vec{E}}^{(sc)}(\vec{r},s) \;,\quad \tilde{\vec{h}}_a(\vec{1}_o,s)\cdot\vec{1}_o = 0 \tag{3.3}$$

Reciprocity [3] relates the transmission and reception properties of an antenna by

$$\tilde{\vec{F}}(\vec{1}_i,s) = \frac{s\mu_0}{2\pi R}\tilde{\vec{h}}_a(-\vec{1}_i,s) \tag{3.4}$$

where R is the assumed input impedance (which we take as a constant resistance) and load resistance (in reception) of the antenna of concern. In time domain, we have an operator equation

$$\vec{F}(\vec{1}_i,t) \circ = \frac{\mu_0}{2\pi R}\vec{h}_a(-\vec{1}_i,t)\frac{d}{dt} \tag{3.5}$$

For convenience we also have

$$\frac{R}{Z_0} = f_g \equiv \text{geometrical impedance factor}$$

$$Z_0 = \left[\frac{\mu_0}{\varepsilon_0}\right]^{\frac{1}{2}} \equiv \text{wave impedance of free space } (\simeq 377\,\Omega) \tag{3.6}$$

The transmitted field then can also be written as

$$\tilde{\vec{E}}^{(sc)}(\vec{0},s) = \frac{s e^{-\gamma r}}{2\pi c r f_g}\,\vec{h}_a(-\vec{1}_i,s)\,\tilde{V}^{(\text{trans})}(s)$$
$$\vec{E}^{(inc)}(\vec{0},t) = \frac{1}{2\pi c r f_g}\,\vec{h}_a(-\vec{1}_i,t)\circ\frac{dV^{(\text{trans})}\left(t-\frac{r}{c}\right)}{dt} \tag{3.7}$$

Combining transmission and reception, perhaps with two antennas near $\vec{r}$ (the observer) we have [4, 19]

$$\tilde{V}^{(rec)}(s) = \frac{s e^{-2\gamma r}}{8\pi^2 c f_g r^2}\,\tilde{\vec{h}}_{rec}(-\vec{1}_i,s)\cdot\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s)\cdot\tilde{\vec{h}}_{trans}(-\vec{1}_i,s)\tilde{V}^{(trans)}(s)$$
$$\tilde{V}^{(rec)}(t) = \frac{1}{8\pi^2 c f_g r^2}\,\vec{h}_{rec}(-\vec{1}_i,t)\overset{\circ}{\cdot}\overleftrightarrow{\Lambda}_b(\vec{1}_i,t)\overset{\circ}{\cdot}\tilde{\vec{h}}_{trans}(-\vec{1}_i,t)\circ\frac{dV^{(trans)}(t-2\frac{r}{c})}{dt} \tag{3.8}$$

as our basic radar equation in both frequency and time domains.

For an impulse radiating antenna [22] we have a constant aperture height $\vec{h}_a$ across a broad band of frequencies (a few decades [20]) on the main beam (boresight). Let us assume that the signals of interest are dominated by this band of frequencies so that we need not consider other details of the antenna response (prepulse, etc.). There is also a delay in the propagation of signals through the antenna, but this is a simple correction which need not influence our present discussion.

For a simple version of the radar equation, let the same (or an identical) antenna be used for both transmission and reception. Let $\vec{h}_a$ be frequency independent. Choose some particular linear polarization (h or v) so that we are considering $\Lambda_{b_{h,h}}$ or $\Lambda_{b_{v,v}}$.

Then a scalar form of (3.8) is

$$\tilde{V}^{(rec)}(s) = \frac{s h_a^2}{8\pi^2 c f_g r^2}\,\tilde{\Lambda}_b(s)\;\tilde{V}^{(trans)}(s)$$
$$\tilde{V}^{(rec)}(t) = \frac{h_a^2}{8\pi^2 c f_g r^2}\,\Lambda_b(t)\circ\frac{d\tilde{V}^{(trans)}(t)}{dt} \tag{3.9}$$

where the delay of 2 r/c has been deleted for simplicity. For the special case that the scattering dyadic is proportional to γ^n as in (2.5) the above reduces to

$$\begin{aligned} \tilde{V}^{(rec)}(s) &= \frac{h_a^2}{8\pi^2 f_g r^2} \gamma^{n+1} \mathrm{K} \; \tilde{V}^{(trans)}(s) \\ V^{(rec)}(t) &= \frac{h_a^2}{8\pi^2 f_g r^2} c^{-n-1} \mathrm{K} \frac{d^{n+1} V^{(trans)}(t)}{dt^{n+1}} \end{aligned} \tag{3.10}$$

4. Flat Surface with Normal Incidence

Consider a perfectly conducting flat disk of area A. For early-time purposes, this also includes other materials extending back from the disk (away from the radar [18 (Section 3.2.6)].

