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# Design Aspects of a Dual Conical Lens between aHigh-Voltage Pulser and a Helical Antenna 

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#### Abstract

High- voltage pulse generators (ex: Marx type) has been used to drive a helical antenna [1-3] to radiate a moderate band of frequencies [4]. Quite often, we will need a transition section between the last output switch of the pulse generator and the feed terminal of the helical antenna. If the pulse generator is of the coaxial type, where we have a flaring coaxial line at the end of the generator, the diameter of the outer conductor needs to be reduced after the flare to efficiently energize the helical antenna. This note addresses this issue by considering a dual conical transition section.


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## 1. Introduction

As an example, we briefly review the work of Mayes et al., in [1], where they applied the output of a Marx generator to a helical antenna. Their geometry and the experimental parameters are as follows:


Figure 1. Output voltage waveform from the Marx generator in [1]
Table 1. Parameters of the helical antenna in [1] designed for 1 GHz

| Parameter | Mayes' Helix |
| :--- | :--- |
| Axial length L | 77.0 cm |
| Diameter D | 9.5 cm |
| Circumference: $\mathrm{C}=$ <br> $\pi \mathrm{D}$ | 30.24 cm |
| Spacing (pitch) S | 7.70 cm |
| Pitch angle $\alpha$ | $14.29^{\circ}$ |
| Length of 1 turn $\mathrm{L}_{\mathrm{T}}$ | 31.20 cm |
| Number of turns N | 10 |
| Wire radius r | 1.5 mm (guess) |



Figure 2. Radiated electric field at a distance of 100 m

High- voltage pulse generators (ex: Marx type) have also been used to drive a helical antenna [2 - 3] to radiate a moderate band of frequencies [4]. Quite often, we will need a transition section between the last output switch of the pulse generator and the feed terminal of the helical antenna. If the pulse generator is of the coaxial type, where we have a flaring coaxial line at the end of the generator, the diameter of the outer conductor needs to be reduced after the flare to efficiently energize the helical antenna. The flare in the coaxial geometry converts a planar TEM wave into a spherical TEM wave. Baum et al., addressed the issue of converting a spherical TEM wave in a conical coaxial line into a planar TEM wave in a cylindrical coaxial line. The flaring section at the output of the pulse generator needs to be connected to the feed terminal of a helical antenna which means the diameter of the coax needs to be small. This is the hole in the ground plane in driving the helical antenna as shown in Figure 1. This note addresses the issue of connecting the flaring or cylindrical coaxial line output of the pulse generator to the feed point of a helical antenna.

## 2. Review of a Prolate Spheroidal Lens

Baum et al., [5] have considered matching a conical coaxial line with a cylindrical coaxial line at high frequencies. They have accomplished this by filling the conical region with a uniform, isotropic dielectric with frequency independent dielectric constant (lossless and dispersion less up to some frequency). The geometry is shown in Figure 3.


Figure 3. Prolate Spheroidal Lens with Circular Conical Transmission Line Feeding Circular coaxial Line as in [5]

The design equations for the above prolate-spheroidal lens are given below. They come from matching impedances, geometrical considerations and equal transit times of outermost and innermost rays.

| $(50 / 60) \sqrt{\varepsilon_{r}}$ |  |
| :---: | :---: |
| $\tan \left(\frac{\beta_{1}}{2}\right)=e \quad \tan \left(\frac{\alpha_{1}}{2}\right)$ | Impedance consideration |
| $60 \ln (b / a)=50$ | Impedance consideration |
| $\sqrt{\varepsilon_{r}} \frac{b}{}+\Delta^{\prime}=l \sqrt{\varepsilon_{r}}$ | Equal transit time on outermost ray |
|  | Equal transit time on innermost ray |
| $a=(l-\Delta) \tan \alpha_{1}$ | Geometrical consideration |
| $b=\left(l-\Delta^{\prime}\right) \tan \beta_{1}$ | Geometrical consideration |
| unknowns (7) $\alpha_{1} \beta_{1} a \mathrm{a}$ b l $\quad \Delta \& \Delta^{\prime}$ |  |
| Set $b=1$ |  |
| unknowns (6) $\quad \alpha, \beta, a, l, \Delta \& \Delta^{\prime}$ |  |

