Interaction Notes

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Renormalization of the BLT Equation

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Abstract

By a renormalization procedure involving the square roots of the characteristic impedance matrices of the multiconductor transmission lines, the wave variables (combined voltages and currents) are put in a special form. This allows one to bound the scattering matrices in 2-norm sense. This gives a convergence criterion for the geometric-series expansion of the inverse of the interaction supermatrix in the BLT equation.

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1. Introduction

Since the introduction of the BLT equation [1], various alternate forms have been developed based on the alternate ways that multiconductor transmission lines can be modeled as tubes or junctions in the overall electromagnetic topology of a complex electronic system [4, 6, 7]. In the modeling of uniform multiconductor transmission lines (MTLs), such as in BLT 1, one constructs linear combinations of voltage and current vectors with the latter multiplied by the characteristic impedance matrix of the MTL. This separates the solution into two uncoupled waves propagating in opposite directions (these being coupled if the MTL is nonuniform (NMTL)).

More recently it has been found useful to multiply such waves by the square root of the characteristic admittance matrix [5, 8]. This has advantages when considering power relationships. In this paper we explore the use of such renormalized waves in the formulation of BLT1. This will prove useful when we consider bounds based on the 2-norm.

2. Renormalized BLT1 Equation

As in [8] let us convert our wave variables to renormalized ones, this time for the simple case of a uniform N-conductor (plus reference) transmission line. Recall the telegrapher equations

$$
\begin{aligned}
&\frac{d}{dz}\left(\tilde{V}_n(z,s)\right) = -\left(\tilde{Z}'_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) + \left(\tilde{V}_n^{(s)'}(z,s)\right) \\
&\frac{d}{dz}\left(\tilde{I}_n(z,s)\right) = -\left(\tilde{Y}'_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) + \left(\tilde{I}_n^{(s)'}(z,s)\right) \\
&s = \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency} \\
&\left(\tilde{Z}'_{n,m}(s)\right) = \left(\tilde{Z}'_{n,m}(s)\right)^{\mathrm{T}} \\
&\left(\tilde{Y}'_{n,m}(s)\right) = \left(\tilde{Y}'_{n,m}(s)\right)^{\mathrm{T}}
\end{aligned}
\tag{2.1}
$$

where reciprocity is assumed.

Based on the above we form

$$
\begin{aligned}
\left(\tilde{\gamma}_{n,m}(s)\right) &= \left[\left(\tilde{Z}'_{n,m}(s)\right) \cdot \left(\tilde{Y}'_{n,m}(s)\right)\right]^{1/2} \quad \text{(p.r. square root)} \\
&\equiv \text{propagation matrix} \\
\left(\tilde{Z}_{c_{n,m}}(s)\right) &= \left(\tilde{\gamma}_{n,m}(s)\right) \cdot \left(\tilde{Y}'_{n,m}(s)\right)^{-1} = \left(\tilde{\gamma}_{n,m}(s)\right)^{-1} \cdot \left(\tilde{Z}'_{n,m}(s)\right) \\
&= \left(\tilde{Y}_{c_{n,m}}(s)\right)^{-1} = \left(\tilde{Z}_{c_{n,m}}(s)\right)^{\mathrm{T}} \\
&\equiv \text{characteristic impedance matrix}
\end{aligned}
\tag{2.2}
$$

This is followed by

$$
\begin{aligned}
\left(\tilde{z}_{c_{n,m}}(s)\right) &= \left(\tilde{Z}_{c_{n,m}}(s)\right)^{1/2} = \left(\tilde{z}_{c_{n,m}}(s)\right)^{\mathrm{T}} \\
\left(\tilde{y}_{c_{n,m}}(s)\right) &= \left(\tilde{Y}_{c_{n,m}}(s)\right)^{1/2} = \left(\tilde{y}_{c_{n,m}}(s)\right)^{\mathrm{T}} = \left(\tilde{z}_{c_{n,m}}(s)\right)^{-1}
\end{aligned}
\tag{2.3}
$$

which are used for renormalization.