For the simple case of normal incidence we can evaluate the backscattering dyadic by treating the disk as an antenna aperture S_a with tangential electric field given by $-\tilde{\vec{E}}^{(inc)}$. Then like an IRA [2], we can evaluate the scattered far field as

$$-\tilde{\vec{E}}_f^{(sc)}(\vec{r},s) = \frac{se^{-\gamma r}}{2\pi cr} \int_{S_a} \left[-\tilde{\vec{E}}^{(inc)}(\vec{0},s) \right] dS = -\frac{\gamma e^{-\gamma r} A}{2\pi\, r} \tilde{\vec{E}}^{(inc)}(\vec{0},s) \tag{2.8}$$

Then from (2.2) we identify

$$\begin{aligned} \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) &= -2\gamma A \overleftrightarrow{1}_i \\ \overleftrightarrow{1}_i &= \overleftrightarrow{1} - \vec{1}_i \vec{1}_i \equiv \text{transverse identity} \\ \overleftrightarrow{1}_i &\equiv \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \equiv \text{identity} \end{aligned} \tag{2.9}$$

This has the form in (2.5) with the additional property of no depolarization. In time domain we then have

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,t) \circ = -\frac{2A}{c} \overleftrightarrow{1}_i \frac{d}{dt} \tag{2.10}$$

5. Curved Surface with Specular Scattering

Now let there be a convex perfectly conducting scattering surface S. Let $\vec{r} = \vec{0}$ be the first point on the scatterer touched by the incident wave. As a smooth convex (incident-wave side) scatterer this point is a specular point, i.e.,

$$\vec{1}_i \perp S \quad \text{at} \quad \vec{r} = \vec{0} \tag{5.1}$$

The high-frequency, early-time scattering near a specular point is governed by the curvature of the wavefront as it leaves the scatterer [16(Section 1.4.3.5)].

The curvature of S at such a point is given by the two principal radii of curvature r_1 and r_2. We also have the total or Gaussian curvature [14] as the reciprocal of the corresponding radius of curvature as

$$r_0 = \left[r_1 r_2\right]^{\frac{1}{2}} \tag{5.2}$$

Now the wave leaving the specular point has curvatures exactly twice those of the surface. An incident plane wave (zero curvatures) has an extra distance to reach S away from $\vec{r} = \vec{0}$ and an equal extra distance to return from S.

The high-frequency, early-time scattered field then takes the form

$$\tilde{\vec{E}}_f^{(sc)}(\vec{r},s) = e^{-\gamma r}\frac{r_0}{2r}\left[-\tilde{\vec{E}}^{(inc)}(\vec{0},s)\right] \tag{5.3}$$

From (2.2) we identify

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = -2\pi r_0 \overleftrightarrow{1}_i \tag{5.4}$$

This has the form in (2.5) with no depolarization. In time domain the scattering dyadic operator becomes

$$\vec{\Lambda}_b(\vec{1}_i,t)\circ \; = -2\pi r_0 \overleftrightarrow{1}_i \tag{5.5}$$

(i.e., just dot multiplication). As a simple example this formula applies to a perfectly conducting sphere of radius a.

6. Circular Cone on Axis

Moving on to scatterers with yet smaller backscattering consider a perfectly conducting circular cone as in Fig. 6.1. From [18] we have the general result

$$\begin{aligned} \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; s) &= \gamma^{-1}\,\overleftrightarrow{K}(\vec{1}_o, \vec{1}_i) \\ \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t)\circ &= c\,\overleftrightarrow{K}(\vec{1}_o, \vec{1}_i)\int_{-\infty}^{t}(\quad)dt' \end{aligned} \tag{6.1}$$

So we need to have some estimate of $\overleftrightarrow{K}$.

For backscattering we have

$$\overleftrightarrow{K}_b(\vec{1}_i) = \overleftrightarrow{K}(-\vec{1}_i, \vec{1}_i) \tag{6.2}$$

Referring to Fig. 6.1 we have axial backscattering defined by

$$\vec{1}_o = -\vec{1}_i = \vec{1}_z \tag{6.3}$$

on a perfectly conducting circular cone defined by the angle θ_1 from the z axis with $\pi/2 < \theta_1 < \pi$. This is treated in some detail in [17 (Section 18.4.3)]. For a wide cone (θ_1 near $\pi/2$) we have

$$\overleftrightarrow{K}_b(-\vec{1}_z) \simeq -\pi\cos^{-2}(\theta_1)\,\overleftrightarrow{1}_i = -\pi\sin^{-2}\left(\theta_1 - \frac{\pi}{2}\right)\overleftrightarrow{1}_i \tag{6.4}$$

and for a thin cone (θ_1 near π) we have

$$\overleftrightarrow{K}_b(-\vec{1}_z) \simeq -4\pi\sin^{2}\left(\frac{1}{2}[\pi-\theta_1]\right)\overleftrightarrow{1}_i \simeq -\pi\left[\pi-\theta_1\right]^{2}\overleftrightarrow{1}_i \tag{6.5}$$

For off-axis backscattering the scattering is greater and the reader may consult the reference.

