Interaction Notes

Note 563

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The Physical-Optics Approximation and Early-Time Scattering

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Abstract

The physical optics approximation is often used to describe the scattering from perfectly conducting targets before information from the shadow boundary can reach the observer. In some cases, including bodies of revolution, it seems often to be a good approximation for axial backscattering. However, for other target shapes, it is sometimes a (very) bad approximation. The validity is discussed in the context of several selected scatterers.

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1. Introduction

An interesting question in electromagnetic scattering concerns the validity of physical optics (PO) to describe the early-time scattering. More precisely, as one goes out in time the approximation might degrade, even before the shadow boundary is seen in the scattering. By looking at various canonical examples with known exact solutions, or about which some exact statements can be made concerning features of the scattering, we can test the PO approximation. The present paper concentrates on backscattering (or monostatic scattering).

For our present purposes we have an incident plane wave

$$\vec{E}^{(inc)}(\vec{r},t) = E_0 f\left(t - \frac{\vec{1}_i \cdot \vec{r}}{c}\right)\vec{1}_e , \quad \tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_0 \tilde{f}(s) e^{-\gamma \vec{1}_i \cdot \vec{r}}\,\vec{1}_e$$

$$\vec{H}^{(inc)}(\vec{r},t) = \frac{E_0}{Z_0} f\left(t - \frac{\vec{1}_i \cdot \vec{r}}{c}\right)\vec{1}_h , \quad \tilde{\vec{H}}^{(inc)}(\vec{r},s) = \frac{E_0}{Z_0} \tilde{f}(s) e^{-\gamma \vec{1}_i \cdot \vec{r}}\,\vec{1}_h$$

$$\vec{1}_e \cdot \vec{1}_i = 0 = \vec{1}_h \cdot \vec{1}_i$$

$$\vec{1}_i \times \vec{1}_e = \vec{1}_h$$

$\vec{1}_i \equiv$ direction of incidence

$\vec{1}_e \equiv$ polarization

$f(t) \equiv$ waveform

$c = [\mu_0 \varepsilon_0]^{-\frac{1}{2}} \equiv$ speed of light

$Z_0 = \left[\frac{\mu_0}{\varepsilon_0}\right]^{\frac{1}{2}} \equiv$ wave impedance of free space

$\sim \equiv$ two-sided Laplace transform over time t

$s = \Omega + j\omega \equiv$ Laplace-transform variable or complex frequency

$\gamma = \frac{s}{c} \equiv$ propagation constant

$\overleftrightarrow{1}_i \equiv \overleftrightarrow{1} - \vec{1}_i \vec{1}_i \equiv$ transverse dyadic for incidence

$\overleftrightarrow{1}_o \equiv \overleftrightarrow{1} - \vec{1}_o \vec{1}_o \equiv$ transverse dyadic for scattering (1.1)

$\overleftrightarrow{1} \equiv \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \equiv$ three-dimensional identity

The scatterer (target) is located such that $\vec{r} = \vec{0}$ corresponds to the point on the target which the incident wave first touches. For backscattering this will give a convenient reference for $t = 0$.

The scattered far field is given by [7]

$$\vec{E}_r^{(sc)}(\vec{r},t) = \frac{1}{4\pi r}\overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i;t) \overset{\circ}{\cdot} \vec{E}^{(inc)}\left(\vec{0}, t - \frac{r}{c}\right)$$

$$\tilde{\vec{E}}_r^{(sc)}(\vec{r},s) = \frac{e^{-\gamma r}}{4\pi r}\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i;s) \overset{\circ}{\cdot} \tilde{\vec{E}}^{(inc)}(\vec{0}, s)$$

$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i;s) \equiv$ scattering dyadic

$\overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i;t)\circ \equiv$ scattering dyadic operator

$\circ \equiv$ convolution with respect to time (1.2)

$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i;t) \equiv$ scattering-dyadic impulse response

$$r \equiv |\vec{r}|$$

For backscattering we have

$$\vec{1}_o = -\vec{1}_i$$

$$\overleftrightarrow{\Lambda}_b(\vec{1}_i,t) \equiv \overleftrightarrow{\Lambda}_b(-\vec{1}_i, \vec{1}_i;t) \tag{1.3}$$

Reprocity implies

$$\overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i;t) = \overleftrightarrow{\Lambda}^T(-\vec{1}_i, -\vec{1}_o;t)$$

$$\overleftrightarrow{\Lambda}_b(\vec{1}_i,t) = \overleftrightarrow{\Lambda}_b^T(\vec{1}_i,t) \tag{1.4}$$

These are general relations pertaining to a wide variety of scatterers.

