EMP Theoretical Notes
Note XXV

18 February 1967

The Reflection of Pulsed Waves From the Surface of a Conducting Dielectric

Capt Carl E. Baum

Air Force Weapons Laboratory

Abstract

With numerical inverse Fourier transforms, the reflection of a step function plane wave from a flat ground or water surface is calculated, using simplified forms of the soil or water conductivity. Using convolution integral techniques with these step response functions, one can extend these results to various pulse shapes for the incident wave.

I. Introduction

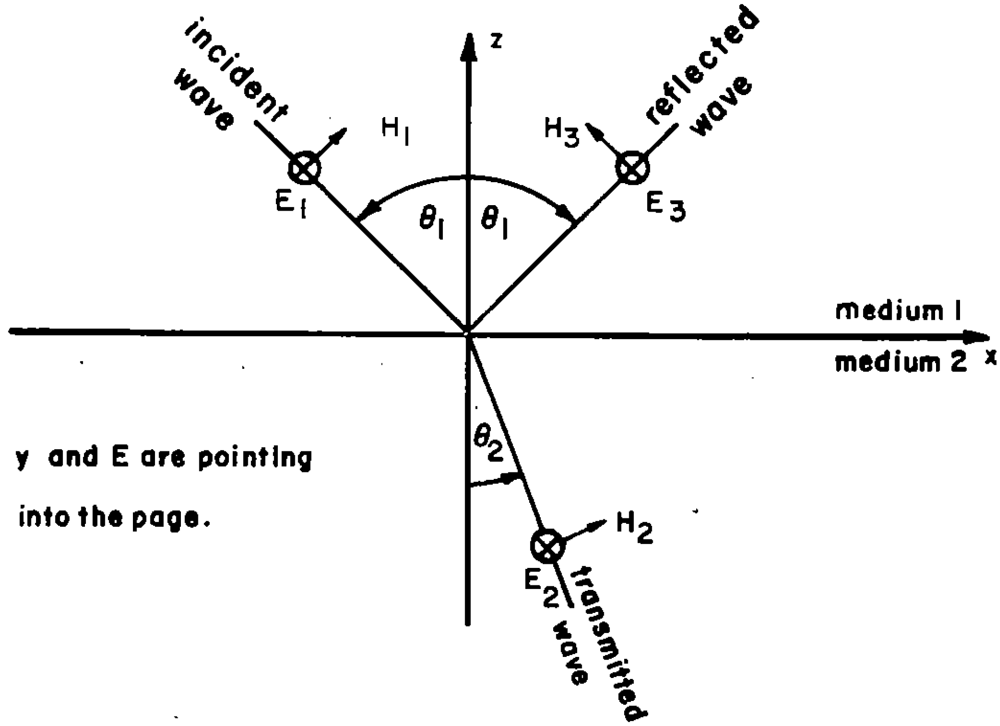
An electromagnetic wave in the air propagating toward the ground or water surface undergoes reflection and refraction at that surface. We may be interested in this phenomenon from various points of view, including calculation of the reflected and transmitted fields, measurement of the fields at the ground or water surface, and design of devices to produce desired field distributions near a ground or water surface. The reflection and transmission coefficients in the frequency domain are given by Fresnel's equations. For this note we use a numerical inverse Fourier transform technique to calculate the reflection of a step function electromagnetic plane wave from the ground or water surface.[1] With the aid of convolution integral techniques these results can be extended to pulse shapes other than step functions for the incident plane wave. Alternatively one can take a Fourier transform of the incident pulse shape, multiply it by the reflection coefficient in the frequency domain, and take the inverse Fourier transform.