While a thin cone has reduced scattering, real cones are not semiinfinite, but are truncated or mounted on other structures which can give larger scattering.

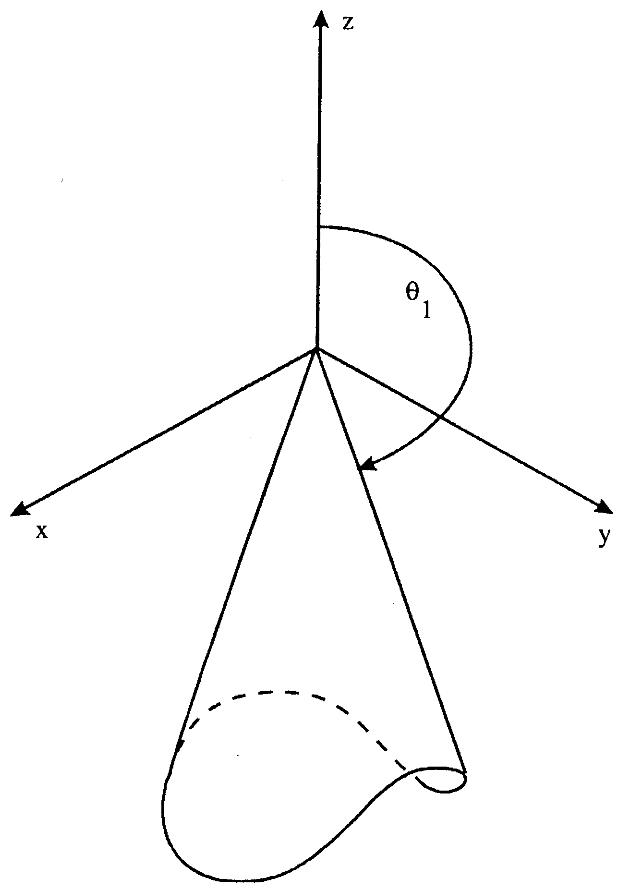

Fig. 6.1 Circular-Cone Scatterer

7. Thin-Wire at Broadside

Previous examples are associated with early-time scattering. For late-time scattering resonances are a feature of interest for which the singularity expansion method (SEM) is the appropriate way to view this phenomenon. In this form the backscattering dyadic can be expressed as [7, 9]

$$\tilde{\overleftrightarrow{\Lambda}}_b(\overrightarrow{1}_i,s) = \sum_\alpha [s-s_\alpha]^{-1}\,\overrightarrow{c}_\alpha(\overrightarrow{1}_i)\,\overrightarrow{c}_\alpha(\overrightarrow{1}_i) \;+\; \text{possible entire function} \tag{7.1}$$

The entire-function contribution is an early-time contribution, which we neglect here to look at the amplitude of the resonances. For calculating $\overrightarrow{c}_\alpha$ we have the general formulae

$$\begin{aligned}
\overrightarrow{c}_\alpha(\overrightarrow{1}_i) &= w_\alpha\,\overrightarrow{C}_\alpha(\overrightarrow{1}_i)\\
\overrightarrow{C}_\alpha(\overrightarrow{1}_i) &= \left\langle \overrightarrow{1}_i\, e^{-\gamma_\alpha \overrightarrow{1}_i\cdot\overrightarrow{r}'}\,;\, \overrightarrow{j}_\alpha(\overrightarrow{r}')\right\rangle\\
w_\alpha^2 &= -s_\alpha\mu_0\left\langle \overrightarrow{j}_\alpha(\overrightarrow{r});\frac{\partial}{\partial s}\tilde{\overleftrightarrow{Z}}(\overrightarrow{r},\overrightarrow{r}';s)\Big|_{s=s_\alpha};\overrightarrow{j}_\alpha(\overrightarrow{r}')\right\rangle^{-1}\\
s_\alpha &\equiv \text{natural frequency}\\
\overrightarrow{j}_\alpha(\overrightarrow{r}') &\equiv \text{natural current mode}\\
\left\langle \tilde{\overleftrightarrow{Z}}(\overrightarrow{r},\overrightarrow{r}';s_\alpha);\, \overrightarrow{j}_\alpha(\overrightarrow{r}')\right\rangle &= \overrightarrow{0}\\
\tilde{\overleftrightarrow{Z}}(\overrightarrow{r},\overrightarrow{r}';s) &\equiv \text{integral equation kernel}
\end{aligned} \tag{7.2}$$

Consider the thin wire of length ℓ and radius b, as in Fig. 7.1, which has been the subject of numerous investigations. The lowest order natural frequency is [5, 15]

$$\frac{s_1\ell}{\pi c} \simeq -0.082 + j\,0.93 \;\text{ for }\; \frac{\ell}{2b} = 100\,,\quad \ell n\left(\frac{\ell}{b}\right) \simeq 5.3 \tag{7.3}$$