By normalizing all linear dimensions to the outer radius b, one has 6 equations and 6 unknowns, which can be solved on a computer. The above set of equations was solved using a simple code in MathCAD and the results are found to be:

$$
\alpha 1=11.287 \quad \beta 1=38.159 \quad \mathrm{a}=0.435 \quad \mathrm{~L}=2.306 \quad \Delta=0.128 \quad \Delta \mathrm{p}=1.033
$$

The above numerical result agrees well with the results in Table B. 3b on page 37 of [5].
However, Baum et al., do not proceed to find the frequency response of the lens via S-parameter estimation. We have done this using HFSS and the S-parameter calculations are shown in Figure 4.



Figure 4. S-parameter estimation of the prolate Spheroidal lens of Figure 5.
It is observed from Figure 2, that the prolate-spheroidal lens works well into $>500 \mathrm{MHz}$ and a small amount of the TEM mode in the conical input line is converted to TM modes. We can now proceed to extend this lens analysis and customize this for the problem at hand.

## 3. Dual-Cone Lens (Extension of the Prolate Spheroidal lens)

In the case of prolate-spheroidal lens the output is a cylindrical coax with a large outer diameter. In the problem at hand of feeding a helical antenna, we need to minimize the size of the hole in the ground plane or the feed point. The required geometry is shown in Figure 5.

The last output switch of the generator could be at the left apex and we have a ground plane and the helical antenna on the right side. It is observed that if the Length L1 on the right is made to be infinite, this problem reduces to the prolate spheroidal case of the previous section.


Figure 5. Dual Cone Lens as an interface between the last output switch and the feed point of the antenna

As before, we have extended the design equations for the dual cone lens and obtain the following set of equations.

$$
\begin{align*}
& \text { Given } \\
& \tan \left(\frac{\beta 1}{2}\right)=\exp \left(5 \cdot \frac{\sqrt{\varepsilon}}{6}\right) \cdot \tan \left(\frac{\alpha 1}{2}\right)  \tag{1}\\
& \tan \left(\frac{\beta 2}{2}\right)=2 \cdot 30 \cdot \tan \left(\frac{\alpha 2}{2}\right)  \tag{2}\\
& \frac{\mathrm{a} \sqrt{\varepsilon}}{\sin (\alpha 1)}+\left(\frac{\Delta}{\cos (\alpha 2)}\right)+\mathrm{L} \cdot\left(\frac{1}{\cos (\alpha 2)}-1\right)=\mathrm{L} \cdot \sqrt{\varepsilon}  \tag{3}\\
& \frac{\mathrm{~b} \sqrt{\varepsilon}}{\sin (\beta 1)}+\left(\frac{\Delta \mathrm{p}}{\cos (\beta 2)}\right)+\mathrm{L} 1 \cdot\left(\frac{1}{\cos (\beta 2)}-1\right)=\mathrm{L} \cdot \sqrt{\varepsilon}  \tag{4}\\
& \mathrm{a}=(\mathrm{L}-\Delta) \cdot \tan (\alpha 1)  \tag{5}\\
& \mathrm{b}=(\mathrm{L}-\Delta \mathrm{P}) \cdot \tan (\beta 1)  \tag{6}\\
& \mathrm{b}=(\mathrm{L} 1+\Delta \mathrm{p}) \cdot \tan (\beta 2)  \tag{7}\\
& \mathrm{a}=(\mathrm{L} 1+\Delta) \cdot \tan (\alpha 2)
\end{align*}
$$

The unknowns are
$\alpha 2 \beta 2$ Output Cone Angles

$$
L, a, \Delta \Delta^{\prime} \text { (Four Linear Dimensions) }
$$

The design equations are governed by impedance matching, equal transit time of outermost and innermost rays and geometrical considerations. We have verified that by letting L1 tend to infinity, the equations reduce to that of the prolate spheroidal lens. In this sense, the prolate spheroidal lens is a limiting case of the dual conical lens.