For these calculations the upper medium or medium one (air), in which the incident plane wave propagates, is assumed to have permittivity, ϵ_0, permeability, μ_0, and zero conductivity. The lower medium or medium two (ground or water) is assumed to have permittivity, ϵ_2, permeability, μ_0, and conductivity, σ_2. These parameters are all taken as independent of frequency. Actually, the permittivity of soil is quite dependent on frequency and to a small extent so is the conductivity.[2] However, as frequency is increased the permittivity appears to level out. Perhaps for radian frequencies of the order of or greater than σ_2/ϵ_2 the permittivity is somewhat constant. For radian frequencies much less than σ_2/ϵ_2 (which is itself frequency dependent) the role of the permittivity is small compared to that of the conductivity. Since we do not know the details of the permittivity (and conductivity) variation with frequency near the relaxation frequency, σ_2/ϵ_2, of the soil, an accurate calculation for a particular case is not feasible. Thus, we assume parameters independent of frequency and calculate for several cases. Specifically, we normalize the time to the relaxation time, ϵ_2/σ_2, and choose different values of ϵ_2/ϵ_0 ranging from 10 to 80. If, for a particular case of interest, the frequency dependence of ϵ_2 and σ_2 were known accurately, then the pulse reflection could be accurately calculated for that case. The present study should, however, give some of the significant features of the pulse reflection.

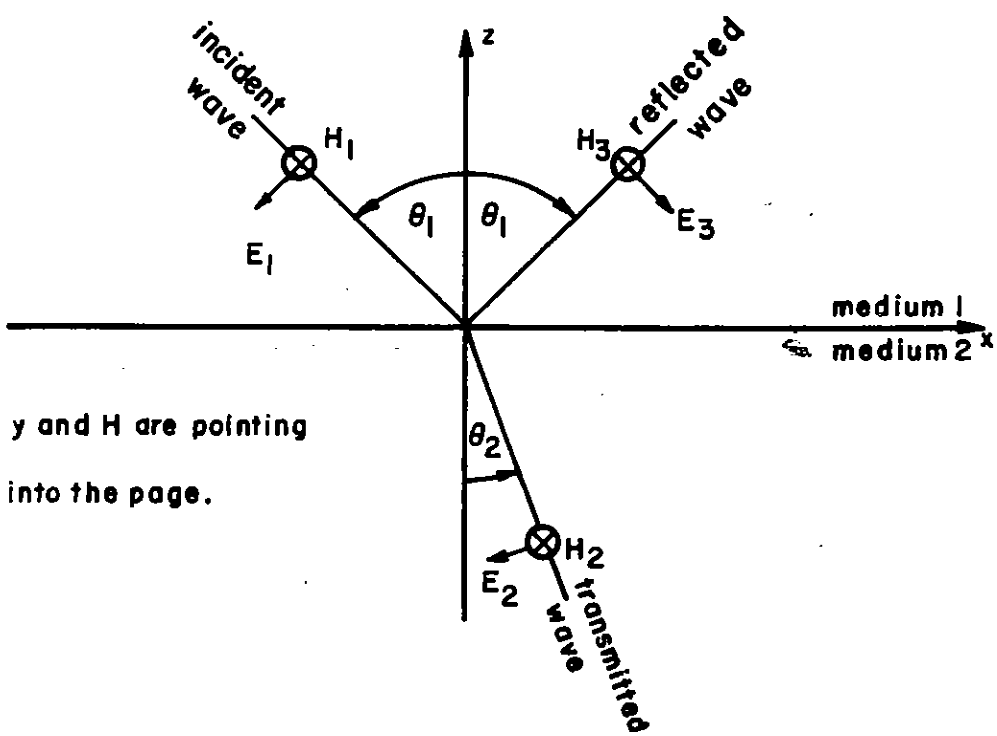
II. Plane Wave Reflection

Figure 1 gives the geometry for the calculations. The incident and reflected waves propagate at an angle, θ_1, with the positive z axis and the transmitted wave propagates at an angle, θ_2, with the negative z axis. The incident, transmitted, and reflected waves are designated by subscripts 1, 2, and 3, respectively. Two cases, based on two polarizations of the incident wave, are considered. The case of electric field parallel to the interface between the two media (figure 1A) is designated by a subscript, e, for the transmission and reflection coefficients; the case of magnetic field parallel to the interface (figure 1B) is similarly designated by a

1. Frank Sulkowski, Mathematics Note II, FORPLEX: A Program to Calculate Inverse Fourier Transforms, November 1966.
2. James H. Scott, EMP Theoretical Note XVIII, Electrical and Magnetic Properties of Rock and Soil, May 1966.



