

EMP Theoretical Notes

Note XII
16 July 1965

Electron Thermalization and Mobility in Air

1/Lt Carl E. Baum
Air Force Weapons Laboratory

CLEARED
FOR PUBLIC RELEASE
AFRL/DEO-PA
29 AUG 04

Abstract

The electron thermalization time is calculated showing that, under normal, sea-level atmospheric conditions, this phenomenon can be neglected in any calculations of electron parameters (e.g., mobility, attachment to O_2 , etc.). The equilibrium electron temperature and the electron mobility in air are also calculated as functions of the electric field. The effect of water vapor is included in the analyses while that of ions is not included.

AFRL/DE 04-414

I. Introduction

As discussed in EMP Theoretical Note VI (TN VI) the temperature of the electrons in ionized air has a significant effect on the electron parameters such as the mobility and attachment and recombination coefficients. The electron temperature tends toward an equilibrium temperature which depends on the electric field. However, the electrons produced during ionization have some fraction of the 32 e.V. per electron-ion pair and take some time to reach the equilibrium temperature. Thus, the average electron temperature is generally higher than the equilibrium temperature. In this note we will first, calculate the time required for the electrons to approach this equilibrium temperature, and second, calculate this equilibrium temperature (together with the electron mobility) as a function of the electric field. The basic equations for these results are based on those derived in TN VI.

II. Electron Thermalization

From the basic equation for the electron temperature developed in TN VI¹

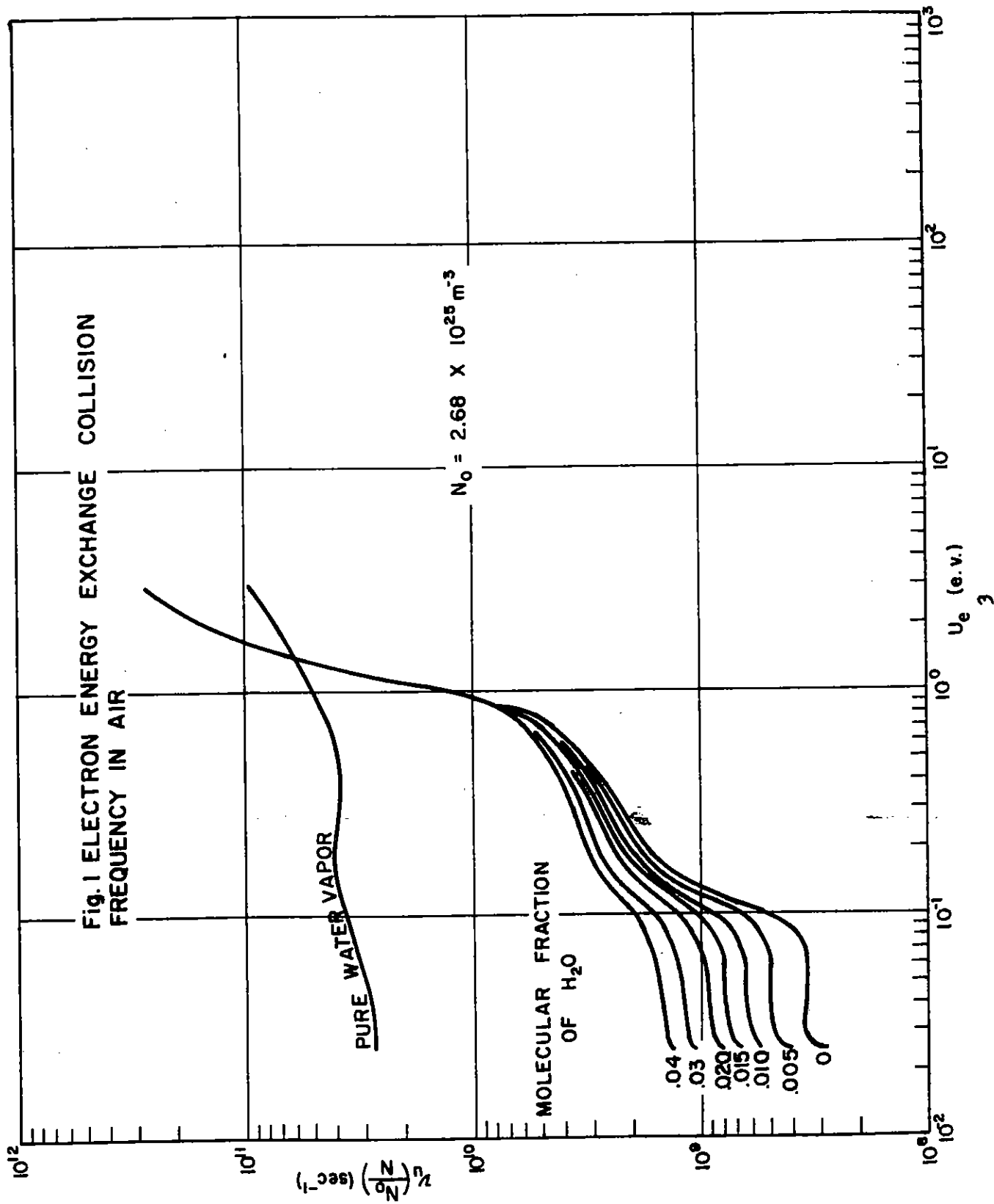
$$\frac{\partial U_e}{\partial t} = \frac{2}{3} \frac{e}{m} \frac{E^2}{v_m} - (U_e - U_0) v_u \quad (1)$$

