> Sensor and Simulation Notes Note 92 9 August 1969 $\begin{gathered}\text { A Parametric Study of the Uniform Surface } \\ \text { Transmission Line Driven by a Step Function Voltage } \\ \text { and Terminated in its High -Frequency Characteristic Impedance } \\ \text { Capt Carl E. Baum } \\ \text { Air Force Weapons Laboratory }\end{gathered}$


#### Abstract

This note presents a parametric study of the early time behavior of the surface transmission based on a model developed in a previous note. The ground conductivity and permittivity, and the height and length of the transmission line are all varied over some ranges of interest. The transmission line is driven by a step function voltage at one end and is terminated in its highfrequency characteristic impedance at the other end..


## Foreword

Most of this note consists of the numerous graphs showing the early time waveforms on the simulator. Hopefully these and the accompanying tables showing the cases considered will help the reader obtain a rapid estimate of the performance of the uniform surface transmission line as a simulator in many cases of interest. We would like to thank Mr. Terry I. Brown of Dikewood for the computer calculations and for the bulk of the graphs. In this latter effort he was assisted by Mr. Joe P. Martinez of Dikewood and by A2C Richard T. Clark, Mr. Lawrence M. Berg, and Mr. Richard M. Calderon of AFWL.

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## I. Introduction

One of the common simulators for the nuclear electromagnetic pulse (EMP) over a ground surface (earth) is a uniform surface transmission linel driven by parallel transition sections ${ }^{2,3}$ from a capacitive generator and terminated in the resistive highfrequency impedance of the transmission line. In reference 1 an analytical model (in frequency domain) is developed for the response of this kind of simulator; numerical inverse Fourier transforms are used to obtain time domain waveforms. The purpose of this note is to make a parametric study of a wide variety of cases using the model developed in reference 1.

In order to limit the number of parameters to be varied we consider only the first part of the time domain waveform for an assumed step function voltage source. This type of source is a good approximation for the early time behavior of a capacitive generator. At late times and for sufficiently large generator capacitance the time domain waveform is basically an exponential decay due to the capacitor discharge into the resistive termination of the surface transmission line. The present calculations then apply to the case that the capacitor discharge time is large compared to the time for any initial transients on the surface transmission line to die down. The purpose of the present note is to parametrically study this early-time behavior, limited by the accuracy of the model developed in reference 1 .

For the present parametric study we let the step voltage source be present at the beginning of the surface transmission line. A length of transmission line with characteristic impedance equal to the high-frequency characteristic impedance of the surface transmission line may be used in a typical application. However to avoid an excessive number of graphs we do not include the length of the connecting transmission line for transition section) as one of the parameters to be varied in this note; for simplicity we choose this length to be zero. In this parametric study we vary the ground conductivity, ground permittivity, length of the surface transmission Iine, and height of the perfectly

1. Capt Car1 E. Baum, Sensor and Simulation Note 46, The SingleConductor, Planar, Uniform Surface Transmission Line Driven from One End, July 1967.
2. Capt Carl E. Baum, Sensor and Simulation Note 31, The Conical Transmission Line as a Wave Launcher and Terminator for a Cylindrical Transmission Line, January 1967.
3. Guy W. Carlisle, Sensor and Simulation Note 54, Matching the Impedance of Multiple Transitions to a Parallel-Plate Transmission Line, April 1968.
conducting sheet (or wire array) above the ground. Perhaps the parametric study can be extended in some future note(s) to include a few other lengths (non zero) for the input transition.

## II. Model for Surface Transmission Line

The surface transmission line, including its schematic transmission-line representation, is illustrated in figure 1. Figure lA shows a side view of the surface transmission line including the relative locations of sources, input transition, conductors extending into the ground, and termination. The medium above the ground has permittivity ${ }^{4} \varepsilon_{0}$, permeability $\mu_{0}$, and zero conductivity; the ground has permittivity $\varepsilon_{2}$, permeability $\mu_{0}$, and conductivity $\sigma_{2}$. The surface transmission line has length $d$, height $h$, and width $w$ where we assume $w \gg h$; the input transition has length $\mathrm{d}_{\mathrm{i}}$.

Figure 1 B shows a schematic representation of the simulator. The input transition is represented as a transmission line of length $\mathrm{d}_{i}$ and of characteristic impedance $\mathrm{Z}_{i}$; this characteristic impedance is roughly chosen to be

$$
\begin{equation*}
z_{i} \simeq z_{0} \frac{h}{w} \tag{1}
\end{equation*}
$$

where the wave impedance of free space is

$$
\begin{equation*}
z_{0}=\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}} \tag{2}
\end{equation*}
$$

More accurately, $z_{i}$ would be chosen to match the high-frequency characteristic impedance of the surface transmission line, including effects associated with the finite width $w$ of the surface transmission line by using results for finite-width parallel plate transmission lines.5,6 The termination impedance $Z_{t}$ is
4. Rationalized MKSA units are used throughout this note.
5. Lt Carl E. Baum, Sensor and Simulation Note 21 , Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.
6. Terry L. Brown and Kenneth D. Granzow, Sensor and Simulation Note 52, A Parameter Study of Two Parallel Plate Transmission Line Simulators of EMP Sensor and Simulation Note 2l, April 1968.