We also include the conjugate natural frequency s_1^*. This fits into a damped sinusoid of the form

$$\frac{1}{2}\left[e^{s_\alpha t+\chi} + e^{s_\alpha^* t+\chi^*}\right]u(t) = e^{\mathrm{Re}(s_\alpha t+\chi)}\cos(\mathrm{Im}(s_\alpha t+\chi))u(t) \tag{7.4}$$

$e^{s_\alpha^* t+\chi^*}$

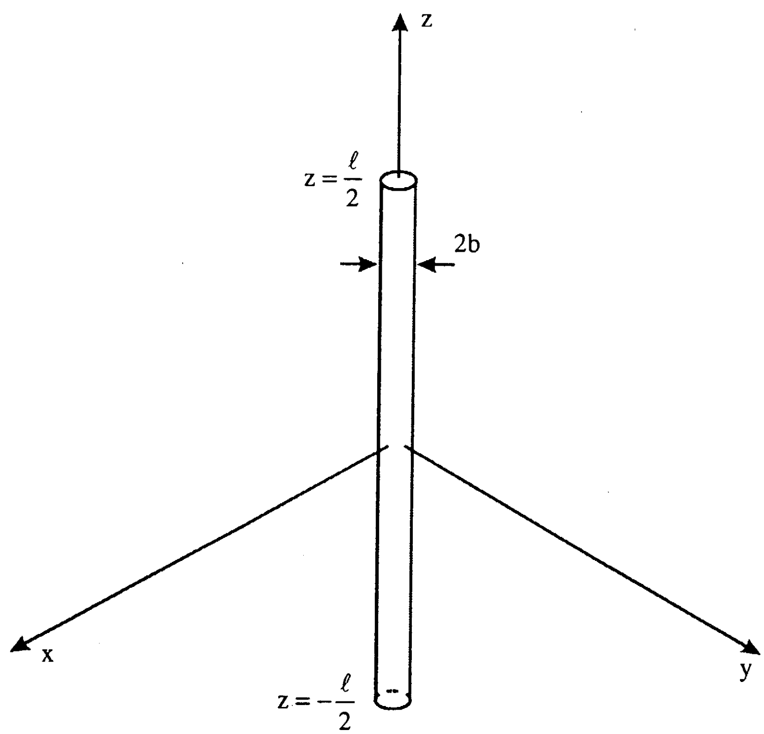

Fig. 7.1 Thin-Wire Scatterer

From this we find the number of cycles to decay to e^{-1} as [13]

$$\Delta^{-1} = \frac{Q}{\pi} = -\frac{1}{2\pi}\frac{\mathrm{Im}[s_1]}{\mathrm{Re}[s_1]} \simeq 1.8 \tag{7.5}$$

where Δ is also called the logarithmic decrement. For purposes of estimating the transient amplitude of such a resonance we can use the value at $t = 0$.

For a simple analytic model of the scattering (consistent with the more accurate numerical models) we have the current on the wire (first pole pair) as [11]

$$\tilde{I}(z,s) \simeq \tilde{\vec{E}}^{(inc)}(\vec{0},s) \cdot \vec{1}_z \frac{4c}{Z_0 \ell n\left(\frac{\ell}{b}\right)} \cos\left(\frac{\pi z}{\ell}\right)\left[\left[s-s_1\right]^{-1} + \left[s-s_1^*\right]^{-1}\right] \tag{7.6}$$

where the incident field is taken as parallel to the wire (i.e., to $\vec{1}_z$) and $\vec{1}_i \perp \vec{1}_z$ (broadside). One can extend this to other angles as approximately $\sin(\theta_i)$ where θ_i is the direction of incidence as measured from the z axis.

The current produces a scattered far field (broadside) as [7]

$$\begin{aligned}
\tilde{\vec{E}}_f^{(sc)}(\vec{r},s) &= -\frac{s\mu_0 e^{-\gamma r}}{4\pi r}\int_V e^{\gamma \vec{1}_o \cdot \vec{r}\,'} \overleftrightarrow{1}_i \cdot \tilde{\vec{J}}(\vec{r},s)dV \\
&= -\frac{s\mu_0 e^{-\gamma r}}{4\pi r}\vec{1}_z \int_{-\ell/2}^{\ell/2} \tilde{I}(z,s)dz \\
&= -\frac{s}{r}\frac{2\ell e^{-\gamma r}}{\pi^2 \ell n(\ell/b)}\left[\left[s-s_1\right]^{-1} + \left[s-s_1^*\right]^{-1}\right]\tilde{\vec{E}}^{(inc)}(\vec{0},s)
\end{aligned} \tag{7.7}$$

The scattering dyadic is then

$$\begin{aligned}
\overleftrightarrow{\Lambda}_b(s) &= -\frac{8\ell}{\pi \ell n(\ell/b)} s\left[\left[s-s_1\right]^{-1} + \left[s-s_1^*\right]^{-1}\right]\vec{1}_z\vec{1}_z \\
&\simeq -\frac{8\ell}{\pi \ell n(\ell/b)}\left[\frac{s_1}{s-s_1} + \frac{s_1^*}{s-s_1^*}\right]\vec{1}_z\vec{1}_z
\end{aligned} \tag{7.8}$$

with magnitude about 5.4 ℓ at first resonance ($s = j\omega = j\,\mathrm{Im}(s_1)$). This can be extended off broadside with an approximate $\sin^2(\theta_i)\,\vec{1}_{\theta_i}\,\vec{1}_{\theta_i}$ factor.