where:

U_e	electron temperature
U_0	thermal temperature (,025e.V.)
e	electron charge (magnitude)
m	electron mass
E	electric field
v_u	electron energy exchange collision frequency
v_m	electron momentum transfer collision frequency
t	time

we can calculate this temperature under various conditions. Assuming a value for the electric field in equation (1), we can calculate the time required for an electron to approach equilibrium temperature from some higher temperature. For this calculation to be valid the fractional energy (or temperature) loss per collision must be small compared to 1. Figures 1 and 2 depict the energy exchange and momentum transfer collision frequencies, respectively, for electrons in air (neglecting collisions with ions), and show that the

¹. All units are taken in the rationalized m,K,s. system unless otherwise specified. The temperatures are taken in electron volts (e.V.).



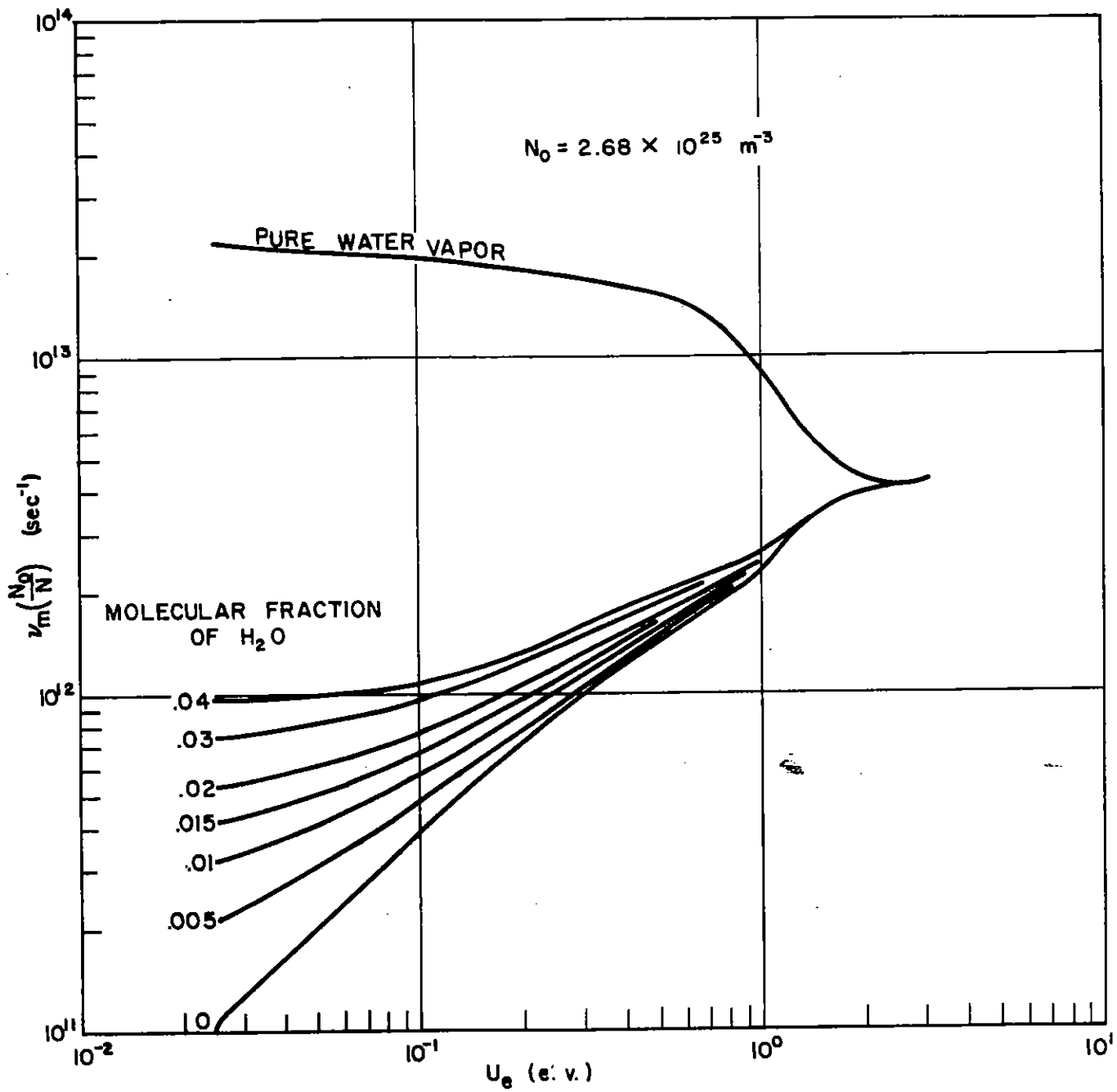


Fig. 2. ELECTRON MOMENTUM TRANSFER COLLISION FREQUENCY IN AIR.

fractional temperature loss per collision, v_u/v_m , is at most about 10^{-1} at the higher temperatures and approximately 10^{-3} at thermal temperatures.²

Since we are here interested in an approximate solution, we will ignore any effects produced by:

1. the electron temperature distribution function.
2. the relationship between the average fractional temperature loss per collision and the typical fractional energy loss per collision.
3. electron attachment and recombination times.
4. electron-ion collisions.

The results which we obtain will show that the thermalization time is much less than the attachment time of 10^{-6} sec at normal atmospheric densities. And, since the attachment time increases more rapidly than the thermalization time as density decreases, the assumption (3) above, is validated.