## A. SIDE VIEW



## B. SCHEMATIC REPRESENTATION

Figure 1. SURFACE TRANSMISSION LINE
also chosen to match the high-frequency characteristic impedance of the surface transmission line, thereby giving

$$
\begin{equation*}
z_{t}=z_{i} \tag{3}
\end{equation*}
$$

From reference 1 we now quote some important equations. The propagation constant $\gamma_{x}$ in the $\pm x$ direction (ref. 1, eqn. 67) is

$$
\begin{equation*}
\frac{\gamma_{x}}{\gamma_{1}} \simeq\left\{1+\frac{1}{\gamma_{2} h}\left[1-\left(\frac{\gamma_{1}}{\gamma_{2}}\right)^{2}\right]^{1 / 2}\right\}^{1 / 2} \tag{4}
\end{equation*}
$$

where the propagation constants for free space and ground are

$$
\begin{align*}
& \gamma_{1}=\frac{s}{c}, \quad c=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}} \\
& \gamma_{2}=\left[s \mu_{0}\left(\sigma+s \varepsilon_{2}\right)\right]^{1 / 2}=\frac{s}{c}\left[\varepsilon_{x}+\frac{\sigma}{s \varepsilon_{0}}\right]^{1 / 2} \tag{5}
\end{align*}
$$

with

$$
\begin{equation*}
c \equiv \frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}}, \quad \varepsilon_{r} \equiv \frac{\varepsilon_{2}}{\varepsilon_{0}} \tag{6}
\end{equation*}
$$

and where $s$ is the Laplace transform variable which can be set equal to $i \omega$ for frequency-domain considerations where $\omega$ is the radian frequency. The characteristic impedance of the surface transmission line (ref. 1 , eqn. 92) is

$$
\begin{equation*}
z_{L_{\infty}} \simeq \frac{h}{w} \frac{\gamma_{x}}{\gamma_{1}} z_{0}=\frac{\gamma_{x}}{\gamma_{1}} z_{i} \tag{7}
\end{equation*}
$$

Note that $\gamma_{X} / \gamma_{1} \rightarrow 1$ as $s \rightarrow \infty$. These equations are approximate; some of the restrictions on their validity are discussed later.

Referring to figure $1 B r_{e}$ is the reflection coefficient for the voltage or electric field at the termination. The input
impedance to the surface transmission line is $Z_{L}$ and the voltage reflection coefficient there is re. The input impedance to the input transition is $Z_{L}^{\prime}$.

From section VI B of reference 1 for our present case of resistive termination we have a reflection coefficient

$$
\begin{equation*}
r_{e} \simeq \frac{1-\frac{\gamma_{x}}{\gamma_{1}}}{1+\frac{\gamma_{x}}{\gamma_{1}}} \tag{8}
\end{equation*}
$$

and an impedance

$$
\begin{equation*}
z_{L}=z_{L_{\infty}} \frac{1+r_{e^{e}} e^{-2 \gamma} x^{d}}{1-r_{e} e^{-2 \gamma} x^{d}} \tag{9}
\end{equation*}
$$

The transformed vertical electric field on the surface transmission line (in the +2 direction) is

$$
\begin{equation*}
\tilde{E}(x)=\tilde{E}(0) \frac{1+r_{e} e^{-2 \gamma_{x}(d-x)}}{1+r_{e} e^{-2 \gamma_{x} d}} e^{-\gamma_{x} x+\gamma_{1} x} \tag{10}
\end{equation*}
$$

and the magnetic field on the surface transmission line (in the -y direction) is

$$
\begin{equation*}
\tilde{B}(x)=\frac{\tilde{E}(0)}{c} \frac{\gamma_{1}}{\gamma_{x}} \frac{1-r_{e} e^{-2 \gamma_{x}(d-x)}}{1+r_{e^{e}}^{-2 \gamma_{x}}} e^{-\gamma_{x} x+\gamma_{1} x} \tag{11}
\end{equation*}
$$

where the inclusion of the factor $e^{\gamma 1 x}$ makes the inverse transforms be expressed in terms of retarded time $t$ * where

$$
\begin{equation*}
t^{*} \equiv t-\frac{x}{c} \tag{12}
\end{equation*}
$$

The tilde, ~, is used to indicate the Laplace transform.