In order to estimate the transient backscattered signal consider the simple scalar form in (3.9) with $\vec{h}_a // \vec{1}_z$ and, of course, $\tilde{\Lambda}_b(s)$ being the z, z component. Then for a resonant scatterer we need to evaluate all terms except $[s - s_1]^{-1}$ and $[s - s_1^*]^{-1}$ (which become damped sinusoids) at the resonance giving

$$\max_t V^{(rec)}(t) \simeq \frac{h_a^2}{8\pi^2 c f_g r^2}\left[\frac{8\ell}{\pi \ell n(\ell / b)}\right] 2|s_1|\left|\tilde{V}^{(trans)}(s_1)\right| \tag{7.9}$$

We have already assumed that h_a is approximately frequency independent for frequencies of interest (around s_1 in this case). This leaves the characteristics of $V^{(trans)}(s)$ to be evaluated near s_1. For a simple case of interest let

$$V^{(trans)}(t) = V_o^{(trans)} u(t)\,,\ \tilde{V}^{(trans)}(s) = \frac{V_o^{(trans)}}{s} \tag{7.10}$$

Then (7.9) can be rewritten as

$$\max_t V_t^{(rec)} \simeq \frac{h_a^2}{8\pi^2 f_g r^2}\left[\frac{8\ell}{\pi \ell n(\ell / b)}\right] 2\frac{|s_1|}{c} V_0^{(trans)} \tag{7.11}$$

This requires that the rise time (t_{mr}) based on the slope of $V^{(trans)}(t)$ be sufficiently short (i.e., $<< |s_1|^{-1}$). This allows us to define an effective backscattering dyadic to be used with $V_0^{(trans)}$ in a formula as in (3.10) as

$$\overleftrightarrow{K}_b^{(eff)} \equiv \frac{16}{\ell n(\ell / b)}\,\frac{|s_1|\ell}{\pi c}\,\vec{1}_z\,\vec{1}_z \simeq 2.8\,\vec{1}_z\,\vec{1}_z \tag{7.12}$$

which is dimensionless and independent of ℓ. This applies only to the magntitude of the first resonance and neglects any early-time contribution.

8. Two-Way Transmission through Interface

An additional complication occurs when the scatterer is in a separate medium (e.g., soil) from that (e.g., air) surrounding the antenna(s). Such is the case in searching for buried targets such as mines and unexploded ordnance (UXO) [21].

Figure 8.1 shows the problem geometry with an incident plane wave incident on a plane interface. We are interested in the transmission coefficient for both vertical and horizontal polarization as indicated. The scattering from the buried target will follow the same path back to the radar and is similarly evaluated. So we define

$$\begin{aligned} \tilde{T}^{(in)}(\psi_i,s) &= \begin{pmatrix} \tilde{T}_v^{(in)}(\psi_i,s) & 0 \\ 0 & \tilde{T}_h^{(in)}(\psi_i,s) \end{pmatrix} \\ \tilde{T}^{(out)}(\psi_i,s) &= \begin{pmatrix} \tilde{T}_v^{(out)}(\psi_i,s) & 0 \\ 0 & \tilde{T}_h^{(out)}(\psi_i,s) \end{pmatrix} \end{aligned} \tag{8.1}$$

in two-dimensional form, since we have only transverse electric fields. Here ψ_i is the direction of incidence measured from the z axis with

$$\cos(\psi_i) = -\vec{1}_i \cdot \vec{1}_z \tag{8.2}$$

Note the direction of incidence $\vec{1}_t$ on the scatterer in the lower medium. For this purpose ψ_t is needed when evaluating the scattering dyadic there. For convenience, however, both $\tilde{T}^{(in)}$ and $\tilde{T}^{(out)}$ will be referenced to ψ_i.