³ In addition, by restricting the discussion to low ionic densities, we can avoid the problem of electron-ion collisions, (4) above, which may significantly lower the electron temperature and mobility at high ionic densities. The electron thermalization time may also be decreased slightly by the presence of ions, but we shall neglect this as a slight correction.

Moreover, we should note that in these curves and those to follow, the quantities have been normalized to the STP (760 mm Hg pressure and 273° K temperature) molecular density of air, $N_0 = 2.68 \times 10^{25}$ meter⁻³, so that the factor N/N_0 to some power may be used to apply these results to any density. The measurements were made at 300° K but at various pressures.

Since the presence of an electric field raises the equilibrium electron temperature and thus shortens the thermalization time for a conduction electron, the electric field will be set to zero as a worst case. A calculation of the time history of the temperature of a conduction electron provides a plot of the thermal and higher temperatures, as shown in figure 3. We shall now indicate how this calculation is done. Without the electric field equation (1) reduces to

$$\frac{\partial U_e}{\partial t} = - (U_e - U_0) \nu_u \quad (2)$$

We can normalize the time and the energy exchange collision frequency to the STP molecular density to give

2. The curves in this note have been constructed from the data supplied by Dr. Phelps of Westinghouse. The influence of water vapor has been included by a linear combination (based on molecular fractions) of the collision frequencies for electrons in pure air and in pure water vapor.

3. For an analysis including these effects but using lower energy exchange collision frequencies for near-thermal electron temperatures see: Air Conductivity Produced by Nuclear Explosions, W. J. Karzas and R. Latter, Memorandum RM-3671-PR, May 1963, Rand Corporation.