We find it notationally convenient to define multiple time integrals of the scattering-dyadic impulse response by

$$\int_{-\infty}^{t}\int_{-\infty}^{t_1}\cdots\int_{-\infty}^{t_{n-1}} \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t_n)\, dt_n dt_{n-1}\cdots dt_1$$

$$= \int_{0_-}^{t}\int_{0_-}^{t_1}\cdots\int_{0_-}^{t_{n-1}} \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t_n)\, dt_n dt_{n-1}\cdots dt_1$$

$$\equiv \overleftrightarrow{\Lambda}^{(n)}(\vec{1}_o, \vec{1}_i; t) \tag{1.5}$$

$$\overleftrightarrow{\Lambda}^{(0)}(\vec{1}_o, \vec{1}_i; t) \equiv \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t)$$

$$\tilde{\overleftrightarrow{\Lambda}}^{(n)}(\vec{1}_o, \vec{1}_i; s) = s^{-n}\, \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s)$$

and similarly in the case of backscattering. In the general case the temporal integrals are from $-\infty$. However, for our choice of temporal origin the first response in the case of backscattering is at $t = 0$ with 0_- to include integration through singularities (e.g., delta functions) at the origin. For bistatic scattering the beginning of temporal integration needs to be appropriately adjusted.

A parameter related to the scattering dyadic is the (scattering) cross section (area) defined by

$$\tilde{A}^{(sc)}(j\omega) \equiv 4\pi r^2 \frac{|\tilde{\vec{E}}_r^{(sc)}(\vec{r}, j\omega) \cdot \vec{1}_m|^2}{|\tilde{\vec{E}}_r^{(inc)}(\vec{0}, j\omega) \cdot \vec{1}_e|^2}$$

$\vec{1}_m$ = polarization measured of scattered far field

$$\tilde{A}^{(sc)}(j\omega) = \frac{1}{4\pi}\, |\vec{1}_m \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; j\omega) \cdot \vec{1}_e|^2$$

$$= \frac{1}{4\pi}\, \vec{1}_m \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; j\omega)\, \vec{1}_e \vec{1}_e \cdot \tilde{\overleftrightarrow{\Lambda}}^{\dagger}(\vec{1}_o, \vec{1}_i; j\omega) \cdot \vec{1}_m \tag{1.6}$$

$\dagger$ = adjoint = $T*$ = transpose conjugate

Note the suppression of phase, $\tilde{A}^{(sc)}(j\omega)$ being real valued. Trying to infer $\tilde{\overleftrightarrow{\Lambda}}$ from $\tilde{A}^{(sc)}$ even has an ambiguity of sign. The scattering dyadic is then the more fundamental parameter. The cross section can, however, be analytically continued into the complex s plane as

$$\tilde{A}^{(sc)}(s) = \frac{1}{4\pi}\, \vec{1}_m \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \vec{1}_e \vec{1}_e \cdot \tilde{\overleftrightarrow{\Lambda}}^{T}(\vec{1}_o, \vec{1}_i; -s) \cdot \vec{1}_m$$

$$= \frac{1}{4\pi}\, \vec{1}_m \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \vec{1}_e \vec{1}_e \cdot \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i, -\vec{1}_o; -s) \cdot \vec{1}_m \tag{1.7}$$

2. Physical-Optics Backscattering

The physical optics (PO) approximation, applicable to perfectly conducting scatterers, has the surface current density on the surface S of the scatterer (coordinates $\vec{r}_s$) given by

$$\vec{J}_s^{(po)}(\vec{r}_s,t) = 2\,\vec{1}_S(\vec{r}_s) \times \vec{H}^{(inc)}(\vec{r}_s,t)$$

$$\vec{1}_S \equiv \text{outward pointing normal to S at } \vec{r}_s \qquad (2.1)$$

As in Fig. 2.1 the coordinate origin is at $\vec{r} = 0$ on S with the z axis and $\vec{1}_i$ there perpendicular to S and pointing into the scatterer. The PO approximation applies only to illuminated portions of the target, which implies, among other things

$$\vec{1}_S(\vec{r}_s) \cdot \vec{1}_z = \vec{1}_S(\vec{r}_s) \cdot \vec{1}_i \geq 0 \qquad (2.2)$$

More generally we restrict our considerations to times only up to when the wave reaches the shadow boundary (including the extra time for the signal from the shadow boundary to reach the observer).