The fields are now expressed in terms of $\tilde{E}(0)$, the vertical electric field at the input of the surface transmission line. At the input of the surface transmission line we have a voltage reflection coefficient

$$
\begin{equation*}
r_{e}^{\prime}=\frac{z_{I}-z_{i}}{z_{I}+z_{i}} \tag{13}
\end{equation*}
$$

At the input of the input transition the impedance is

$$
\begin{equation*}
z_{L}^{\prime}=z_{i} \frac{1+r_{e}^{\prime} e^{-2 \gamma_{1} d_{i}}}{1-r_{e}^{\prime} e^{-2 \gamma_{1} d_{i}}} \tag{14}
\end{equation*}
$$

If $\tilde{\mathrm{V}}_{0}$ is the voltage output of the generator then we have

$$
\begin{equation*}
\tilde{E}(0)=\frac{\tilde{v}_{0}}{h} \frac{1+r_{e}^{\prime}}{1+r_{e}^{\prime} e^{-2 \gamma_{1} d_{i}}} \tag{15}
\end{equation*}
$$

A factor of $e^{-\gamma 1 d_{i}}$ is not included in this expression, again to keep the time domain results in retarded time.

For purposes of the present parametric study we set

$$
\begin{align*}
& \mathrm{d}_{i} \equiv 0 \\
& \tilde{\mathrm{~V}}_{0}=\frac{E_{0}}{\mathrm{~h}} \tag{16}
\end{align*}
$$

so that the effects associated with the length of the input transition are not included in the present results. Then we also have

$$
\begin{equation*}
\tilde{E}(0)=\frac{E_{0}}{s}, \quad E(0)=E_{0} u\left(t^{*}\right) \tag{17}
\end{equation*}
$$

where $u$ is the unit step function.

In reference 1 there are several dimensionless parameters used for this case. These include

$$
\begin{equation*}
\varepsilon_{r} \equiv \frac{\varepsilon_{2}}{\varepsilon_{0}}, \quad p_{3} \equiv \frac{t_{r}^{\prime} t_{d}}{t_{h}^{2}}, \quad x^{\prime} \equiv \frac{x}{d} \tag{18}
\end{equation*}
$$

where we also have the characteristic times

$$
\begin{array}{ll}
t_{h} \equiv \frac{h}{c}, & t_{d} \equiv \frac{d}{c} \\
t_{r} \equiv \frac{\varepsilon_{2}}{\sigma}, & t_{r}^{\prime} \equiv \frac{\varepsilon_{0}}{\sigma} \tag{19}
\end{array}
$$

In reference 1 a normalized Laplace transform variable and normalized retarded time are defined as

$$
\begin{equation*}
s_{d} \equiv s t_{d}, \quad \tau_{d} \equiv \frac{c t-x}{d} \tag{20}
\end{equation*}
$$

For purposes of the present calculations the inverse Fourier transforms were done in terms of these normalized quantities and then the time scale converted to $t$ *.

In reference 1 (equation 121) the magnitude of the initial rise was determined to be

$$
\begin{equation*}
h_{d}(0+)=e_{z_{d}}(0+)=e^{-\frac{x^{\prime}}{2}\left[p_{3} \frac{t_{d}}{t_{r}}\left(1-\frac{1}{\varepsilon_{r}}\right)\right]^{1 / 2}} \tag{21}
\end{equation*}
$$

In terms of our present field quantities this becomes

$$
\begin{equation*}
\left.\frac{B(x)}{B_{0}}\right|_{t *=0+}=\left.\frac{E(x)}{E_{0}}\right|_{t *=0+}=e^{-\frac{x}{2 h}\left[\frac{1}{\varepsilon_{r}}\left(1-\frac{1}{\varepsilon_{r}}\right)\right]^{1 / 2}} \tag{22}
\end{equation*}
$$

where

$$
\begin{equation*}
B_{0} \equiv \frac{E_{0}}{C} \tag{23}
\end{equation*}
$$

This result also applies to the case that $d_{i}>0$ because $Z_{I_{\infty}} \rightarrow Z_{i}$ as $s \rightarrow \infty$.

There are some general restrictions on the ranges of various parameters for the previous results to be accurate. From reference 1 (equation 64) we have restriction 1 as

$$
\begin{equation*}
\frac{1}{10.4 \varepsilon_{r}^{3}}\left(\frac{t_{r}}{t_{h}}\right)^{2} \ll 1 \tag{24}
\end{equation*}
$$

or

$$
\begin{equation*}
\left[10.4 \varepsilon_{r} z_{o}^{2} h^{2} \sigma^{2}\right]^{-1} \equiv r_{1} \ll 1 \tag{25}
\end{equation*}
$$

Also from reference 1 (equation 73) we have restriction 2 as
$\omega \ll \frac{\sqrt{\varepsilon_{r}}}{t_{h}}=\frac{c \sqrt{\varepsilon_{r}}}{h} \equiv \frac{1}{t_{2}}$

Note from restriction 2 that in the time domain the present model is perhaps only accurate for characteristic times for changes in the waveform large compared to $t_{2}=t_{h} / \sqrt{\varepsilon_{r}}$. Thus the initial rise discontinuity as in equation 22 is only an approximate concept because the model does not apply to arbitrarily large $\omega$.