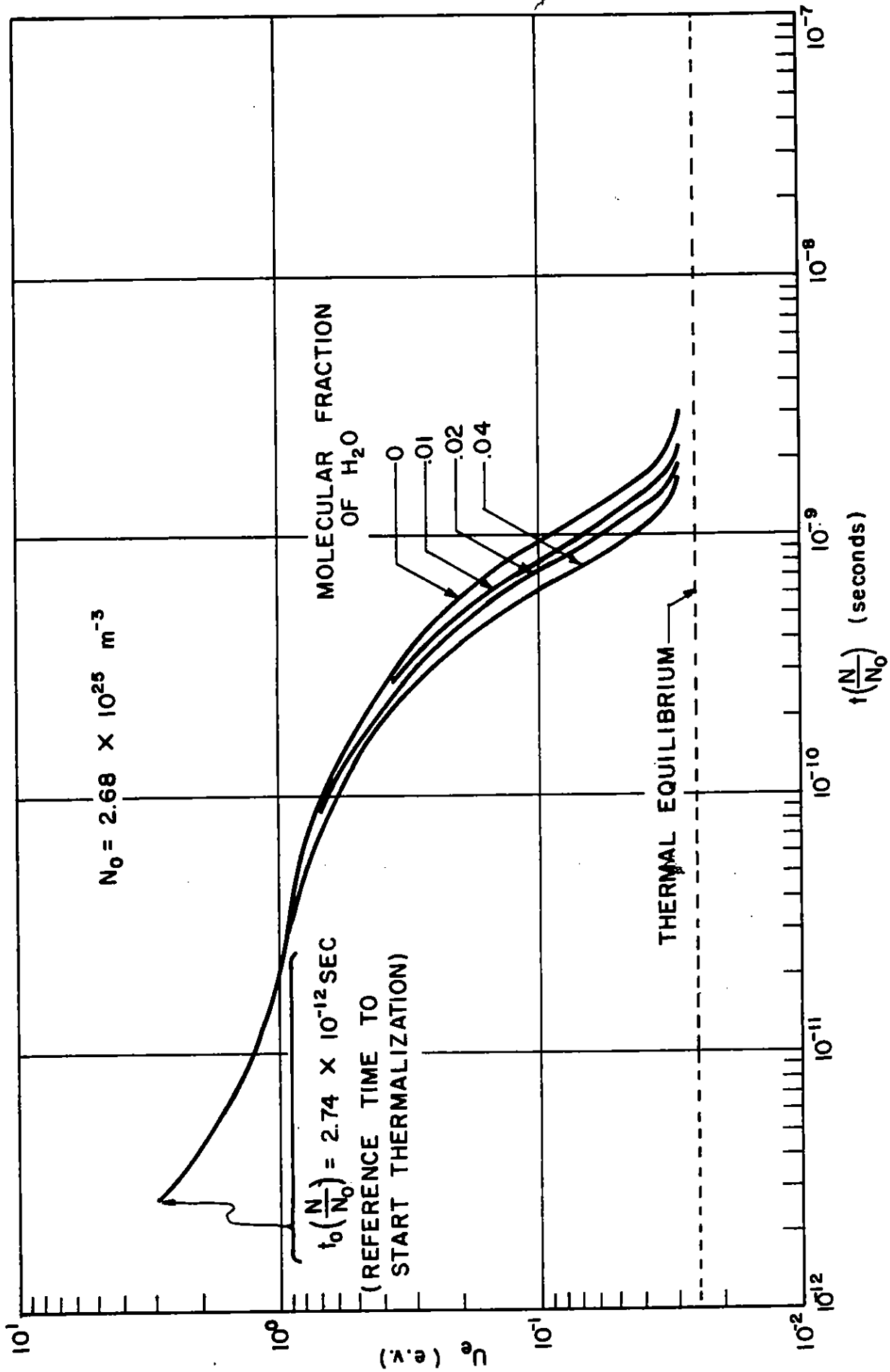


Fig. 3. ELECTRON THERMALIZATION TIME IN AIR

$$\frac{\partial U_e}{\partial (t \frac{N}{N_0})} = - (U_e - U_0) \left(v_{u \frac{N}{N_0}} \right) \quad (3)$$

For a thermalization calculation we must choose a temperature at which to start. Since the data extend up to 3 e.V, we might start there and call that time zero. However, the collision frequency increases rapidly with temperature, and we may extrapolate this plot of energy exchange collision frequency vs. temperature with a straight line on log-log scales. This is equivalent to assuming that

$$v_{u \left(\frac{N}{N_0} \right)} = \left(v_{u_{\max} \frac{N}{N_0}} \right) \left(\frac{U_e}{U_{e_{\max}}} \right)^n \quad (4)$$

where $(v_{u_{\max} \frac{N}{N_0}})$ and $U_{e_{\max}}$ are the largest values on the data curve and the exponent n is given by the slope at this data point.