A. E PARALLEL TO INTERFACE



B. H PARALLEL TO INTERFACE

FIGURE I. REFLECTION OF A PLANE WAVE AT THE SURFACE OF A CONDUCTING DIELECTRIC subscript, h. More general polarizations can be considered as combinations of these two cases.

The electric field in the three waves is taken positive in the directions indicated for the two cases in figure 1. It has the forms for the incident wave

$$\tilde{E} = \tilde{E}_1 e^{-\gamma_1[x\sin(\theta_1)-z\cos(\theta_1)]} \tag{1}$$

for the transmitted wave

$$\tilde{E} = \tilde{E}_2 e^{-\gamma_2[x\sin(\theta_2)-z\cos(\theta_2)]} \tag{2}$$

and for the reflected wave

$$\tilde{E} = \tilde{E}_3 e^{-\gamma_1[x\sin(\theta_1)+z\cos(\theta_1)]} \tag{3}$$

where in this formulation we use the Laplace transformed quantities. The field quantities and transmission and reflection coefficients have a tilde, $\sim$, over them to indicate the Laplace transform of the quantities. Similar expressions apply for the magnetic field in the three waves.

The propagation constants and wave impedances are of the forms

$$\gamma = \sqrt{s\mu(\sigma+s\varepsilon)} \tag{4}$$

and

$$Z = \sqrt{\frac{s\mu}{\sigma+s\varepsilon}} \tag{5}$$

respectively, and they are appropriately subscripted to apply to a specific medium. There is a convenient relation which is later used that

$$\frac{\gamma_1}{\gamma_2} = \frac{\mu_1}{\mu_2}\frac{Z_2}{Z_1} \tag{6}$$

and since we take $\mu_2 = \mu_1 = \mu_o$ for these calculations the expression further simplifies.

In equations (1) through (3) there is already the result that the angles for the incident and reflected waves are equal. Another result of requiring the three waves to match along the interface at $z = 0$ is

$$\gamma_1 \sin(\theta_1) = \gamma_2 \sin(\theta_2) \tag{7}$$

For a frequency domain analysis replace s by $j\omega$. In the frequency domain, then, θ_2 is in general a complex number for a chosen real θ_1, since γ_1/γ_2 is a complex number due to the finite conductivity of the lower medium. This does not affect the validity of the results but makes the form of the fields in the lower medium a little more complicated.[3]

3. J. A. Stratton, Electromagnetic Theory, Chap. IX, 1941.

For convenience we define certain parameters. The relaxation time of the lower medium is

$$t_r = \frac{\varepsilon_2}{\sigma_2} \tag{8}$$

Then define a normalized Laplace transform variable

$$s_r = st_r \tag{9}$$

which simplifies the expressions somewhat. In this form an inverse Fourier transform gives the pulse in terms of a normalized time which we define by

$$\tau_r = \frac{t}{t_r} \tag{10}$$

where t is time. Finally, there is the relative dielectric constant for the lower medium as

$$\varepsilon_r = \frac{\varepsilon_2}{\varepsilon_o} \tag{11}$$

Now consider the reflection and transmission of the incident wave at the interface between the two media. The reflection and transmission coefficients are developed in another note and can be found in several places.[3,4] Consider first the case of the electric field parallel to the interface (figure 1A). The reflection coefficient is given by

$$\tilde{r}_e = \frac{\tilde{E}_3}{\tilde{E}_1} = \frac{\tilde{H}_3}{\tilde{H}_1} = \frac{1 - \dfrac{Z_1}{Z_2}\dfrac{\cos(\theta_2)}{\cos(\theta_1)}}{1 + \dfrac{Z_1}{Z_2}\dfrac{\cos(\theta_2)}{\cos(\theta_1)}} \tag{12}$$

and the transmission coefficient by

$$\tilde{t}_e = \frac{\tilde{E}_2}{\tilde{E}_1} = \frac{Z_2}{Z_1}\frac{\tilde{H}_2}{\tilde{H}_1} = 1 + \tilde{r}_e \tag{13}$$

4. Capt Carl E. Baum, Sensor and Simulation Note XXXII, A Lens Technique for Transitioning Waves Between Conical and Cylindrical Transmission Lines, January 1967.

