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EMP Theoretical Notes

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Later Time Sources of EMP

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Abstract

The later time EMP sources are here defined as the charge currents and ionization rates due to neutron interactions with ground or water beneath a nuclear detonation. This note summarizes the representation of these later time sources.

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A distributed source of gamma radiation is produced within a few tens of centimeters of the surface of the ground (or water) beneath a nuclear detonation, through capture and inelastic collisions of neutrons. This gamma source creates charge currents and ionization in the vicinity of the detonation through Compton collisions. These currents and ionization are negligible in early stages of EMP generation. However, after the 14 Mev neutrons have passed any point, the later time sources are dominant and effective in generating significant EMP fields. The distributed source of gammas in the ground is treated in this analysis as if it could be represented by point source and surface source components rather than the actual volume sources.

A. General Expressions.

Expressions for the currents and ionization rates from point isotropic and surface sources of monoenergetic gamma rays are given here (for first collisions of the gamma rays).

1. Current from a point source

$$J_c(R,t)\hat{r} = \frac{qR_e}{\lambda_\gamma} \cdot \frac{e^{-R/\lambda_\gamma}}{4\pi R^2} S_0(t-R/c) \quad \frac{\text{amps}}{\text{meter}^2} \tag{1}$$

where $q = 1.6\ (10^{-19})$ coulombs

S_o = the number of gammas per second leaving the point source

λ_γ = the gamma ray mean free path (the total cross section very nearly equals the Compton collision cross section in the anticipated energy region and is assumed identical)

R_e = the mean forward electron range for Compton recoil electrons

2. Ionization rate from a point source

$$Q(R,t) = \frac{E_e(10^6)\, e^{-R/\lambda_\gamma}}{34 \cdot \lambda_\gamma \cdot 4\pi R^2} S_0(t-R/c) \quad \frac{\text{ion-pairs}}{\text{m}^3\text{-sec}} \tag{2}$$

where Ee is the average recoil electron energy in Mevs.

3. Currents from a surface source

$$J_c(R,\theta,t)\hat{z} = \frac{qR_e}{\lambda_\gamma} e^{-R\cos\theta/\lambda_\gamma} S_s\left(R\sin\theta,\ t-\frac{R\cos\theta}{c}\right) \quad \frac{\text{amps}}{\text{m}^2} \tag{3}$$

where θ is the polar angle measured from the normal,

S_s is the surface source expressed in gammas/meter^2-sec in the Z direction (up).

$$J_c(R,\theta,t)\hat{r} = J_c(R,\theta,t)\hat{z}\cos\theta$$

$$J_c(R,\theta,t)\hat{\theta} = J_c(R,\theta,t)\hat{z}\sin\theta$$

4. Ionization rate from a surface source

$$Q(R,\theta,t) = \frac{E_e(10^6)}{34} \cdot \frac{1}{\lambda_\gamma} \cdot e^{-\frac{R\cos\theta}{\lambda_\gamma}} \cdot S_s\left(R\sin\theta, t - \frac{R\cos\theta}{c}\right) \ \frac{\text{ion-pairs}}{m^3\text{-sec}} \qquad (4)$$

B. Multiply Scattering Gamma Rays.

What should be done about the fact that gamma rays scatter and continue to contribute for more than one collision? Since the point and surface gamma sources, S_o and S_s, vary slowly with respect to time (decay times are typically hundreds of microseconds), we can multiply the results of expressions 2 and 4 of section A by the build-up-factors for energy deposition as functions of R and Z respectively, to obtain the true ionization rates. Expressions 1 and 3 can be multiplied by the current build-up-factors to obtain the true currents.

C. The Point and Surface Sources.

The point and surface sources S_o and S_s arise from neutron interactions with the ground. In reality the sources are volume sources near the burst point with distribution into the ground. Approximately equivalent surface sources can be computed as indicated in Chapter II, Section 4 of the Reactor Handbook, Vol III, Part B, by E. P. Blizard.

The ground inelastic collisions of fast neutrons form a source relatively localized around the burst point (or the projection of the burst point on the ground plane). This source can be considered to be a point source for EMP calculations.