With the incident field as in (1.1) we have

$$\vec{J}_s^{(po)}(\vec{r},t) = 2\frac{E_0}{Z_0} f\left(t - \frac{\vec{1}_i \cdot \vec{r}_s}{c}\right) \vec{1}_S(\vec{r}_s) \times [\vec{1}_z \times \vec{1}_e] \qquad (2.3)$$

The scattered far field can be written as an integral over the surface current density on the scatterer as [6]

$$\vec{E}_f^{(sc)}(\vec{r},t) = -\frac{\mu_0}{4\pi r}\frac{\partial}{\partial t}\left[\overleftrightarrow{1}_o \cdot \int_S \vec{J}\left(\vec{r}_s, t + \frac{\vec{1}_o \cdot [\vec{r}_s - \vec{r}]}{c}\right) dS\right] \qquad (2.4)$$

Using PO and specializing to backscattering we have

$$r = -z \quad (z \text{ negative})$$

$$\vec{E}_f^{(sc,po)}(z\,\vec{1}_z,t) = -\frac{\mu_0}{2\pi r}\frac{E_0}{Z_0}\frac{\partial}{\partial t}\left[\overleftrightarrow{1}_z \cdot \int_S \vec{1}_S(\vec{r}_s) \times [\vec{1}_z \times \vec{1}_e] f\left(t - \frac{2z_s + r}{c}\right) dS\right]$$