Rearranging equation (12) and substituting from equations (6) and (7) gives

$$\tilde{r}_e = \frac{\cos(\theta_1) - \sqrt{\left(\frac{\gamma_2}{\gamma_1}\right)^2 - (\sin(\theta_1))^2}}{\cos(\theta_1) + \sqrt{\left(\frac{\gamma_2}{\gamma_1}\right)^2 - (\sin(\theta_1))^2}} \tag{14}$$

where from equations (4), (8), and (9)

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2 = \frac{s\mu_o(\sigma_2+s\varepsilon_2)}{s^2\mu_o\varepsilon_o} = \varepsilon_r \frac{1+s_r}{s_r} \tag{15}$$

Letting the incident wave be of the form $1/s_r$, which is a step function in the time domain, we then have a step response function for the reflected wave as

$$\tilde{R}_e = \frac{\tilde{r}_e}{s_r} = \frac{1}{s_r} \frac{\cos(\theta_1) - \sqrt{\varepsilon_r \frac{1+s_r}{s_r} - (\sin(\theta_1))^2}}{\cos(\theta_1) + \sqrt{\varepsilon_r \frac{1+s_r}{s_r} - (\sin(\theta_1))^2}} \tag{16}$$

Substituting $j\omega$ for s_r and performing the numerical inverse transform gives R_e as a function of τ_r with θ_1 and ε_r as parameters to be chosen. Note that the pulse shape of the reflected wave is preserved as it propagates away from the interface. The results calculated here then apply for various heights above the interface with an appropriate time delay introduced. The step response function for the transmitted wave is

$$\tilde{T}_e = \frac{\tilde{t}_e}{s_r} = \frac{1}{s_r} + \tilde{R}_e \tag{17}$$

This last function, however, only applies at the interface, $z = 0$, since the pulse shape is distorted for negative z due to the influence of σ_2 on γ_2 and on θ_2.

Second, consider the case of the magnetic field parallel to the interface (figure 1B). The reflection coefficient is given by

$$\tilde{r}_h = \frac{\tilde{H}_3}{\tilde{H}_1} = \frac{\tilde{E}_3}{\tilde{E}_1} = \frac{1 - \frac{Z_2}{Z_1}\frac{\cos(\theta_2)}{\cos(\theta_1)}}{1 + \frac{Z_2}{Z_1}\frac{\cos(\theta_2)}{\cos(\theta_1)}} \tag{18}$$

and the transmission coefficient by

$$\tilde{t}_h = \frac{\tilde{H}_2}{\tilde{H}_1} = \frac{Z_1}{Z_2} \frac{E_2}{E_1} = 1 + \tilde{r}_h \tag{19}$$

Rearranging equation (18) and substituting from equations (6) and (7) gives

$$\tilde{r}_h = \frac{\cos(\theta_1) - \frac{\gamma_1}{\gamma_2} \sqrt{1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2 (\sin(\theta_1))^2}}{\cos(\theta_1) + \frac{\gamma_1}{\gamma_2} \sqrt{1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2 (\sin(\theta_1))^2}} \tag{20}$$

The step response function for the reflected wave is then

$$\tilde{R}_h = \frac{\tilde{r}_h}{s_r} = \frac{1}{s_r} \frac{\cos(\theta_1) - \sqrt{\frac{1}{\varepsilon_r} \frac{s_r}{1+s_r} \left[1 - \frac{1}{\varepsilon_r} \frac{s_r}{1+s_r} (\sin(\theta_1))^2\right]}}{\cos(\theta_1) + \sqrt{\frac{1}{\varepsilon_r} \frac{s_r}{1+s_r} \left[1 - \frac{1}{\varepsilon_r} \frac{s_r}{1+s_r} (\sin(\theta_1))^2\right]}} \tag{21}$$

The step response function for the transmitted wave is

$$\tilde{T}_h = \frac{\tilde{t}_h}{s_r} = \frac{1}{s_r} + \tilde{R}_h \tag{22}$$

Again the last function only applies at the interface while R_h applies for various heights above the interface with an appropriate time delay.