Another approximation should be mentioned at this point, and this relates to the finite width of the surface transmission line. Even though we have $w \gg h$ if we consider small enough w such that the magnetic field penetration into the ground goes to depths comparable to $w$, then the finite width can alter the results somewhat, depending to some extent on the relative sizes of $w$ and $d$. The fields in the ground for low frequencies and at depths larger than $w$ and $d$ are fixed by the geometry of the conductors in the ground. 7
7. It Carl E. Baum, Sensor and Simulation Note 22, A Transmission Line EMP Simulation Technique for Buried Structures, June 1966.

## III. Numerical Results

For this parametric study a wide range of parameters is used, specifically

$$
\begin{align*}
& \sigma=10^{-1}, 10^{-2}, 10^{-3}, 10^{-4} \text { mho/meter } \\
& \varepsilon_{r}=2,10,20  \tag{27}\\
& a=20,50,100 \text { meters }
\end{align*}
$$

For each combination of these three parameters 4 appropriate values of $h$ are chosen such that for each combination of parameters the initial time-domain characteristics vary from poor to rather good response. Both magnetic field and vertical electric field are considered for

$$
\begin{equation*}
\frac{x}{d}=0, .5,1 \tag{28}
\end{equation*}
$$

The electric field at $\mathrm{x}=0$ is simply a step function and is not included in the graphs.

All the cases considered are tabulated in Table 1 which is located just before the graphs. The graphs are not numbered sequentially but are labeled with a special code which identifies some of the major parameter values used for the graph. This code has the form

$$
\left(a_{1}, a_{2}, a_{3}, a_{4}\right)
$$

where

$$
\begin{align*}
& a_{1}=\log _{10}(\sigma) \\
& a_{2}=\varepsilon_{r}  \tag{29}\\
& a_{3}=d
\end{align*}
$$

and where $a_{4}$ is $B$ or $E$ to indicate which field is being graphed. In another form we might summarize this code as

$$
\left(\log _{10}(\sigma), \varepsilon_{r}, d, \quad \begin{array}{l}
\mathrm{B}
\end{array}\right)
$$

Each page contains four graphs corresponding to 4 values of $h$ which are labeled A, B, C, D. The four values of $h$ are chosen so that the shoulder in the magnetic field waveform ( $B / B_{0}$ ) at $t$ * of the order of $t_{d}$ has values which are approximately .2, .4, .6, .8 in that order. The values of $h$ used for the electric field waveforms are the same as those used for the associated magnetic field waveforms. As one would expect, increasing h improves the early-time performance of the surface transmission line. Note that the time scales chosen are

| $d$ | time scale (t*) |
| :---: | :---: |
| 20 m | 0 to $.6 \mu \mathrm{~s}$ |
| 50 m | 0 to $1.5 \mu \mathrm{~s}$ |
| 100 m | 0 to $3 \mu \mathrm{~s}$ |

In each case the maximum t* corresponds to about 9 transit times ( $9 \mathrm{t}_{\mathrm{d}}$ ) on the surface transmission line. This shows the earlytime behavior of the pulses and the first portion of the asymptotic rise of the waveform to its final value.

Referring back to restriction 1 (equation 25) note that for some of the graphs this restriction is not met, particularly for the cases with smallest $h, \varepsilon_{r}$ and $\sigma$. For example the case ( $-4,2$, 20 , a4) with $h=2.5 \mathrm{~m}$ has a value for rl of roughly 6 which is certainly not small compared to l. For an opposite extreme the case ( $-1,20,100$, a4) with $h=4.5 \mathrm{~m}$ has a value for rl of roughly $2 \times 10-7$ which is rather small compared to 1 . Thus for many of the cases considered the model should be rather accurate, within the high-frequency limitation of restriction 2 (equation 26). In some cases restriction $l$ is not met; these cases are included for completeness but should be used with caution because the accuracy of the model is in doubt for these cases.

As an assistance to the reader the values of $r_{1}$ and $t_{2}$ are included with each graph to indicate some of the limitations on the results. rl is dimensionless and should be small compared to 1 for good results; $t_{2}$ is a characteristic time indicating the short-time or high-frequency limits of the model in the particular case.