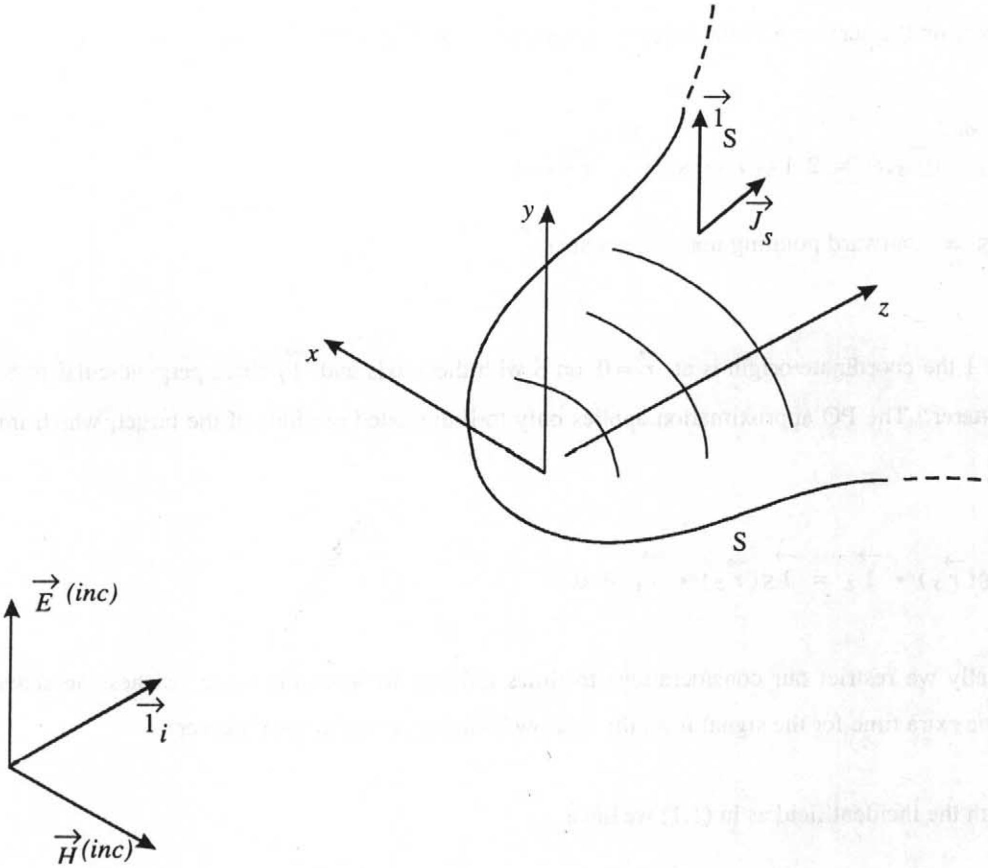

Fig. 2.1. Portion of Perfectly Conducting Scatterer Initially Illuminated

$$= -\frac{E_0}{2\pi c r} \vec{1}_e \frac{\partial}{\partial t} \int_S \vec{1}_S(\vec{r}_s) \cdot \vec{1}_z f\left(t - \frac{2z_s + r}{c}\right) dS \tag{2.5}$$

Note the proportionality to $\vec{1}_e$, i.e., no depolarization, even though we have not assumed z as a rotation axis of the scatterer. Choosing $f(t)$ as a delta function $\delta(t)$ we can write the backscattering dyadic impulse response from (1.2) as

$$\overleftrightarrow{\Lambda}_b^{(po)}(\vec{1}_z, t) = -\frac{2}{c} \vec{1}_z \frac{\partial}{\partial t} \int_S \delta\left(t - \frac{2z_s}{c}\right) \vec{1}_z \cdot \vec{1}_S(\vec{r}_s) dS \tag{2.6}$$

This fairly simple formula can be further manipulated for convenience in interpretation. On a plane of constant z_s S has a contour C and we define

$$\begin{aligned} A(z_s) &\equiv \text{area contained within the projection of C on the } xy \text{ plane (a disk)} \\ &\equiv \text{cross-sectional area of the scatterer on plane of constant } z_s \\ A(z_s) &= 0 \text{ for } z_s < 0 \end{aligned} \tag{2.7}$$

Notice that

$$\vec{1}_z \cdot \vec{1}_S(\vec{r}_s) = \text{projection of unit surface area of S at } \vec{r}_s \text{ on } xy \text{ plane} \tag{2.8}$$

Consider an incremental part of S between z_s and $z_s + \Delta z_s$ and integrate around C to give

$$\int_C \vec{1}_z \cdot \vec{1}_S(\vec{r}_s) dC = \frac{dA(z_s)}{dz_s} \tag{2.9}$$

with the remaining integration over z_s. This gives

$$\begin{aligned} \overleftrightarrow{\Lambda}_b^{(po)}(\vec{1}_z, t) &= -\frac{2}{c} \overleftrightarrow{1}_z \frac{\partial}{\partial t} \int_0^{z_{max}} \delta\left(t - \frac{2z_s}{c}\right) \frac{dA(z_s)}{dz_s} dz_s \\ z_{max} &= \text{minimum } z_s \text{ on shadow boundary} \end{aligned} \tag{2.10}$$

Noting the symmetry of the delta function this gives

$$\overset{\leftrightarrow}{\Lambda}_b^{(po)}(\vec{1}_z,t) = -\overset{\leftrightarrow}{1}_z \frac{\partial}{\partial t} \int_0^{\frac{2z_{\max}}{c}} \delta\left(\frac{2z_s}{c}-t\right)\frac{dA(z_s)}{dz_s} d\left(\frac{2z_s}{c}\right) = -\overset{\leftrightarrow}{1}_z \frac{\partial}{\partial t}\left[\left.\frac{dA(z_a)}{dz_s}\right|_{z_s=\frac{ct}{2}}\right] \tag{2.11}$$