The renormalized voltage and current variables take the form

$$
\begin{aligned}
\left(\tilde{\mathrm{v}}_n(z,s)\right) &= \left(\tilde{y}_{c_{n,m}}(s)\right) \bullet \left(\tilde{V}_n(z,s)\right) \\
\left(\tilde{i}_n(z,s)\right) &= \left(\tilde{z}_{c_{n,m}}(s)\right) \bullet \left(\tilde{I}_n(z,s)\right) \\
\left(\tilde{\mathrm{v}}_n^{(s)'}(z,s)\right) &= \left(\tilde{y}_{c_{n,m}}(s)\right) \bullet \left(\tilde{V}_n^{(s)'}(z,s)\right) \\
\left(\tilde{i}_n^{(s)'}(z,s)\right) &= \left(\tilde{z}_{c_{n,m}}(s)\right) \bullet \left(\tilde{I}_n^{(s)'}(z,s)\right)
\end{aligned}
\tag{2.4}
$$

Since the square roots of the characteristic impedance matrices are not functions of z, the telegrapher equations reduce to [8]

$$
\begin{aligned}
\frac{d}{dz}\left(\tilde{\mathrm{v}}_n(z,s)\right) &= \left(\tilde{g}_{n,m}(s)\right) \bullet \left(\tilde{\mathrm{v}}_n(z,s)\right) + \left(\tilde{\mathrm{v}}_n^{(s)'}(z,s)\right) \\
\frac{d}{dz}\left(\tilde{i}_n(z,s)\right) &= \left(\tilde{g}_{n,m}(s)\right) \bullet \left(\tilde{i}_n(z,s)\right) + \left(\tilde{i}_n^{(s)'}(z,s)\right) \\
\left(\tilde{g}_{n,m}(s)\right) &= -\left(\tilde{y}_{c_{n,m}}(s)\right) \bullet \left(\tilde{\gamma}_{n,m}(s)\right) \bullet \left(\tilde{z}_{c_{n,m}}(s)\right) \\
&= -\left(\tilde{z}_{c_{n,m}}(s)\right) = \left(\tilde{\gamma}_{n,m}(s)\right)^{\mathrm{T}} \bullet \left(\tilde{y}_{c_{n,m}}(s)\right) \\
&= \left(\tilde{g}_{n,m}(s)\right)^{\mathrm{T}}
\end{aligned}
\tag{2.5}
$$

which is a symmetric form.

The combined normalized voltages (waves) are

$$
\left(\tilde{\mathrm{v}}_n(z,s)\right)_{\substack{1\\2}} = \left(\tilde{\mathrm{v}}_n(z,s)\right) \pm \left(\tilde{i}_n(z,s)\right)
\tag{2.6}
$$

and similarly for the sources. This leads to a supervector/supermatrix equation

$$
\begin{aligned}
\frac{d}{dz}\left(\left(\tilde{\mathrm{v}}_n(z,s)\right)_{\nu}\right) &= \left(\left(\tilde{g}_{n,m}(s)\right)_{\nu,\nu'}\right) \odot \left(\left(\tilde{\mathrm{v}}_n(z,s)\right)_{\nu,\nu'}\right) + \left(\left(\tilde{\mathrm{v}}_n^{(s)'}(z,s)\right)_{\nu}\right) \\
\left(\left(\tilde{g}_{n,m}(s)\right)_{\nu,\nu'}\right) &= \begin{pmatrix} \left(\tilde{g}_{n,m}(s)\right) & \left(0_{n,m}\right) \\ \left(0_{n,m}\right) & -\left(\tilde{g}_{n,m}(s)\right) \end{pmatrix} = \left(\left(\tilde{g}_{n,m}(z,s)\right)_{\nu,\nu'}\right)^{\mathrm{T}}
\end{aligned}
\tag{2.7}
$$

At this point we can write down the solution for the two uncoupled waves, noting that

$$\prod_{z_0}^{z} e^{\left(\left(g_{n,m}(s)\right)_{\nu,\nu'}\right)dz'} = e^{[z-z_0]\left(\tilde{g}_{n,m}(s)\right)} \oplus e^{-[z-z_0]\left(\tilde{g}_{n,m}(s)\right)} \tag{2.8}$$

For constructing BLT1, following [1], we note that waves need only propagate in the $+z$ direction by reversing the z coordinate for the opposite direction, accounting for this by two values of the u,v variables labeling the waves on a tube (MTL). Define a renormalized delay matrix for the uth wave as

$$\left(\tilde{G}_{n,m}(s)\right)_u \equiv e^{\left(\tilde{g}_{n,m}(s)\right)_u L_u} = \left(\tilde{G}_{n,m}(s)_u\right)^{\mathrm{T}}$$
$$u = 1,2,\ldots,N_w\,,\ N_w = \text{ number of waves (twice number of tubes)} \tag{2.9}$$