In the radar equation in Section 3 we need to include $\tilde{T}^{(in)}$ to modify the incident field and $\tilde{T}^{(out)}$ to modify the scattered field giving

$$\tilde{V}^{(rec)}(s) = \frac{se^{-2\gamma r}}{8\pi^2 cfgr^2} \tilde{\vec{h}}^{(rec)}(-\vec{1}_i,s) \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_t,s) \cdot \tilde{T}^{(in)}(\psi_i,s) \cdot \tilde{\vec{h}}^{(trans)}(-\vec{1}_i,s)\tilde{V}^{(trans)}(s) \tag{8.3}$$

where the depth of burial is small compared to r to avoid a correction for the wavefront curvature in passing through the interface. Thus we have an effective backscattering dyadic

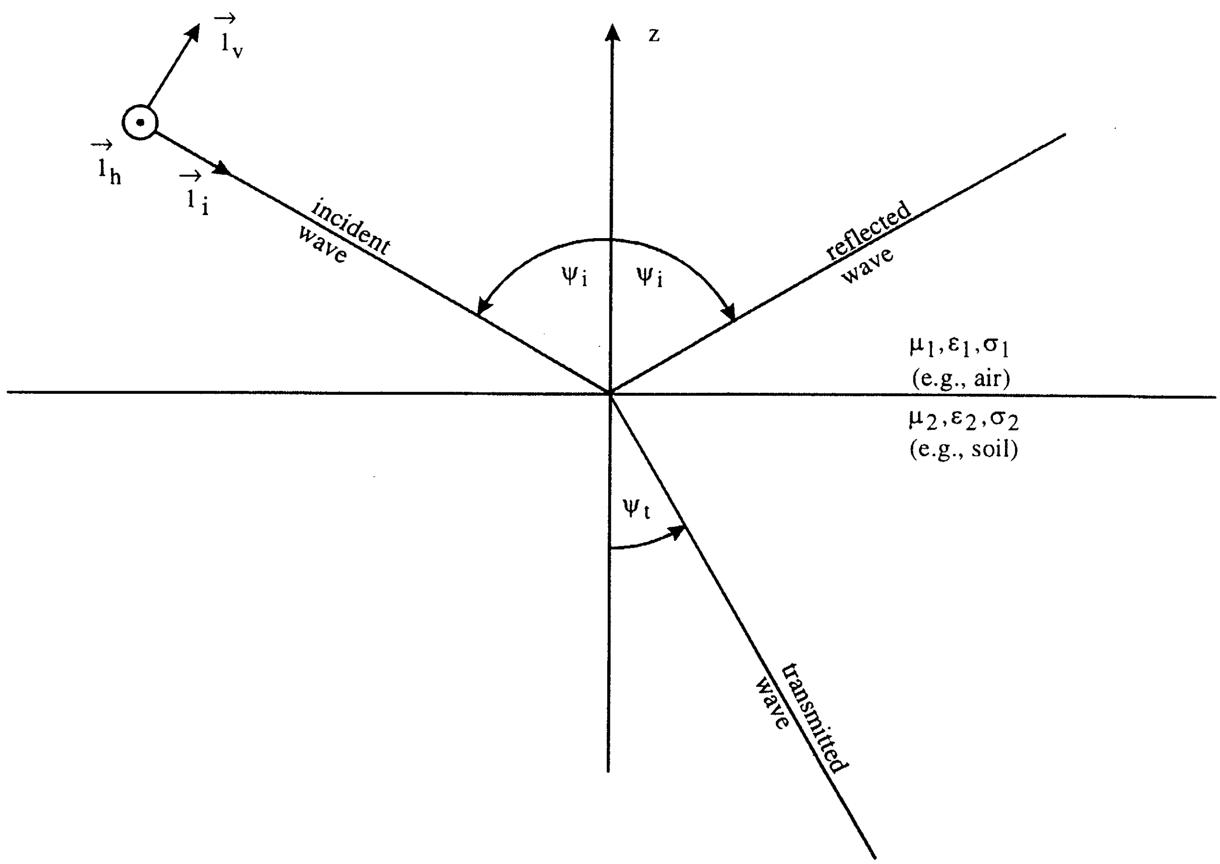

Fig. 8.1 Plane-Wave Transmission through Interface.

$$\tilde{\overleftrightarrow{\Lambda}}_b^{(eff)}(\vec{1}_i,s) = \tilde{T}^{(out)}(\psi_i,s) \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_t,s) \cdot \tilde{T}^{(in)}(\psi_i,s) \tag{8.4}$$

Reciprocity requires

$$\tilde{\overleftrightarrow{\Lambda}}_b^{(eff)}(\vec{1}_i,s) = \tilde{\overleftrightarrow{\Lambda}}^{(eff)T}(\vec{1}_i,s) = \tilde{T}^{(in)}(\psi_i,s) \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_t,s) \cdot \tilde{T}^{(out)}(\psi_i,s) \tag{8.5}$$

showing that $\tilde{T}^{(in)}$ and $\tilde{T}^{(out)}$ can be interchanged.