Using the temporal-integration notation introduced in (1.5) gives

$$\begin{aligned}
\overset{\leftrightarrow}{\Lambda}_b^{(1,po)}(\vec{1}_z,t) &= -\overset{\leftrightarrow}{1}_z \left.\frac{dA(z_s)}{dz_s}\right|_{z_s=\frac{ct}{2}} = \text{step response} \\
\overset{\leftrightarrow}{\Lambda}_b^{(2,po)}(\vec{1}_z,t) &= -\overset{\leftrightarrow}{1}_z \int_{0_-}^{t} \left.\frac{dA(z_s)}{dz_s}\right|_{z_s=\frac{ct'}{2}} dt' \\
&= -\overset{\leftrightarrow}{1}_z \int_{0_-}^{z_s=\frac{ct}{2}} \frac{dA(z_s)}{dz_s} d\left(\frac{2z_s}{c}\right) \\
&= -\frac{2}{c}\overset{\leftrightarrow}{1}_z A(z_s)\Big|_{z_s=\frac{ct}{2}} = \text{ramp response}
\end{aligned} \tag{2.12}$$

in agreement with [3]. From a measurement of the backscattering one has a PO approximation of $A(z_s)$, a rather simple geometric interpretation.

3. Paraboloid Axial Backscattering

Now let the perfectly conducting scatterer be a paraboloid (a body of revolution). Introduce, for bodies of revolution, cylindrical coordinates (Ψ, ϕ, z) with

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \tag{3.1}$$

With subscripts these apply on S.

The paraboloid surface is given by

$$\begin{aligned} z_s &= \frac{\Psi_s^2}{4\mathrm{F}} \\ \mathrm{F} &= \text{focal length} \\ (\Psi, z) &= (0, \mathrm{F}) \equiv \text{focal point} \\ A(z_s) &= \pi \Psi_s^2 = 4\pi \mathrm{F}\, z_s \end{aligned} \tag{3.2}$$

The ramp response is then

$$\overleftrightarrow{\Lambda}_b^{(2,po)}(\vec{1}_z, t) = -\frac{2}{c}\vec{1}_z A(z_s)\Big|_{z_s = \frac{ct}{2}} = -4\pi \mathrm{F}\, tu(t)\, \overleftrightarrow{1}_z \tag{3.3}$$

The incident field being a ramp as well as the scattered field, then an arbitrary waveform scatters with the same waveform (up to the shadow boundary limit).

As discussed in [2], the PO solution is exact in this special case. In [1] this is shown to be exact in the near field provided the focal point is taken as the coordinate origin. It is to be noted that this applies bistatically as long as there is axial incidence. By reciprocity the result applies for off-axis incidence as well provided the observer is on the negative z axis and far away. Furthermore, it is shown in [1] that the axially incident wave can be an inhomogeneous TEM plane wave (such as on a TEM transmission line) and the exact scattered field can be readily calculated via a stereographic transformation, with the scattered waveform again replicating the incident waveform. (Shadow boundary and transmission-line conductors still limit the time of applicability.)

4. Circular-Cone Axial Backscattering

The pecfectly-conducting circular-conical surface is given by

$$\begin{aligned}
\Psi_s &= z_s \tan(\psi) \\
\psi &\equiv \text{angle of surface from } z \text{ axis} \\
&\equiv \text{interior semivertex angle} \\
A(z_s) &= \pi\Psi_s^2 = \pi z_s^2 \tan^2(\psi)
\end{aligned} \tag{4.1}$$

The ramp resonse is then

$$\overset{\leftrightarrow}{\Lambda}{}_b^{(2,po)}(\vec{r},t) = -\frac{2}{c}\overset{\leftrightarrow}{1}_z A(z_s)\Big|_{z_s=\frac{ct}{2}} = -\frac{\pi}{2}\tan^2(\psi)ct^2u(t)\overset{\leftrightarrow}{1}_z \tag{4.2}$$

As discussed in [4] the PO approximation has been shown to be asymptotically correct for $\psi \to 0$ and $\psi \to \pi/2$. In between these limiting cases the accuracy of the PO approximation is not yet well understood. Perhaps a numerical summation of the eigenfunctions representing the solution in [4] could help establish this.