The graphs following table 1 are arranged four on a page with the four values of $h$ associated with a particular set of values for the code (a1, a2, a3, a4) all on the same page. The pages
are ordered by indexing first $a_{4}$ ( $B$ then $E$ ), second a3 (20, 50 , 100 in this order), third a2 ( $2,10,20$ in this order), and fourth al ( $-1,-2,-3,-4$ in this order).

As mentioned before, the magnetic and vertical electric field waveforms were calculated in terms of normalized frequency and time variables just as was done in reference l; the time axes were then converted to microseconds. Numerical inverse Fourier transforms ${ }^{8}$ were used and the rms errors between successive calculations (see reference 8 ) ranged between about $4.8 \times 10^{-3}$ and $7.8 \times 10^{-4}$.
8. Frank J. Sulkowski, Mathematics Note 2, FORPLEX: A Program to Calculate Inverse Fourier Transforms, November 1966.

| $\sigma$ | $\varepsilon_{r}$ | d | h |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | A (.2) | B (.4) | C(.6) | D (.8) |
| -1 | 2 | 20 | 0.1 | 0.2 | 0.7 | 1.7 |
|  |  | 50 | 0.1 | 0.4 | 1.0 | 2.5 |
|  |  | 100 | 0.2 | 0.6 | 1.6 | 4.5 |
|  | 10 | 20 | 0.1 | 0.2 | 0.6 | 1.5 |
|  |  | 50 | 0.1 | 0.4 | 1.0 | 2.5 |
|  |  | 100 | 0.2 | 0.5 | 1.5 | 4.5 |
|  | 20 | 20 | 0.1 | 0.2 | 0.6 | 1.5 |
|  |  | 50 | 0.1 | 0.4 | 1.0 | 2.5 |
|  |  | 100 | 0.2 | 0.5 | 1.5 | 4.5 |
| -2 | 2 | 20 | 0.2 | 0.8 | 2.0 | 6.0 |
|  |  | 50 | 0.5 | 1.0 | 3.0 | 10.0 |
|  |  | 100 | 0.5 | 2.0 | 5.0 | 14.0 |
|  | 10 | 20 | 0.2 | 0.7 | 2.0 | 5.0 |
|  |  | 50 | 0.5 | 1.0 | 3.0 | 10.0 |
|  |  | 100 | 0.5 | 1.5 | 5.0 | 15.0 |
|  | 20 | 20 | 0.2 | 0.6 | 2.2 | 5.0 |
|  |  | 50 | 0.5 | 1.0 | 3.0 | 10.0 |
|  |  | 100 | 0.5 | 1.5 | 5.0 | 15.0 |
| -3 | 2 | 20 | 1.0 | 2.0 | 6.0 | 15.0 |
|  |  | 50 | 1.5 | 3.0 | 12.0 | 25.0 |
|  |  | 100 | 2.5 | 5.0 | 15.0 | 40.0 |
|  | 10 | 20 | 0.7 | 2.0 | 5.0 | 12.0 |
|  |  | 50 | 1.5 | 3.0 | 10.0 | 20.0 |
|  |  | 100 | 2.0 | 5.0 | 14.0 | 35.0 |
|  | 20 | 20 | 0.7 | 2.0 | 5.0 | 12.0 |
|  |  | 50 | 1.5 | 3.0 | 8.0 | 15.0 |
|  |  | 100 | 2.0 | 5.0 | 12.0 | 35.0 |
| -4 | 2 | 20 | 2.5 | 6.0 | 15.0 | 40.0 |
|  |  | 50 | 3.5 | 11.0 | 30.0 | 80.0 |
|  |  | 100 | 6.0 | 16.0 | 40.0 | 100.0 |
|  |  | 20 | 1.5 | 4.5 | 8.0 | 20.0 |
|  | 10 | 50 | 3.0 | 8.0 | 20.0 | 60.0 |
|  |  | 100 | 5.0 | 14.0 | 35.0 | 80.0 |
|  |  | 20 | 1.0 | 3.0 | 8.0 | 20.0 |
|  | 20 | 50 | 2.5 | 6.0 | 15.0 | 40.0 |
|  |  | 100 | 4.0 | 12.0 | 30.0 | 65.0 |

Table 1. Summary of Parameters Used for Graphs (all units rationalized MKSA)