For cases of diagonal $\tilde{\overleftrightarrow{\Lambda}}_b$ then $\tilde{\overleftrightarrow{\Lambda}}_b^{(eff)}$ is also diagonal and we have

$$\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}_b^{(eff)}(\vec{1}_i,s) &= \tilde{T}(\psi_i,s) \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_t,s) = \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_t,s) \cdot \tilde{T}(\psi_i,s) \\
\tilde{T}(\psi_i,s) &= \tilde{\overleftrightarrow{T}}^{(out)}(\psi_i,s) \cdot \tilde{\overleftrightarrow{T}}^{(in)}(\psi_i,s) = \tilde{\overleftrightarrow{T}}^{(in)}(\psi_i,s) \cdot \tilde{\overleftrightarrow{T}}^{(out)}(\psi_i,s) \\
&= \begin{pmatrix} \tilde{T}_h(\psi_i,s) & 0 \\ 0 & \tilde{T}_v(\psi_i,s) \end{pmatrix} \\
&= \begin{pmatrix} \tilde{T}_h^{(out)}(\psi_i,s)\tilde{T}_h^{(in)}(\psi_i,s) & 0 \\ 0 & \tilde{T}_v^{(out)}(\psi_i,s)\tilde{T}_v^{(in)}(\psi_i,s) \end{pmatrix}
\end{aligned} \tag{8.6}$$

Such occurs in the case of scatterers with a symmetry plane containing the z axis and parallel to $\vec{1}_i$ [8].

Transmission and reflection of plane waves is a well understood problem [10, 12]. Referring to Fig. 8.1 we have for the two media

$$\begin{aligned}
\tilde{\gamma}_n &= \left[s\mu_n\left[\sigma_n + s\varepsilon_n\right]\right]^{\frac{1}{2}} \equiv \text{propagation constants} \\
\tilde{Z}_n &= \left[\frac{s\mu_n}{\sigma_n + s\varepsilon_n}\right]^{\frac{1}{2}} \equiv \text{wave impedances} \\
n &= 1,2
\end{aligned} \tag{8.7}$$

and for later use we have the "simple dielectric" case as

$$\mu_1 = \mu_2 = \mu_0 \ , \quad \sigma_2 = \sigma_1 = 0 \ , \quad \varepsilon_r \equiv \frac{\varepsilon_2}{\varepsilon_1} > 1 \tag{8.8}$$

Matching the propagation of the incident and transmitted waves along the interface gives (Snell's law)

$$\tilde{\gamma}_1 \sin(\psi_i) = \tilde{\gamma}_2 \sin(\psi_t) \tag{8.9}$$

relating $\tilde{\psi}_t$ to ψ_i through the media parameters and frequency.

Following [10] we have for vertical polarization

$$\begin{aligned}
\tilde{T}_{\text{v}}^{(in)}(\psi_i,s) &= \frac{\tilde{E}_2}{\tilde{E}_1} = \frac{\tilde{Z}_2}{\tilde{Z}_1}[1+\tilde{r}_{\text{v}}] = \frac{\tilde{Z}_2}{\tilde{Z}_1}2[1+\tilde{\chi}_{\text{v}}]^{-1} \\
\tilde{\chi}_{\text{v}} &= \frac{\tilde{Z}_2}{\tilde{Z}_1}\frac{\cos(\tilde{\psi}_t)}{\cos(\psi_i)} \\
&= \frac{\tilde{Z}_2}{\tilde{Z}_1}\sec(\psi_i)\left[1-\left[\frac{\tilde{\gamma}_1}{\tilde{\gamma}_2}\right]^2 \sin^2(\psi_i)\right]^{\frac{1}{2}} \\
&= \sec(\psi_i)\left[\left[\frac{\tilde{Z}_2}{\tilde{Z}_1}\right]^2 - \left[\frac{\sigma_1+s\varepsilon_1}{\sigma_1+s\varepsilon_2}\right]^2 \sin^2(\psi_i)\right]^{\frac{1}{2}} \\
&= \varepsilon_r^{-1}\sec(\psi_i)\left[\varepsilon_r - \sin^2(\psi_i)\right]^{\frac{1}{2}} \quad \text{(simple dielectric)}
\end{aligned} \tag{8.10}$$

Interchanging the roles of incident and transmitted waves gives

$$\tilde{T}_{\text{v}}^{(out)}(\psi_i,s) = \frac{\tilde{Z}_1}{\tilde{Z}_2}2\left[1+\chi_{\text{v}}^{-1}\right]^{-1} \tag{8.11}$$

these can be combined to give

$$\begin{aligned}
\tilde{T}_{\text{v}}(\psi_i,s) &= 4[1+\tilde{\chi}_{\text{v}}]^{-1}\left[1+\tilde{\chi}_{\text{v}}^{-1}\right]^{-1} \\
&= 4\left[2+\tilde{\chi}_{\text{v}}+\tilde{\chi}_{\text{v}}^{-1}\right]^{-1}
\end{aligned} \tag{8.12}$$