Once again, we have solved the above set of equations on a computer, by setting $\mathrm{b}=1$, and obtained a family of solutions listed in Table 1.

TABLE 1. Family of Solutions for the Dual Cone Lens Equations

| Case | $\mathbf{b}$ | $\mathbf{L 1}$ | $\boldsymbol{\alpha} \mathbf{1}$ | $\boldsymbol{\alpha} \mathbf{2}$ | $\boldsymbol{\beta} \mathbf{1}$ | $\boldsymbol{\beta} \mathbf{2}$ | $\mathbf{L}$ | $\mathbf{a}$ | $\boldsymbol{\Delta}$ | $\boldsymbol{\Delta}^{\prime}$ | $\mathbf{L}+\mathbf{L 1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1.1 | No | CON | VER | GEN | CE |  |  |  |  |
| 2 | 1 | 1.2 | 0.76 | 2.661 | 12.935 | 29.22 | 22.1 | 0.292 | 0.073 | 0.587 | 23.3 |
| 3 | 1 | 1.5 | 2.24 | 7.83 | 11.00 | 24.99 | 7.91 | 0.30 | 0.077 | 0.645 | 9.41 |
| 4 | 1 | 2.0 | 3.93 | 13.697 | 8.872 | 20.23 | 4.81 | 0.325 | 0.083 | 0.713 | 6.81 |
| 5 | 1 | 2.5 | 5.063 | 17.593 | 7.46 | 17.05 | 3.91 | 0.339 | 0.088 | 0.76 | 6.41 |
| 6 | $\mathbf{1}$ | $\mathbf{2 . 6}$ | 5.248 | $\mathbf{1 8 . 2 2 5}$ | 7.231 | $\mathbf{1 6 . 5 3}$ | 3.80 | $\mathbf{0 . 3 4 1}$ | $\mathbf{0 . 0 8 9}$ | $\mathbf{0 . 7 6 8}$ | $\mathbf{6 . 4 0}$ |
| 7 | 1 | 2.8 | 5.584 | 19.374 | 6.816 | 15.59 | 3.62 | 0.346 | 0.091 | 0.775 | 6.42 |
| 8 | 1 | 3 | 5.884 | 20.393 | 6.448 | 14.76 | 3.48 | 0.35 | 0.093 | 0.795 | 6.48 |
| 9 | 1 | 5 | 7.726 | 26.595 | 4.209 | 9.662 | 2.87 | 0.376 | 0.103 | 0.874 | 7.87 |
| 10 | 1 | 6 | 8.24 | 28.302 | 3.592 | 8.251 | 2.75 | 0.383 | 0.106 | 0.896 | 8.75 |
| 11 | 1 | 50 | 10.87 | 36.847 | 0.488 | 1.123 | 2.34 | 0.427 | 0.125 | 1.014 | 52.34 |

Of all the 11 cases listed above, there was no convergent solution for case 1 . Case 6 above appears to be the optimal, in the sense it minimizes the parameter $\mathrm{L}+\mathrm{L}_{1}$ or the overall length of the lens. We have chosen this solution for further evaluation. This solution is shown inn Figure 6.


Found a Solution where $L+L_{1}$ is minimized


Figure 6. Chosen solution for further evaluation
We can now proceed to evaluate the S-parameters of the dual cone lens of Figure 4. This has been done using HFSS and the results are shown in Figure 7.


Figure 7. Calculated S-parameters of the dual-cone lens

Performance of the Dual Cone Transition Section shown in Figure 5 is for the case of $b=1 \mathrm{~m}$; This dual conical line is seen to perform very well by examining S11 and S12, over an extremely broad band of frequencies.
One has to scale the frequency axis for other values of b. For example: if b=20 cm; multiply the frequency axis by 5. If b (the maximum radius of the outer conductor is 20 cm or less), this lens is seen to work well into a few GHz and thus help get most of the spectral content of the pulser waveform on to the antenna.

## References

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