The step response functions for the reflected wave are plotted in figures 2 through 9. Figures 2 through 5 cover the case of the electric field parallel to the interface and figures 6 through 9 cover the case of the magnetic field parallel to the interface. Each figure takes a particular ε_r (ranging from 10 through 80) and varies $\frac{2}{\pi}\theta_1$ between zero and one as a parameter for the curves. The indicated errors (from the successive calculations of the inverse Fourier transform computer code[5]) are less than $.5 \times 10^{-2}$ for $\frac{2}{\pi}\theta_1 \leq .9$ and about 10^{-2} or less for $\frac{2}{\pi}\theta_1 > .9$.

There are some things which can be noted about these reflection step response functions. At $\tau_r = 0$ there is an immediate rise (or fall) of a certain size depending on the polarization, on θ_1, and on ε_r. The value of this rise (or fall) can be calculated from the initial-value theorem of the Laplace transform, giving for the electric field parallel to the interface

5. See reference 1.

$$R_e(0+) = \lim_{s_r \to \infty} s_r \tilde{R}_e = \frac{\cos(\theta_1) - \sqrt{\epsilon_r - (\sin(\theta_1))^2}}{\cos(\theta_1) + \sqrt{\epsilon_r - (\sin(\theta_1))^2}} \tag{23}$$

and for the magnetic field parallel to the interface

$$R_h(0+) = \lim_{s_r \to \infty} s_r \tilde{R}_h = \frac{\cos(\theta_1) - \frac{1}{\epsilon_r}\sqrt{\epsilon_r - (\sin(\theta_1))^2}}{\cos(\theta_1) + \frac{1}{\epsilon_r}\sqrt{\epsilon_r - (\sin(\theta_1))^2}} \tag{24}$$

Note that R_e starts negative and continues negative, asymptotically approaching -1, and that it takes a time of many t_r (for small $\frac{2}{\pi}\theta_1$) to approach within, say .1, of this limit. On the other hand, R_h starts either positive or negative (somewhere between -1 and +1) and asymptotically approaches +1. For $\frac{2}{\pi}\theta_1$ near one it takes, comparatively, a very long time to approach the limit of plus one, and R_h is more sensitive to θ_1 in this region. Note also that $R_h(0+)$ is zero for some value of θ_1; this is just the Brewster angle phenomenon, which applies only to the polarization with the magnetic field parallel to the interface. Looking at equations (16) and (21) there is a singular case for $\theta_1 = \frac{\pi}{2}$ in that R_e and R_h are both step functions of amplitude -1. In such a case the incident and reflected waves cancel each other, leaving no net fields.

III. Summary

We have then calculated the reflection of step function waves from the flat surface of a uniform ground or water medium under certain simplifying assumptions. In particular, we have taken the permittivity and conductivity of the lower medium as independent of frequency. The conductivity of soil is apparently roughly frequency independent but the permittivity is not, although the permittivity does seem to level out for high frequencies (compared to σ_2/ϵ_2). Using relative permittivities of the lower medium ranging from 10 through 80, the step response functions are calculated for various angles for the incident wave. These step response functions can be used to approximate the reflection of various pulse shapes from a ground or water surface by the use of convolution integral techniques.

We would like to thank Mr. John N. Wood for the numerical calculations and the resulting graphs.