Combining equations (3) and (4) and ignoring U_0 as being small for this calculation we have

$$\frac{\partial U_e}{\partial (t \frac{N}{N_0})} = - \left(v_{u_{\max} \frac{N}{N_0}} \right) \left(\frac{U_e}{U_{e_{\max}}} \right)^n U_e \quad (5)$$

Now we can calculate the time required for an electron to thermalize from an arbitrarily large temperature to this maximum data temperature. Thus, on integrating equation (5)

$$U_{e_{\max}}^n \int_0^{U_{e_{\max}}} \frac{dU_e}{U_e^{n+1}} = - \left(v_{u_{\max} \frac{N}{N_0}} \right) \int_0^{t_0 \frac{N}{N_0}} d(t \frac{N}{N_0}) \quad (6)$$

or

$$-\frac{1}{n} = - \left(v_{u_{\max} \frac{N}{N_0}} \right) \left(t_0 \frac{N}{N_0} \right) \quad (7)$$

or

$$t_0 \frac{N}{N_0} = \left[n \left(v_{u_{\max} \frac{N}{N_0}} \right) \right]^{-1} \quad (8)$$

From the data in figure 1 we find that

$$n = 1.34 \quad (9)$$

and thus

$$t_o \frac{N}{N_o} = 2,74 \times 10^{-12} \text{ sec} \quad (10)$$

This is small compared to the total thermalization time and will have a significant effect only near the beginning of the thermalization curve which can now be calculated from equation (3) with the data of figure 1. This result is given in figure 3. As we can see from this figure the time required for an electron to reach near-thermal temperatures for near-STP densities is about a nanosecond. This justifies our neglect of electron thermalization under such conditions, and permits us to assume that all conduction electrons are at their equilibrium temperature.

III. Equilibrium Electron Temperature

The presence of an electric field establishes an equilibrium electron temperature greater than thermal. If we know this equilibrium electron temperature we can then estimate the time required for a conduction electron to reach this temperature. We can use the graph of electron temperature versus time in figure 3, noting where a horizontal line through a desired temperature intersects the appropriate thermalization curve,

To determine the equilibrium electron temperature, given the electric field, we set the time derivative of the electron temperature to zero in equation (1). Thus,

$$\frac{2}{3} \frac{e}{m} \frac{E^2}{v_m} = (U_e - U_o) v_u \quad (11)$$

Rearranging terms we have

$$E = \left[\frac{3}{2} \frac{m}{e} (U_e - U_o) v_m v_u \right]^{1/2} \quad (12)$$

Including the density normalization factors we have,

$$E \left(\frac{N}{N_o} \right) = \left[\frac{3}{2} \frac{m}{e} (U_e - U_o) \left(v \frac{N_o}{mN} \right) \left(v \frac{N_o}{uN} \right) \right]^{1/2} \quad (13)$$

From this equation and the plots in figures 1 and 2, we can plot the equilibrium electron temperature as a function of the electric field (figure 4). Given the electric field and the atmospheric density, we can use figures 3 and 4 to determine the time required for the typical conduction electron to reach a near-equilibrium temperature. Thus, we may determine the validity in a given case of assuming that the conduction electrons are at a particular equilibrium temperature.