For the renormalized wave we also define a scattering matrix from the vth to the uth wave via

$$\left(\tilde{\mathrm{v}}_n(0,s)\right)_u = \left(\tilde{S}'_{n,m}(s)\right)_{u,\mathrm{v}} = \left(\mathrm{v}_n(L_{\mathrm{v}},s)\right)_{\mathrm{v}}$$
$$L_{\mathrm{v}} = \text{length of tube with label v} \tag{2.10}$$

Note that the previous scattering matrix was defined via

$$\left(\tilde{V}_n(0,s)\right)_u = \left(\tilde{S}_{n,m}(s)\right)_{u,v} \bullet \left(\tilde{V}_n(L_{\mathrm{v}},s)\right)_{\mathrm{v}} \tag{2.11}$$

giving the transformation

$$\left(\tilde{S}_{n,m}(s)\right)_{n,\mathrm{v}} = \left(z_{c_{n,m}}(s)\right)_u \bullet \left(\tilde{S}'_{n,m}(s)\right)_{u,\mathrm{v}} \bullet \left(y_{c_{n,m}}(s)\right)_{\mathrm{v}} \tag{2.12}$$

using the characteristic impedance matrices appropriate to the uth and vth waves.

We can now write down the renormalized BLT equation as

$$\left[\left(\left(1_{n,m}\right)_{u,\text{v}}\right) - \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\text{v}}\right) \odot \left(\left(\tilde{G}_{n,m}(s)\right)_{u,\text{v}}\right)\right] \odot \left(\left(\tilde{\text{v}}_n(0,s)\right)_u\right)$$

$$= \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\text{v}}\right) \odot \left(\left(\tilde{\text{v}}_n^{(s)}(s)\right)_u\right)$$

$$\left(\left(\tilde{\text{v}}_n^{(s)}(s)\right)_u\right) = \int_0^{L_u} e^{\left(g_{n,m}(s)\right)_u\left[L_u - z'\right]} \odot \left(\tilde{\text{v}}_n^{(s)'}(z')\right) dz' \tag{2.13}$$

$$\left(\tilde{G}_{n,m}(s)\right)_{u,\text{v}} = \begin{cases} \left(\tilde{G}_{n,m}(s)\right)_{u,u} & \text{for } u = \text{v} \\ \left(0_{n,m}\right) & \text{otherwise} \end{cases}$$

$$\left(\left(\tilde{G}_{n,m}(s)\right)_{u,\text{v}}\right) = \left(\tilde{G}_{n,m}(s)\right)_1 \oplus \left(\tilde{G}_{n,m}(s)\right)_2 \oplus \ldots \oplus \left(\tilde{G}_{n,m}(s)\right)_{N_w}$$

3. Expansion of BLT1 in Geometric Series

The formal solution of (2.13) can take the form [6]

$$\left(\left(\tilde{\text{v}}_n(0,s)\right)_u\right) = \left(\left(\tilde{K}'_{n,m}(s)\right)_{u,\text{v}}\right) \odot \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\text{v}}\right) \odot \left(\left(\tilde{\text{v}}_n^{(s)}(s)\right)_u\right)$$

$$\left(\left(\tilde{K}'_{n,m}(s)\right)_{u,\text{v}}\right) = \left[\left(\left(1_{n,m}\right)_{u,\text{v}}\right) - \left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\text{v}}\right)\right]^{-1} = \text{inverse of interaction supermatrix}$$

$$= \left(\left(1_{n,m}\right)_{u,\text{v}}\right) + \sum_{\ell=1}^{\infty}\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\text{v}}\right)^{\ell} \tag{3.1}$$

$$\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\text{v}}\right) \equiv \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\text{v}}\right) \odot \left(\left(\tilde{G}_{n,m}(s)\right)_{u,\text{v}}\right)$$

For the geometric series to converge (for any given *s*) requires that $((\tilde{\Sigma}'_{n,m}(s))_{u,\text{v}})$ be sufficiently small compared to the identity. Smallness can be considered in a norm sense. Note that later we specialize *s* to $j\omega$.

We need

$$\left\|\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\text{v}}\right)\right\| \leq \delta(s) < 1 \tag{3.2}$$

so that [11]

$$
\begin{aligned}
\left\|\left(\left(\tilde{K}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)\right\| &\leq \left\|\left(\left(1_{n,m}\right)_{u,\mathrm{v}}\right)\right\| + \sum_{\ell=1}^{\infty}\left\|\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)\right\|^{\ell} \\
&\leq 1 + \sum_{\ell=1}^{\infty}\delta(s) < 1 \qquad (3.3) \\
&= \left[1 - \delta(s)\right]^{-1}
\end{aligned}
$$

Here we note for all associated matrix norms [10] that

$$
\left\|\left(\left(1_{n,m}\right)_{u,\mathrm{v}}\right)\right\| = 1 \qquad (3.4)
$$

Note that $\delta(s)$ is not an analytic function of s.