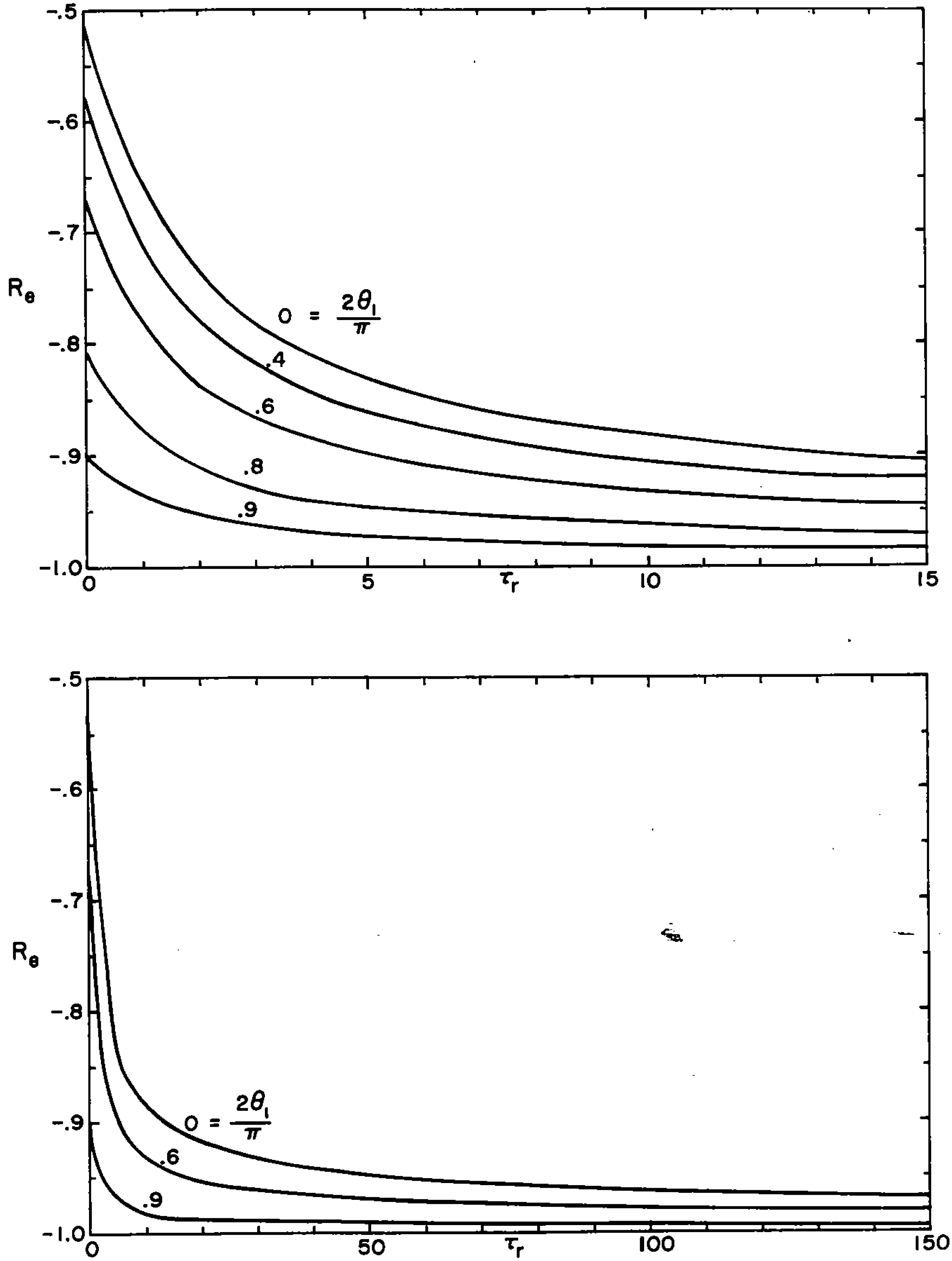


Figure 2. REFLECTION OF STEP FUNCTION WAVE WITH ELECTRIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 10$