Graphs for parameter study Pages 15 to 86



A. $h=0.1 \mathrm{~m}, r_{1} \simeq 3.4 \times 10^{-3}, t_{2} \simeq 0.24 \mathrm{~ns}$

C. $\quad h=1.0 \mathrm{~m}, r_{1} \simeq 3.4 \times 10^{-5}, t_{2} \simeq 2.36 \mathrm{~ns}$

B. $\quad h=0.4 \mathrm{~m}, r_{1} \simeq 2.1 \times 10^{-4}, t_{2} \simeq 0.94 \mathrm{~ns}$

D. $\quad h=2.5 \mathrm{~m}, r_{1} \simeq 5.4 \times 10^{-6}, t_{2} \simeq 5.90 \mathrm{~ns}$ $(-1,2,50, B)$




A. $h=0.1 \mathrm{~m}, r_{1} \simeq 6.8 \times 10^{-4}, t_{2} \simeq 0.11 \mathrm{~ns}$

C. $\quad h=0.6 \mathrm{~m}, r_{1} \simeq 1.9 \times 10^{-5}, t_{2} \simeq 0.63 \mathrm{~ns}$

B. $h=0.2 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-4}, t_{2} \simeq 0.21 \mathrm{~ns}$

D. $h=1.5 \mathrm{~m}, r_{1} \simeq 3.0 \times 10^{-6}, t_{2} \simeq 1.58 \mathrm{~ns}$ $(-1,10,20, B)$
















$(-2,2,100, B)$




$\xrightarrow{\text { i A A. }} \quad h=0.5 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-3}, t_{2} \simeq 0.53 \mathrm{~ns}$

C. $\quad h=3.0 \mathrm{~m}, r_{1} \simeq 7.5 \times 10^{-5}, t_{2} \simeq 3.16 \mathrm{~ns}$

B. $h=1.0 \mathrm{~m}, r_{1} \simeq 6.8 \times 10^{-4}, t_{2} \simeq 1.05 \mathrm{~ns}$

D. $\quad h=10.0 \mathrm{~m}, r_{1} \simeq 6.8 \times 10^{-6}, t_{2} \simeq 10.55 \mathrm{~ns}$

$$
(-2,10,50, B)
$$





$\stackrel{A}{\Delta}$ A. $h=0.5 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-3}, t_{2} \simeq 0.53 \mathrm{~ns}$
B. $\quad h=1.5 \mathrm{~m}, r_{1} \simeq 3.0 \times 10^{-4}, t_{2} \simeq 1.58 \mathrm{~ns}$


C. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-5}, t_{2} \simeq 5.27 \mathrm{~ns}$

$$
\begin{gathered}
\text { D. } h=15.0 \mathrm{~m}, r_{1} \simeq 3.0 \times 10^{-6}, t_{2} \simeq 15.82 \mathrm{~ns} \\
(-2,10,100, E)
\end{gathered}
$$






$\stackrel{\text { is }}{\infty} \quad$ A. $\quad h=0.5 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-3}, t_{2} \simeq 0.37 \mathrm{~ns}$

C. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-5}, t_{2} \simeq 3.73 \mathrm{~ns}$

B. $h=1.5 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-4}, t_{2} \simeq 1.12 \mathrm{~ns}$

D. $\quad h=15.0 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-6}, t_{2} \simeq 11.19 \mathrm{~ns}$

$$
(-2,20,100, B)
$$





$$
\text { A. } \quad h=1.0 \mathrm{~m}, r_{1} \simeq 3.4 \times 10^{-1}, t_{2} \simeq 2.36 \mathrm{~ns}
$$


C. $h=6.0 \mathrm{~m}, \mathrm{r}_{1} \simeq 9.4 \times 10^{-3}, t_{2} \simeq 14.15 \mathrm{~ns}$

B. $h=2.0 \mathrm{~m}, r_{1} \simeq 8.5 \times 10^{-2}, t_{2} \simeq 4.72$ ns

D. $\quad h=15.0 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-3}, t_{2} \simeq 35.38 \mathrm{~ns}$ $(-3,2,20, E)$


${ }_{\omega}^{N}$
A. $\quad h=1.5 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-1}, t_{2} \simeq 3.54 \mathrm{~ns}$
B. $\quad h=3.0 \mathrm{~m}, r_{1} \simeq 3.8 \times 10^{-2}, t_{2} \simeq 7.08 \mathrm{~ns}$


C. $\quad h=12.0 \mathrm{~m}, \mathrm{r}_{1} \simeq 2.4 \times 10^{-3}, t_{2} \simeq 28.30 \mathrm{~ns}$
D. $\quad h=25.0 \mathrm{~m}, \mathrm{r}_{1} \simeq 5.4 \times 10^{-4}, t_{2} \simeq 58.97 \mathrm{~ns}$

$$
(-3,2,50,8)
$$





©r $A . \quad h=0.7 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-1}, t_{2} \simeq 0.74 \mathrm{~ns}$

C. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-3}, t_{2} \simeq 5.27 \mathrm{~ns}$

B. $\quad h=2.0 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-2}, t_{2} \simeq 2.11 \mathrm{~ns}$

D. $\quad h=12.0 \mathrm{~m}, r_{1} \simeq 4.7 \times 10^{-4}, t_{2} \simeq 12.66 \mathrm{~ns}$

$$
(-3,10,20,8)
$$



0

$$
\text { A. } \quad h=0.7 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-1}, t_{2} \simeq 0.74 \mathrm{~ns}
$$