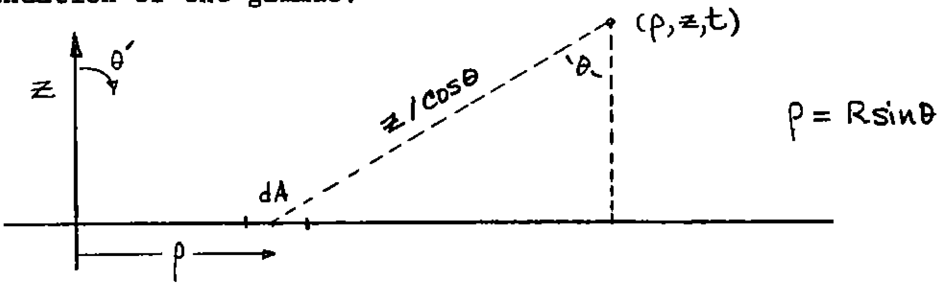
The ground capture collisions contain a term localized around the burst and a term spread over long distances. The localized term can be considered as a point source and the other can be considered as a surface source.

The surface sources (and point sources obtained from these) as described by Blizard have a Cos θ dependence associated with them because more gammas will escape from the ground in the Z direction than at angles to Z because of absorption. How does this alter the expressions 1-4?

D. Cos Theta Correction.

Expressions 1 and 2 can be corrected by merely multiplying them by Cos θ. The build-up factor should also be based on a Cos θ rather than an isotropic source; however, within several mean free paths the isotropic point source build-up-factor is probably adequate.

The surface source expressions (3 and 4) were based on a plane source emitting gamma rays in the Z direction. But we don't have a plane source. We have a source emitting $(S_s/2\pi)$ Cos θ gammas/steradian-meter^2, where θ is the angle from the normal to the ground. S_s gammas/meter^2 was found by integrating one half of the volume source over Z in the ground and ignoring attenuation of the gammas.



The $\hat{z}$ current density at (ρ,z,t) for a uniform source, independent of ρ, is:

$$J_{\hat{z}}(z,t) = \frac{q R_e S_s(\rho, t - z/c)}{\lambda_\gamma \cdot 2\pi} \int_0^{\pi/2} \frac{e^{-\frac{z/\cos\theta}{\lambda_\gamma}}}{(z/\cos\theta)^2} \cos^2\theta \, \overbrace{2\pi\left(\frac{z}{\cos\theta}\right)\sin\theta\left(\frac{z}{\cos\theta}\right)\cos\theta \, d\theta}^{dA \text{ ring}}$$

or

$$J_{\hat{z}}(z,t) = \frac{q R_e S_s(\rho, t - z/c)}{\lambda_\gamma} \int_0^{\pi/2} e^{-\frac{z/\cos\theta}{\lambda_\gamma}} \cos^3\theta \sin\theta \, d\theta$$

If we assume that $\exp(-z\cos\theta/\lambda_\gamma) \cong \exp(-z/\lambda_\gamma)$, we obtain as an overestimate,

$$J_{\hat{z}}(z,t) \cong \frac{q R_e S_s}{\lambda_\gamma} e^{-z/\lambda_\gamma} \int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta = \frac{q R_e S_s}{\lambda_\gamma} e^{-z/\lambda_\gamma} \left[-\frac{\cos^4\theta}{4} \right]_0^{\pi/2}$$

If we substitute for $S_s(\rho, t-z/c)$ the non-uniform value $S_s(R\sin\theta, t-\frac{R\cos\theta}{c})$, we obtain

$$J_{\hat{z}}(R\cos\theta, t) \cong \frac{q R_e}{\lambda_\gamma} \frac{S_s(R\sin\theta', t - \frac{R\cos\theta'}{c})}{4} e^{-R\cos\theta'/\lambda_\gamma} \tag{5}$$

θ' ~~theta~~ is a different theta than that of the above integration; it is the angle from the positive z direction.

Note that the effect of the cosine emission rather than the plane Z directed emission was to decrease $J_{\hat{z}}$ by a factor of at least 4. The surface source S_s is not uniform, but it varies slowly with time and space, therefore, the $J_{\hat{z}}$ of expression 5 is very accurate near the ground.

The corresponding ionization rate is

$$Q(\rho,z,t) = \frac{E_e(10^6)}{34\lambda_\gamma}\frac{S_s(\rho, t - z/c)}{2\pi}\int_0^{\pi/2} \frac{e^{-\frac{z/\cos\theta}{\lambda_\gamma}}}{(z/\cos\theta)^2}\cos\theta \, 2\pi \left(\frac{z}{\cos\theta}\right)^2 \sin\theta\cos\theta \, d\theta$$

which reduces to

$$Q(R,\theta,t) = \frac{E_e(10^6)}{34\lambda_\gamma}\;\frac{S_s(R\sin\theta', t - R\cos\theta'/c)}{3} \qquad (6)$$

An interesting result is that the ionization rate is not decreased as much as the charge current, which is to be expected because the ionization rate is not a vector quantity.