In [6] it is shown that the exact solution for a cone of arbitrary cross section (including certain types which are not perfectly conducting) gives a scattering dyadic of the form

$$\begin{aligned}
\overset{\leftrightarrow}{\tilde{\Lambda}}(\vec{1}_o,\vec{1}_i;s) &= \frac{c}{s}\overset{\leftrightarrow}{K}(\vec{1}_o,\vec{1}_i) \\
\overset{\leftrightarrow}{\Lambda}(\vec{1}_o,\vec{1}_i;t) &= cu(t)\overset{\leftrightarrow}{K}(\vec{1}_o,\vec{1}_i) \\
\overset{\leftrightarrow}{\Lambda}{}^{(1)}(\vec{1}_o,\vec{1}_i;t) &= ctu(t)\overset{\leftrightarrow}{K}(\vec{1}_o,\vec{1}_i) \\
\overset{\leftrightarrow}{\Lambda}{}^{(2)}(\vec{1}_o,\vec{1}_i;t) &= \frac{1}{2}ct^2u(t)\overset{\leftrightarrow}{K}(\vec{1}_o,\vec{1}_i)
\end{aligned} \tag{4.3}$$

This is a very general and exact form for bistatic scattering with $\overset{\leftrightarrow}{K}$ independent of frequency. The form of (4.2) is in agreement with this. So we can say

$$\overset{\leftrightarrow}{K}{}_b^{(po)}(\vec{1}) = -\pi\tan^2(\psi)\overset{\leftrightarrow}{1}_z \tag{4.3}$$

The transverse-dyadic part $(\overset{\leftrightarrow}{1}_z)$ is exact due to the rotation symmetry. But, how accurate is the coefficient for intermediate ψ?

5. Backscattering from Targets Without Full Two-Dimensional Rotation and Reflection Symmetry

Symmetry considerations [5] have shown that targets with O_2 symmetry (all rotations and reflections in two dimensions) do not depolarize in axial backscattering. This is exact and does not rely on the PO approximation. Note that our previous examples have such symmetry and are sometimes called bodies of revolution (BORs). In addition, bodies with C_N symmetry for $N \geq 3$ (N-fold rotation axis, symmetry planes not being required) also do not depolarize in axial backscattering.

An interesting example which has none of the above symmetries is the angular sector shown in Fig. 5.1. For present purposes it is a perfectly conducting sheet confined to the $x = 0$ plane with

$$\begin{aligned} &|y| \leq z \tan(\psi) \quad , \quad z \geq 0 \\ &\psi \equiv \text{interior semivertex angle} \end{aligned} \tag{5.1}$$

This has the $x = 0$ plane and $y = 0$ plane as reflection symmetry planes giving the symmetry group

$$C_{2a} = R_x \otimes R_y \tag{5.2}$$

With $\vec{1}_e = \vec{1}_x$ (i.e., polarized perpendicular to the flat sheet) the scattering is zero. Since this is a special case of a perfectly conducting cone then (4.3) exactly applies. So we can write

$$\overleftrightarrow{K}_b(\vec{1}_z) = \begin{pmatrix} 0 & 0 \\ 0 & K_{b_{y,y}} \end{pmatrix} \tag{5.3}$$

noting that for backscattering we only need the transverse (x, y) coordinates to describe the scattering.

The angular sector has an area function

$$A(z_s) = 0 \tag{5.4}$$

implying

$$\overleftrightarrow{K}_b^{(po)}(\vec{1}_z) = \overleftrightarrow{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{5.5}$$

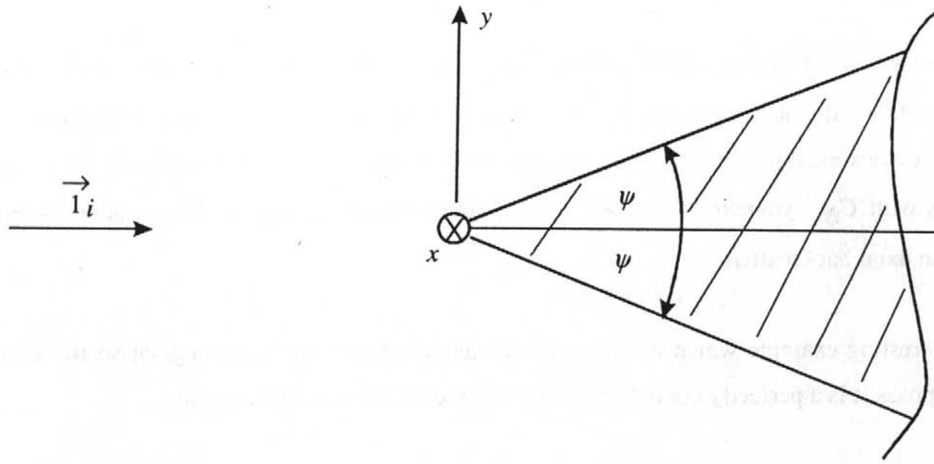

Fig. 5.1. Angular Sector: Side View.