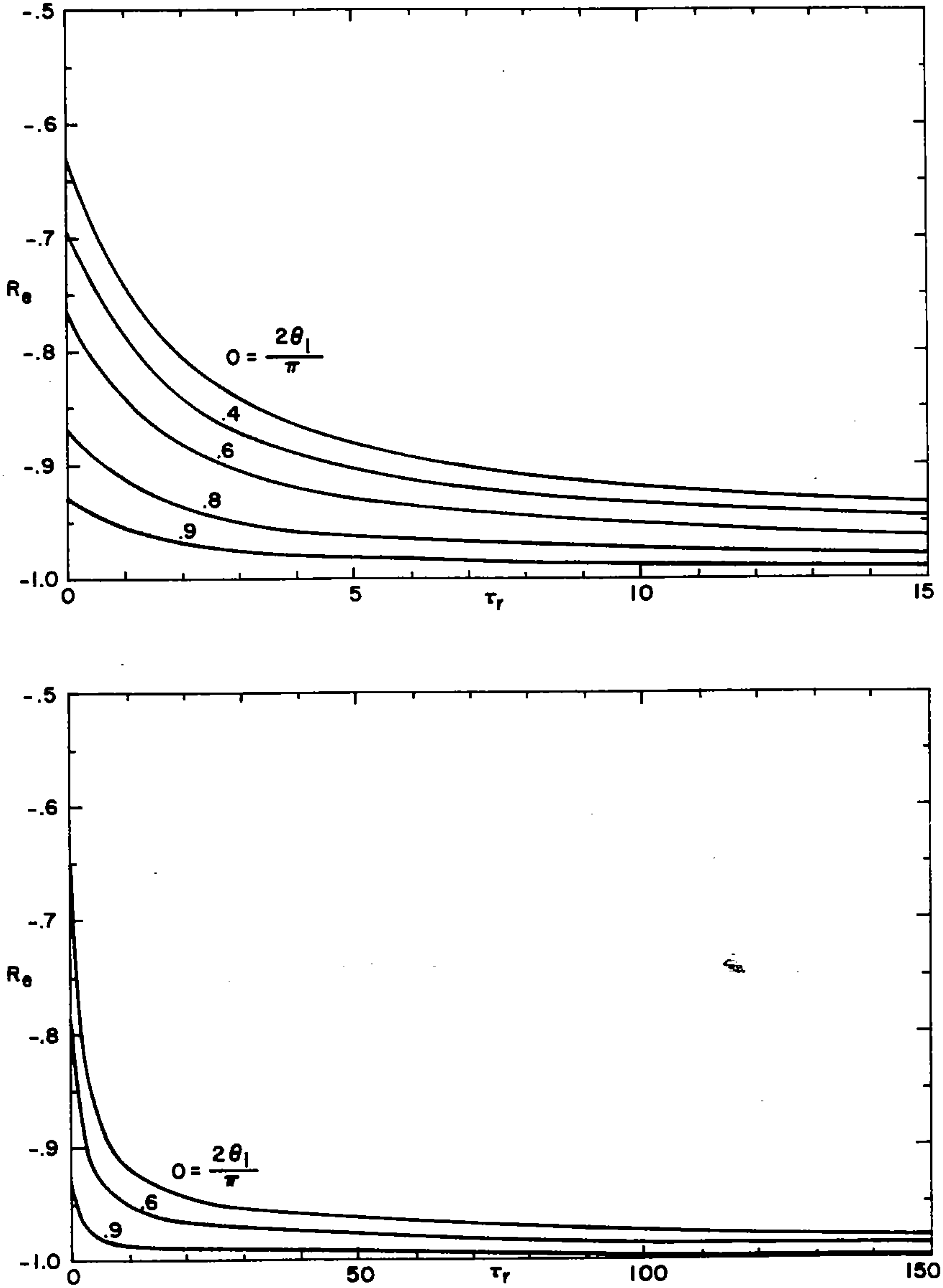


Figure 3. REFLECTION OF STEP FUNCTION WAVE WITH ELECTRIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 20$