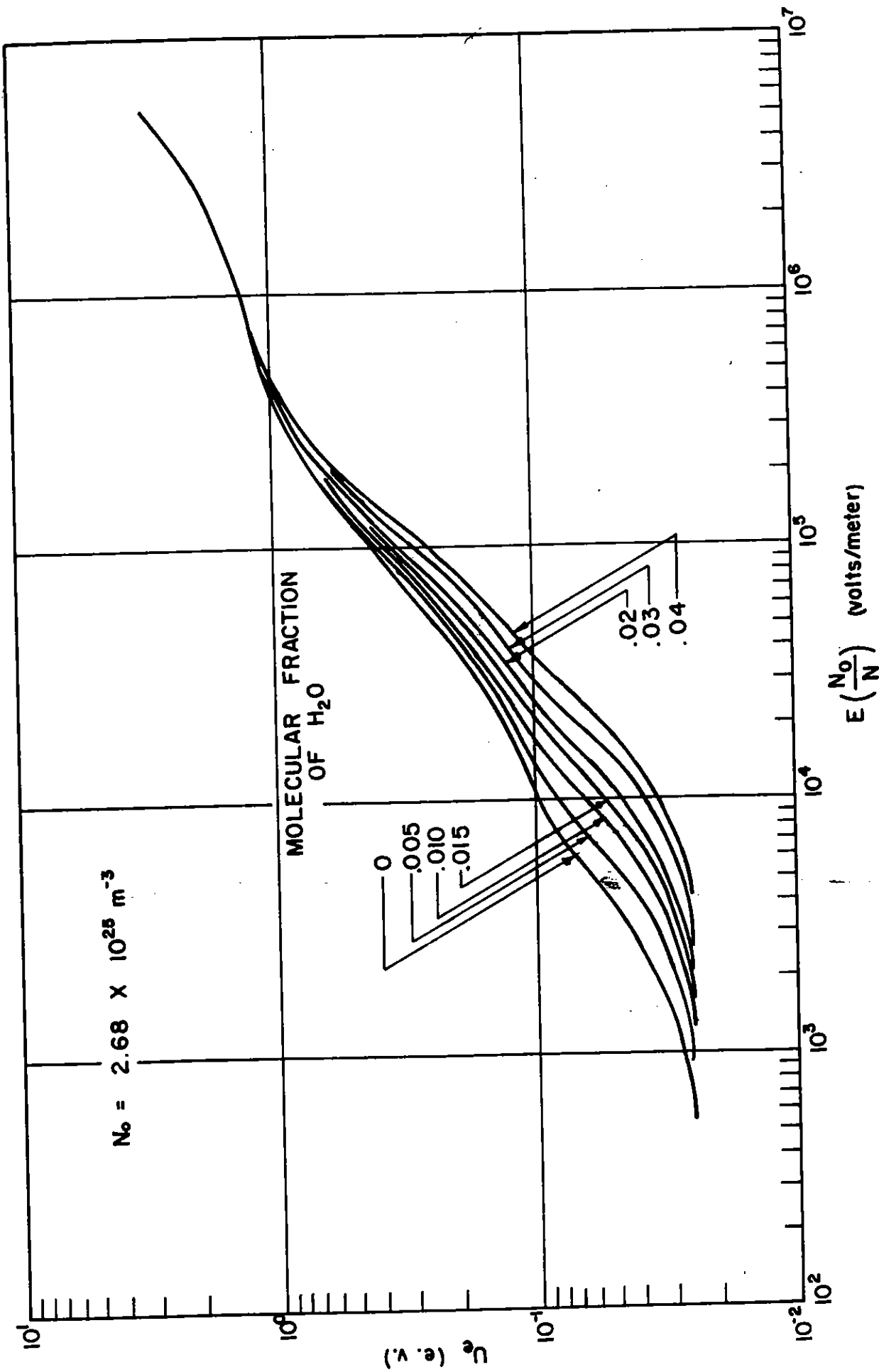


Fig. 4. EQUILIBRIUM ELECTRON TEMPERATURE IN AIR

IV. Electron Mobility

With this last result for the equilibrium electron temperature and with the assumption that the electron thermalization time is short compared with times of interest, we can calculate the electron mobility as a function of the electric field with the molecular fraction of water vapor as a parameter. To do this we need the equation for the electron mobility

$$\mu_e = \frac{e}{m} \frac{1}{\nu_m} \quad (14)$$

or

$$\mu_e \frac{N}{N_0} = \frac{e}{m} \left[\nu_m \frac{N_0}{N} \right]^{-1} \quad (15)$$

Using equation (15) together with figures 2 and 4 we obtain the electron mobility vs. the electric field shown in figure 5.

V. Summary

Thus, by considering the electron energy exchange collision frequency as a function of the electron temperature we can calculate the thermalization time for an electron. This calculation indicates that, for near-STP densities, the thermalization time is about a nanosecond. This justifies our neglect of this time so that mobility calculations as shown in figure 5 are valid. For sufficiently high $E \frac{N_0}{N}$ the equilibrium electron tempera-

ture may also be high enough so that, even for reduced molecular densities, the time to approach equilibrium may also be short enough to neglect.