For horizontal polarization we have

$$\tilde{T}_h^{(in)}(\psi_i,s) = \frac{\tilde{E}_2}{\tilde{E}_1} = 1 + \tilde{r}_h = 2[1+\tilde{\chi}_h]^{-1}$$

$$
\begin{aligned}
\tilde{\chi}_h &= \frac{\tilde{Z}_1 \cos(\tilde{\psi}_t)}{\tilde{Z}_2 \cos(\tilde{\psi}_i)} \\
&= \frac{\tilde{Z}_1}{\tilde{Z}_2} \sec(\psi_i) \left[1 - \left[\frac{\tilde{\gamma}_1}{\tilde{\gamma}_2} \right]^2 \sin^2(\psi_1) \right]^{\frac{1}{2}} \\
&= \sec(\psi_i) \left[\left[\frac{\tilde{Z}_1}{\tilde{Z}_2} \right]^2 - \left[\frac{\mu_1}{\mu_2} \right]^2 \sin^2(\psi_1) \right]^{\frac{1}{2}} \\
&= \sec(\psi_i) \left[\varepsilon_r - \sin^2(\psi_i) \right]^{\frac{1}{2}} \quad \text{(simple dielectric)}
\end{aligned}
\tag{8.13}
$$

$\tilde{Z}_2 \cos(\psi_i)$ ψ_i ψ_i ψ_i

Interchanging the roles of incident and transmitted waves gives

$$
\tilde{T}_h^{(out)}(\psi_i, s) = 2\left[1 + \tilde{\chi}_h^{-1}\right]^{-1}
\tag{8.14}
$$

These can be combined to give

$$
\begin{aligned}
\tilde{T}_h(\psi_i, s) &= 4\left[1 + \tilde{\chi}_h\right]^{-1}\left[1 + \tilde{\chi}_h^{-1}\right]^{-1} \\
&= 4\left[2 + \tilde{\chi}_h + \tilde{\chi}_h^{-1}\right]^{-1}
\end{aligned}
\tag{8.15}
$$

For the case of normal incidence we have

$$
\begin{aligned}
\tilde{\psi}_t &= \psi_i = 0 \\
\tilde{\chi}_v &= \frac{\tilde{Z}_2}{\tilde{Z}_1} = \tilde{\chi}_h^{-1} \\
\tilde{T}_v(0, s) &= \tilde{T}_h(0, s) = 4\left[2 + \frac{\tilde{Z}_1}{\tilde{Z}_2} + \frac{\tilde{Z}_2}{\tilde{Z}_1}\right]^{-1} \\
&= 4\left[2 + \varepsilon_r^{\frac{1}{2}} + \varepsilon_r^{\frac{1}{2}}\right]^{-1} \quad \text{(simple dielectric)}
\end{aligned}
\tag{8.16}
$$

For $\varepsilon_r = 9$ then $\tilde{T}_v = 3/4 = 0.75$ and this not a big effect. There is also the Brewster angle at which there is no reflected field a the interface. For vertical polariztion we have

ψ_i

$$
\begin{aligned}
\tilde{\chi}_{\mathrm{v}} &= 1\ , \quad \tilde{T}_{\mathrm{v}}(\psi_i, s) = 1 \\
\cos^2(\psi_i) &= \left[\frac{\tilde{Z}_2}{\tilde{Z}_1}\right]^2 - \left[\frac{\sigma_1 + s\varepsilon_1}{\sigma_2 + s\varepsilon_2}\right]^2 \sin^2(\psi_i) \\
&= \varepsilon_r^{-1} - \varepsilon_r^{-2}\sin^2(\psi_i) \quad \text{(simple dielectric)} \\
\tan(\psi_i) &= \varepsilon_r^{\frac{1}{2}} = \tan(\psi_{iB}) \quad \text{(simple dielectric)}
\end{aligned}
\tag{8.17}
$$

For horizontal polarization one can have a Brester angle if there is a difference in permeabilities between the two media. For the case of a simple dielectric we have at the Brewster angle

$$
\begin{aligned}
\tilde{\chi}_h &= \sec(\psi_{iB})\left[\varepsilon_r - \sin^2(\psi_{iB})\right]^{\frac{1}{2}} \\
&= \varepsilon_r \qquad \text{(from (8.17))} \\
\tilde{T}_h(\psi_{iB}, s) &= 4\left[2 + \varepsilon_r + \varepsilon_r^{-1}\right]^{-1}
\end{aligned}
\tag{8.18}
$$

For $\varepsilon_r = 9$ then $\tilde{T}_h = 9/25 = 0.36$. So while $\tilde{T}_{\mathrm{v}}$ has been improved at the Brewster angle, $\tilde{T}_h$ has been diminished somewhat.

9. Concluding Remarks

Hopefully this compilation of simple scattering formulae and combination of these with antenna characteristics will prove useful in estimating signal strengths for detection of various scatterers both in the air and buried. One could add various other canonical scatterers, but this should help one bound the problem. The antennas are of the IRA variety with their wideband ratio, and for which the formulae in time domain simplify somewhat. Alternately one can deconvolve the response of other types of antennas (if they have sufficient bandwidth) to obtain similar results.

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IN 559

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Dear Dr. Baum:

The enclosed paper is submitted to you for publication in the EMP Note Series as an Interaction Note. It contains no proprietary or classified information and can be published in the open literature.

Sincerely,

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