Interaction Notes

Note 568

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Smythe's Use of Coulomb Gauge for Waveguides and Cavities

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This short note is prompted by the comments in [1] about Smythe's procedure of calculating the vector potential in the Coulomb gauge due to a current element in a waveguide [2]. It serves to clarify some of the points in the two editions of his book [2,3] regarding this subject.

1. The vector potential $\mathbf{A}$ given by Eq.13.00(2) in Smythe [2] is expressed in terms of two Hertz potentials. Smythe's construction of $\mathbf{A}$ is the same as expanding $\mathbf{A}$ in terms of the $\underline{M}_{emn}$ and $\underline{N}_{omn}$ functions given by Eqs. (3.2) and (3.3) in [1], with h replaced by β'_{mn}. It can be shown, by the procedure given in Section 13.17 [2] and Section 15.12 [3] for a cavity resonator, that $\mathbf{A}$ and $\nabla\phi$ are orthogonal within the waveguide due to the boundary conditions (this is no longer true in infinite space). Thus, $\nabla\phi$ dose not contribute to the expansion coefficients of $\mathbf{A}$. This is the reason why Smythe does not resolve the current element into its solenoidal and irrotational parts when calculating his $\mathbf{A}$.

2. As expected, Smythe's $\mathbf{A}$ will give the total field through $\mathbf{E} = -j\omega\mathbf{A}$, $\mathbf{B} = \nabla \times \mathbf{A}$. The first term in the low-frequency expansion of $-j\omega\mathbf{A}$ gives $-\nabla\phi$. If this term is subtracted from $-j\omega\mathbf{A}$, one then has to add the Coulomb field, $-\nabla\phi$, back to get the total $\mathbf{E}$ field. This is Smythe's implied procedure to obtain the total $\mathbf{E}$, and that procedure leads to identical results in [1] for a rectangular waveguide. But why does Smythe not explicitly state the subtraction ? Or why does he introduce ϕ when $-j\omega\mathbf{A}$ already gives the total $\mathbf{E}$? (Note that in his 2nd edition[3] ϕ is hardly mentioned when treating waveguide excitation.) Perhaps, he does this because he wanted the treatment to be consistent with his treatment of cavities later in the chapter. In his mind, the Coulomb part of the $\mathbf{E}$ field comes from $-\nabla\phi$ and the dynamic part from $-j\omega\mathbf{A}$, as implied on

p.499 and p.544 in [2], and p.521 and p.546 in [3]. The formal procedure in [1] automatically subtracts the Coulomb part of the electric field from $-j\omega\mathbf{A}$.

Note: The 3rd edition of Smythe [2] contains numerous typographical and printing errors. It is advisable to compare with what is in his 2nd edition [3], which is free of errors.

Acknowledgment: Helpful discussions with Dr. Larry K. Warne of Sandia National Laboratories and Dr. Carl E. Baum of AFRL are greatly appreciated.

[1] Robert Nevels and Krzysztof Michalski,"Coulomb Gauge Green's Functions For the Electromagnetic Field," Interaction Note 564, February 2001.

[2] W.R.Smythe, Static and Dynamic Electricity, 3rd ed., Hemisphere, 1989.

[3] W.R.Smythe, Static and Dynamic Electricity, 2nd ed., McGraw-Hill, 1950, Chapter 15 and problems at end of chapter.

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Re. Smthye's Use of Coulomb Gauge for Waveguides and Cavities

Dear Carl,

I am submitting the reference article for possible publication in your Interaction Note series. The work was done entirely on my own time.

Yours sincerely,
Kelvin S.H. Lee