This analysis could also include the effect of ions on the electron thermalization and mobility in the same manner as the effect of water vapor is included. However, this would give two sets of parameters for the curves, and would complicate the presentation. The ions should have little effect on the time required for the electrons to cool to a given temperature, but the presence of large ion densities can change the equilibrium electron temperature and electron mobility,

The author would like to thank Mrs. Beverly Berger for her calculational assistance in the preparation of this note.

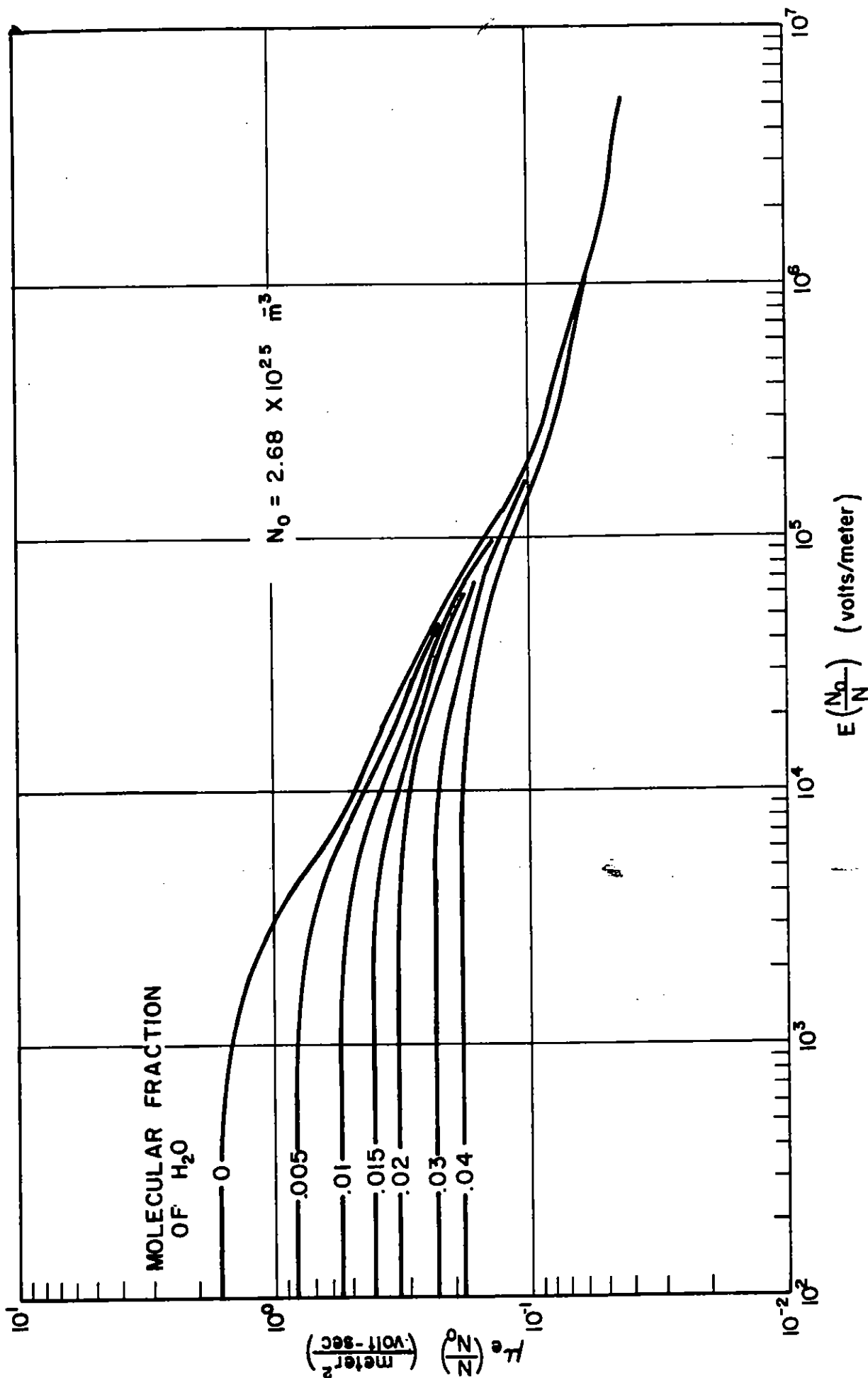


Fig. 5. ELECTRON MOBILITY IN AIR