Consider now the 2-norm for our associated matrix norm [2]. Thus we need

$$
\begin{aligned}
&\left\|\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)\right\|_2 = \left[\lambda_{\max}\left(\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)\right)\right]^{1/2} \\
&\lambda_{\max} \equiv \text{maximum eigenvalue (all eigenvalues nonnegative)} \qquad (3.5) \\
&\dagger = \mathrm{T}* \equiv \text{adjoint = transpose conjugate}
\end{aligned}
$$

Relating the problem to one of the eigenvalues of a Hermitian matrix with positive eigenvalues.

We have

$$
\begin{aligned}
&\lambda_{\max}\left(\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) \\
&= \lambda_{\max}\left(\left(\left(\tilde{G}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\tilde{S}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) \odot \left(\tilde{G}_{n,m}(s)\right)_{u,\mathrm{v}}\right) \qquad (3.6)
\end{aligned}
$$

Consider the delay matrix for the case of a simple lossless MTL for which we have

$$
\begin{aligned}
\left(\tilde{Z}'_{n,m}(s)\right) &= s\left(L'_{n,m}\right) \quad , \quad \left(\tilde{Y}'_{n,m}(s)\right) = s\left(C'_{n,m}(z)\right) \\
\left(\tilde{\gamma}_{n,m}(s)\right) &= s\left[\left(L'_{n,m}\right) \bullet \left(C'_{n,m}\right)\right]^{1/2} \\
\left(Z_{c_{n,m}}\right) &= \left[\left(L'_{n,m}\right) \bullet \left(C'_{n,m}\right)\right]^{-1/2} \bullet \left(L'_{n,m}\right) \\
&= \left[\left(L'_{n,m}\right) \bullet \left(C'_{n,m}\right)\right]^{1/2} \bullet \left(C'_{n,m}\right) \\
&= \left(Z_{c_{n,m}}\right)^{\mathrm{T}} \\
\left(z_{c_{n,m}}\right) &= \left(Z_{c_{n,m}}\right)^{1/2} = \left(z_{c_{n,m}}\right)^{\mathrm{T}} \\
\left(\tilde{g}_{n,m}(s)\right) &= -s\left(y_{c_{n,m}}\right) \bullet \left[\left(L'_{n,m}\right) \bullet \left(C'_{n,m}\right)\right]^{1/2} \bullet \left(z_{c_{n,m}}\right) \\
&= -s\left(z_{c_{n,m}}\right) \bullet \left(C'_{n,m}\right) \bullet \left(z_{c_{n,m}}\right) \\
&= -s\left(y_{c_{n,m}}\right) \bullet \left(L'_{n,m}\right) \bullet \left(y_{c_{n,m}}\right) \\
&= \left(\tilde{g}_{n,m}(s)\right)^{\mathrm{T}}
\end{aligned}
\tag{3.7}
$$

Then the delay matrix in (2.9) has the property for $s = j\omega$

$$
\begin{aligned}
\left(\tilde{G}_{n,m}(j\omega)\right)_u^{\dagger} &= \left[e^{j\omega\left(z_{c_{n,m}}\right)_u \bullet \left(C'_{n,m}\right)_u \bullet \left(z_{c_{n,m}}\right)} \right]^{\dagger} \\
&= e^{-j\omega\left(z_{c_{n,m}}\right) \bullet \left(C'_{n,m}\right)_u \bullet \left(z_{c_{n,m}}\right)} \\
&= \left(\tilde{G}_{n,m}(j\omega)\right)_u^{-1}
\end{aligned}
\tag{3.8}
$$

and is therefore unitary [9]. In (3.6) we then have (for $s = j\omega$), noting the block-diagonal property (2.13) of the delay supermatrix,

$$
\begin{aligned}
&\lambda_{\max}\left(\left(\left(\tilde{\Sigma}'_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\tilde{\Sigma}'_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)\right) \\
&= \lambda_{\max}\left(\left(\left(\tilde{G}_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)^{-1} \odot \left(\left(\tilde{S}'_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\left(\tilde{S}'_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right) \odot \left(\left(\tilde{G}_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)\right)\right) \\
&= \lambda_{\max}\left(\left(\left(\tilde{S}_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)^{\dagger} \odot \left(\left(\tilde{S}_{n,m}(j\omega)\right)_{u,\mathrm{v}}\right)\right)
\end{aligned}
\tag{3.9}
$$

since eigenvalues are preserved in a similarity transformation. This simplifies the result somewhat.