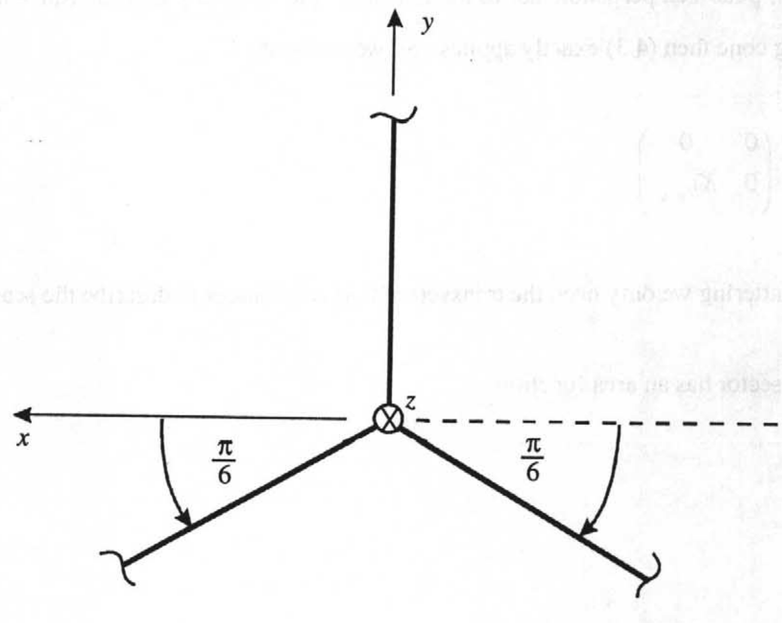

Fig. 5.2. Three Joined Angular Sectors with C_{3a} Symmetry: Front View.

so clearly the PO approximate does not apply to the angular sector in axial backscattering.

Consider another example as in Fig. 5.2. This consist of three angular sectors defined by

$$\Psi_s \leq z_s \tan(\psi) \quad , \quad z \geq 0 \tag{5.6}$$

connected together on the $+z$ axis and lying on planes separated by $\pi/3$ (120°). This has C_{3a} symmetry (a 3-fold rotation axis with 3 axial symmetry planes). Per the theorem [5] this produces no depolarization in axial backscatter, giving

$$\overleftrightarrow{K}_b(\overrightarrow{1}_z) = K_b(\overrightarrow{1}_z)\overleftrightarrow{1} = K_b(\overrightarrow{1}_z)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5.7}$$

Again, however, the area function is zero and the PO approximation gives zero axial backscattering, thereby not applying.

By the way, the solution for the angular sector axial scattering also applies to two such angular sectors intersecting on the z axis due to the common two axial symmetry planes. One merely takes $K_{b_{y,y}}$ as in (5.3) and set $K_{b_{x,x}}$ equal to this provided ψ applies equally to both angular sectors. This latter case has C_{4a} symmetry. If two different semivertex angles ψ_1 and ψ_2 apply to the two angular sectors, then K_b will still be diagonal (in the (x, y) coordinate system), but the two elements will have to be separately evaluated. This case has only C_{2a} symmetry.

The reader might be concerned about the case of zero thickness conductors and the shadow boundary. However, consider a cone of elliptic cross section. This should also depolarize in axial backscattering. As the minor radius of the ellipse shrinks toward zero the area function becomes arbitrarily small, as does the PO backscattering. The angular sector is but a limiting case of the cone with elliptical cross section.

6. Concluding Remarks

The PO scattering dyadic appears to be a useful approximation in some cases, but not others. We need to understand just when it can be used and when not. We need to extend the list of canonical targets with exact solutions from that in [4, 8].

Using modern numerical methods one may obtain "semianalytic" results for which some exact properties are known, but some parameters are calculated numerically (tabulated). An example is the class of cones discussed in [6] for which a constant dyadic coefficient multiples the simple frequency/temporal behavior. The perfectly conducting angular sector is a good example of this. Thereby one may establish a library of quasi-canonical scatterers.

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