B. $h=2.0 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-2}, t_{2} \simeq 2.11 \mathrm{~ns}$


C. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-3}, t_{2} \simeq 5.27 \mathrm{~ns}$
D. $\quad h=12.0 \mathrm{~m}, r_{1} \simeq 4.7 \times 10^{-4}, t_{2} \simeq 12.66 \mathrm{~ns}$ $(-3,10,20, E)$



$$
\text { A. } \quad h=1.5 \mathrm{~m}, r_{1} \simeq 3.0 \times 10^{-2}, t_{2} \simeq 1.58 \mathrm{~ns}
$$


C. $\quad h=10.0 \mathrm{~m}, r_{1} \simeq 6.8 \times 10^{-4}, t_{2} \simeq 10.55 \mathrm{~ns}$

B. $h=3.0 \mathrm{~m}, r_{1} \simeq 7.5 \times 10^{-3}, t_{2} \simeq 3.16 \mathrm{~ns}$

D. $\quad h=20.0 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-4}, t_{2} \simeq 21.10 \mathrm{~ns}$

$$
(-3,10,50, E)
$$




## 


$\stackrel{9}{(8)}$

$$
\text { A. } \quad h=0.7 \mathrm{~m}, r_{1} \simeq 6.9 \times 10^{-2}, t_{2} \simeq 0.52 \text { ns }
$$

B. $\quad h=2.0 \mathrm{~m}, r_{1} \simeq 8.5 \times 10^{-3}, t_{2} \simeq 1.49 \mathrm{~ns}$


c. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-3}, t_{2} \simeq 3.73 \mathrm{~ns}$
D. $\quad h=12.0 \mathrm{~m}, r_{1} \simeq 2.4 \times 10^{-4}, t_{2} \simeq 8.95 \mathrm{~ns}$ $(-3,20,20, B)$


$\stackrel{\otimes}{4}$

$$
\text { A. } \quad h=0.7 \mathrm{~m}, r_{1} \simeq 6.9 \times 10^{-2}, t_{2} \simeq 0.52 \mathrm{~ns}
$$

B. $\quad h=2.0 \mathrm{~m}, r_{1} \simeq 8.5 \times 10^{-3}, t_{2} \simeq 1.49 \mathrm{~ns}$


C. $h=5.0 \mathrm{~m}, r_{1} \simeq 1.4 \times 10^{-3}, t_{2} \simeq 3.73 \mathrm{~ns}$
D. $\quad h=12.0 \mathrm{~m}, r_{1} \simeq 2.4 \times 10^{-4}, t_{2} \simeq 8.95 \mathrm{~ns}$ $(-3,20,20, E)$

$\stackrel{8}{6}$

C. $\quad h=8.0 \mathrm{~m}, r_{1} \simeq 5.3 \times 10^{-4}, t_{2} \simeq 5.97 \mathrm{~ns}$

B. $\quad h=3.0 \mathrm{~m}, r_{1} \simeq 3.8 \times 10^{-3}, t_{2} \simeq 2.24 \mathrm{~ns}$

D. $h=15.0 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-4}, t_{2} \simeq 11.19 \mathrm{~ns}$

$$
(-3,20,50, B)
$$







$\xrightarrow{2}$

$$
\text { A. } \quad h=2.5 \mathrm{~m}, r_{1} \simeq 5.4 \times 10^{0}, t_{2} \simeq 5.90 \mathrm{~ns}
$$

B. $\quad h=6.0 \mathrm{~m}, r_{1} \simeq 9.4 \times 10^{-1}, t_{2} \simeq 14.15 \mathrm{~ns}$


C. $\quad h=15.0 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-1}, t_{2} \simeq 35.38 \mathrm{~ns}$
D. $\quad h=40.0 \mathrm{~m}, r_{1} \simeq 2.1 \times 10^{-2}, t_{2} \simeq 94.35 \mathrm{~ns}$ $(-4,2,20, E)$





C. $\quad h=30.0 \mathrm{~m}, r_{1} \simeq 3.8 \times 10^{-2}, t_{2} \simeq 70.76 \mathrm{~ns}$
D. $\quad h=80.0 \mathrm{~m}, r_{1} \simeq 5.3 \times 10^{-3}, t_{2} \simeq 188.69 \mathrm{~ns}$

$$
(-4,2,50, B)
$$



N

$$
\text { A. } h=3.5 \mathrm{~m}, r_{1} \simeq 2.8 \times 10^{0}, t_{2} \simeq 8.26 \mathrm{~ns}
$$