E. Quantitative Results on a per Neutron Basis Using John Malik's Monte Carlo Results*

E.1. The Ground Inelastic Contributions.

$$S_o = \frac{10\, e^{-t/280}}{t^2}\;\mu(t-6)\;\frac{\text{Mev}}{\text{shake}}, \quad t \text{ in shakes, average } \gamma \text{ energy} = 5 \text{ Mev}.$$

$$\text{or } S_o = 2(10^{-8})\,\frac{e^{-t/2.8(10^{-6})}}{t^2}\;\mu(t - 6(10^{-8}))\;\frac{\gamma\text{'s}}{\text{sec}}, \quad t \text{ in seconds}.$$

$$J_c(R,t)^{\hat{r}} = .658(10^{-29})\,\frac{e^{-\frac{R\cdot\rho_r}{310}}}{R^2}\cdot\frac{e^{-t/2.8(10^{-6})}}{t^2}\;\mu(t - 6(10^{-8}))\cos\theta \qquad -(7)$$

where ρ_r is the relative air density (relative to STP)

$$Q(R,t) = .758(10^{23})\,\rho_r\, J_c(R,t) \qquad (8)$$

*See DASA Data Center Report 41 (SRD), "Electromagnetic Pulse Phenomenology and Effects" (U) Dec 1965, Table 1.1 (U)

E. 2. The Ground Capture Contributions From the Close Term.

$$S_o = 1.15(10^{-8})\, e^{-\frac{R}{15}}\, e^{-\frac{t}{6(10^{+4})}} \frac{\text{Mev}}{\text{M}^2\text{-Shake}}$$ t in shakes, average gamma energy is 3 Mev

or $$S_o = 3.83(10^{-1})\, e^{-\frac{R}{15}}\, e^{-\frac{t}{6(10^{-4})}} \frac{\text{gammas}}{\text{m}^2\text{-second}},$$ t in seconds

We want to consider this as a point source; therefore, we sum (integrate) S_o over R and consider all gammas to be leaving R = 0.

$$S_o = .383 \int_0^\infty e^{-R/15}\, 2\pi R\, dR\, e^{-t/6(10^{-4})}$$

$$S_o = .383(2\pi)(225)\, e^{-t/6(10^{-4})}\ \gamma\text{'s/sec}.$$

finally $$S_o = 5.43(10^{+2})\, e^{-t/6(10^{-4})}\ \gamma\text{'s/sec},\ t \text{ in seconds}$$

thus

$$J_c(R,t)\hat{R} = 1.22(10^{-19})\, e^{-t/6(10^{-4})} \cos\theta\, \frac{e^{-R\cdot\rho_r/220}}{R^2}\ \frac{\text{amps}}{\text{m}^2\text{-neutron}} \tag{9}$$

$$Q(R,t) = 1.01(10^4)\,\rho_r\, e^{-t/6(10^{-4})} \cdot e^{-R\cdot\rho_r/220}\, \frac{\cos\theta}{R^2}\ \frac{\text{ion-pairs}}{\text{m}^3\text{sec-n}} \tag{10}$$

or $$Q(R,t) = .828(10^{23})\,\rho_r\, J_c(R,t)$$

E. 3. The Ground Capture Contributions from the Dispersed Term.

$$S_s = .733(10^{-10})\, e^{-R\sin\theta/113} \cdot e^{-t/6(10^{-4})}\ \frac{\text{Mev}}{\text{m}^2\text{-sh}},\ t \text{ in shakes}$$

$$S_s = 2.44(10^{-3})\, e^{-R\sin\theta/113}\, e^{-t/6(10^{-4})}\ \gamma\text{'s/m}^2\text{-sec},\ t \text{ in seconds}$$

then

$$J_c(R\cos\theta,t)\hat{z} = 1.72(10^{-24})\, e^{-\frac{R\sin\theta}{113} - \frac{R\cos\theta\cdot\rho_r}{220}} \cdot e^{-t/6(10^{-4})}\ \frac{\text{amps}}{\text{m}^2\text{-n}} \tag{11}$$

$$Q(R\cos\theta, t) = .19\,\rho_r\, e^{-\frac{R\sin\theta}{113} - \frac{R\cos\theta\,\rho_r}{220}} \cdot e^{-t/6(10^{-4})} \quad \frac{\text{ion-pairs}}{m^3 - n} \tag{12}$$

$$\text{or} \quad Q(R\cos\theta, t) = 1.108\, J_c(R,t)\,\hat{z} \times 10^{23}$$

F. Summary.

Expressions 7, 8, 9, 10, 11 and 12, when multiplied by their appropriate build-up-factors will yield the approximate current and ionization rate densities in the air around a near surface burst.