Another way to look at this geometric series is to diagonalize $((\tilde{\Sigma}'_{n,m})_{u,\mathrm{v}})$ as

$$\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) = \sum_{\beta=1}^{N} \tilde{\chi}_{\beta}(s)\left(\tilde{r}_n(s)_u\right)_{\beta}\left(\tilde{\ell}_n(s)_u\right)_{\beta}$$

N_u = number of wires on uth tube

N_w = number of waves (twice number of tubes)

$$N = \sum_{u=1}^{N_w} N_u$$

$$\left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) \odot \left(\left(\tilde{r}_n(s)\right)_u\right)_{\beta} = \tilde{\chi}_{\beta}(s)\left(\left(\tilde{r}_n(s)\right)_u\right)_{\beta}$$

$$\left(\left(\tilde{\ell}_n(s)\right)_u\right)_{\beta} \odot \left(\left(\tilde{\Sigma}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) = \tilde{\chi}_{\beta}(s)\left(\left(\tilde{\ell}_n(s)\right)_u\right)_{\beta} \tag{3.10}$$

$$\left(\left(\tilde{r}_n(s)\right)_u\right)_{\beta_1} \odot \left(\left(\tilde{\ell}_n(s)\right)_u\right)_{\beta_2} = 1_{\beta_1,\beta_2} = \begin{cases} 1 & \text{for } \beta_1 = \beta_2 \\ 0 & \text{for } \beta_1 \neq \beta_2 \end{cases}$$

(biorthonormal)

In this form we have

$$\left(\left(\tilde{K}'_{n,m}(s)\right)_{u,\mathrm{v}}\right) = \sum_{\beta=1}^{N} \left[1-\tilde{\chi}_{\beta}(s)\right]^{-1}\left(\left(\tilde{r}_n(s)\right)_u\right)_{\beta}\left(\left(\tilde{\ell}_n(s)\right)_u\right)_{\beta} \tag{3.11}$$

Comparing this to (3.3) we can see the factor $[1-\tilde{\chi}_{\beta}(s)]^{-1}$ corresponding to a geometric series in the individual eigenvalues, as compared to the bound $[1-\delta]^{-1}$.

4. Scattering Matrices

This renormalized form (2.12) of the scattering matrix has appeared previously [3, 9]. It has special properties. In particular for passive cases

$$0 \le \left\| \left(\left(\tilde{S}'_{n,m}(j\omega) \right)_{u,\mathrm{v}} \right) \right\|_2 \le 1 \tag{4.1}$$

with 1 corresponding to the lossless case for this bounded real supermatrix. This is a property following from the renormalization which placed the waves in a more power-like form. Note that the renormalization needs to be in terms of real-valued impedance/admittance matrices, which applies in the case of lossless MTLs (and other special cases). Note that this passivity property also applies to the scattering matrix for each junction separately in the electromagnetic topology.

Revisiting the bound in (3.3) we see that for such a series to be appropriate we need losses in the network to bring the 2-norm, which can be used in (3.3), to less than one. Note that in this case δ is the 2-norm.

There is an important difference between the geometric-series expansions in frequency and time domains. In time domain, causality isolates the leading terms from successive terms, giving an early-time exact representation. In frequency domain, the Laplace/Fourier transform, being a transform over all time, gives different properties. One can compute $((\tilde{K}'_{n,m})_{u,\mathrm{v}})$ (the inverse of the interaction supermatrix) as a matrix inverse (except at natural frequencies of the system). Using the geometric series (3.1) depends on successively smaller correction terms for series convergence. The present formulation in terms of the renormalized variables gives series convergence for sufficiently lossy systems, the more loss giving more rapid convergence.

5. Concluding Remarks

By this renormalization procedure we have found a general bounding result for the BLT equation for passive systems. This allows one to find a convergent geometric series for the inverse of the interaction supermatrix. This applies to frequency domain, the time-domain form having other properties.

While this derivation has been in terms of BLT1 involving junctions and tubes, it applies to other forms of the BLT equation as well. One can replace the tubes (MTLs) by equivalent junctions. These are then absorbed into the scattering supermatrix for which the present results apply.

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