B. $h=11.0 \mathrm{~m}, r_{1} \simeq 2.8 \times 10^{-1}, t_{2} \simeq 25.95 \mathrm{~ns}$


D. $h=80.0 \mathrm{~m}, r_{1} \simeq 5.3 \times 10^{-3}, t_{2} \simeq 188.69 \mathrm{~ns}$ $(-4,2,50, E)$

A. $h=6.0 \mathrm{~m}, r_{1} \simeq 9.4 \times 10^{-1}, t_{2} \simeq 14.15 \mathrm{~ns}$

C. $\quad h=40.0 \mathrm{~m}, r_{1} \simeq 2.1 \times 10^{-2}, t_{2} \simeq 94.35 \mathrm{~ns}$

B. $h=16.0 \mathrm{~m}, r_{1} \simeq 1.3 \times 10^{-1}, t_{2} \simeq 37.74 \mathrm{~ns}$

D. $h=100.0 \mathrm{~m}, r_{1} \simeq 3.4 \times 10^{-3}, t_{2} \simeq 235.87 \mathrm{~ns}$

$$
(-4,2,100, B)
$$




تr

$$
\text { A. } \quad h=1.5 \mathrm{~m}, r_{1} \simeq 3.0 \times 10^{0}, t_{2} \simeq 1.58 \mathrm{~ns}
$$


C. $\quad h=8.0 \mathrm{~m}, r_{1} \simeq 1.1 \times 10^{-1}, t_{2} \simeq 8.44 \mathrm{~ns}$

B. $\quad h=4.5 \mathrm{~m}, \mathrm{r}_{1} \simeq 3.3 \times 10^{-1}, t_{2} \simeq 4.75 \mathrm{~ns}$

D. $\quad h=20.0 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-2}, t_{2} \simeq 21.10 \mathrm{~ns}$ $(-4,10,20, B)$



A. $\quad h=3.0 \mathrm{~m}, r_{1} \simeq 7.5 \times 10^{-1}, t_{2} \simeq 3.16 \mathrm{~ns}$
B. $\quad h=8.0 \mathrm{~m}, r_{1} \simeq 1.1 \times 10^{-1}, t_{2} \simeq 8.44 \mathrm{~ns}$


C. $h=20.0 \mathrm{~m}, r_{1} \simeq 1.7 \times 10^{-2}, t_{2} \simeq 21.10 \mathrm{~ns}$
D. $\quad h=60.0 \mathrm{~m}, r_{1} \simeq 1.9 \times 10^{-3}, t_{2} \simeq 63.29 \mathrm{~ns}$ $(-4,10,50,8)$

$\stackrel{\rightharpoonup}{6}$

A. $\quad h=5.0 \mathrm{~m}, r_{1} \simeq 2.7 \times 10^{-1}, t_{2} \simeq 5.27 \mathrm{~ns}$

C. $\quad h=35.0 \mathrm{~m}, r_{1} \simeq 5.5 \times 10^{-3}, t_{2} \simeq 36.92 \mathrm{~ns}$

B. $\quad h=14.0 \mathrm{~m}, r_{1} \simeq 3.5 \times 10^{-2}, t_{2} \simeq 14.77 \mathrm{~ns}$

D. $\quad h=80.0 \mathrm{~m}, r_{1} \simeq 1.1 \times 10^{-3}, t_{2} \simeq 84.39 \mathrm{~ns}$ $(-4,10,100,8)$



$\stackrel{\infty}{\sim}$

$$
\text { A. } \quad h=1.0 \mathrm{~m}, r_{1} \simeq 3.4 \times 10^{0}, t_{2} \simeq 0.75 \mathrm{~ns}
$$

B. $\quad h=3.0 \mathrm{~m}, r_{1} \simeq 3.8 \times 10^{-1}, t_{2} \simeq 2.24 \mathrm{~ns}$


C. $\quad h=8.0 \mathrm{~m}, r_{1} \simeq 5.3 \times 10^{-2}, t_{2} \simeq 5.97 \mathrm{~ns}$
D. $\quad h=20.0 \mathrm{~m}, \mathrm{r}_{1} \simeq 8.5 \times 10^{-3}, t_{2} \simeq 14.92 \mathrm{~ns}$
$(-4,20,20, B)$


$\stackrel{\infty}{\omega}$
A. $\quad h=2.5 \mathrm{~m}, r_{1} \simeq 5.4 \times 10^{-1}, t_{2} \simeq 1.86 \mathrm{~ns}$

C. $\quad h=15.0 \mathrm{~m}, r_{1} \simeq 1.5 \times 10^{-2}, t_{2} \simeq 11.19 \mathrm{~ns}$

B. $\quad h=6.0 \mathrm{~m}, r_{1} \simeq 9.4 \times 10^{-2}, t_{2} \simeq 4.48 \mathrm{~ns}$

D. $\quad h=40.0 \mathrm{~m}, r_{1} \simeq 2.1 \times 10^{-3}, t_{2} \simeq 29.83 \mathrm{~ns}$