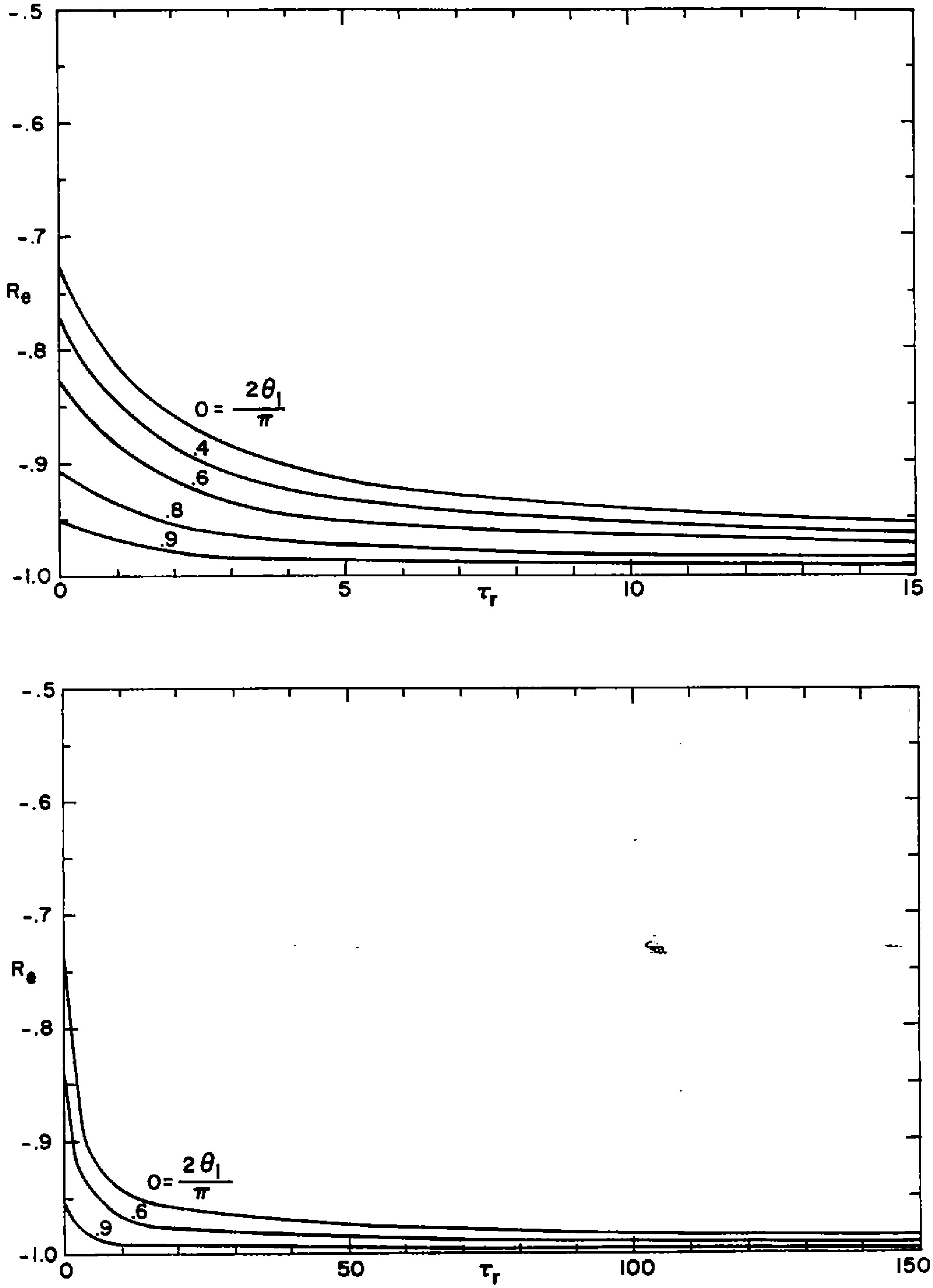
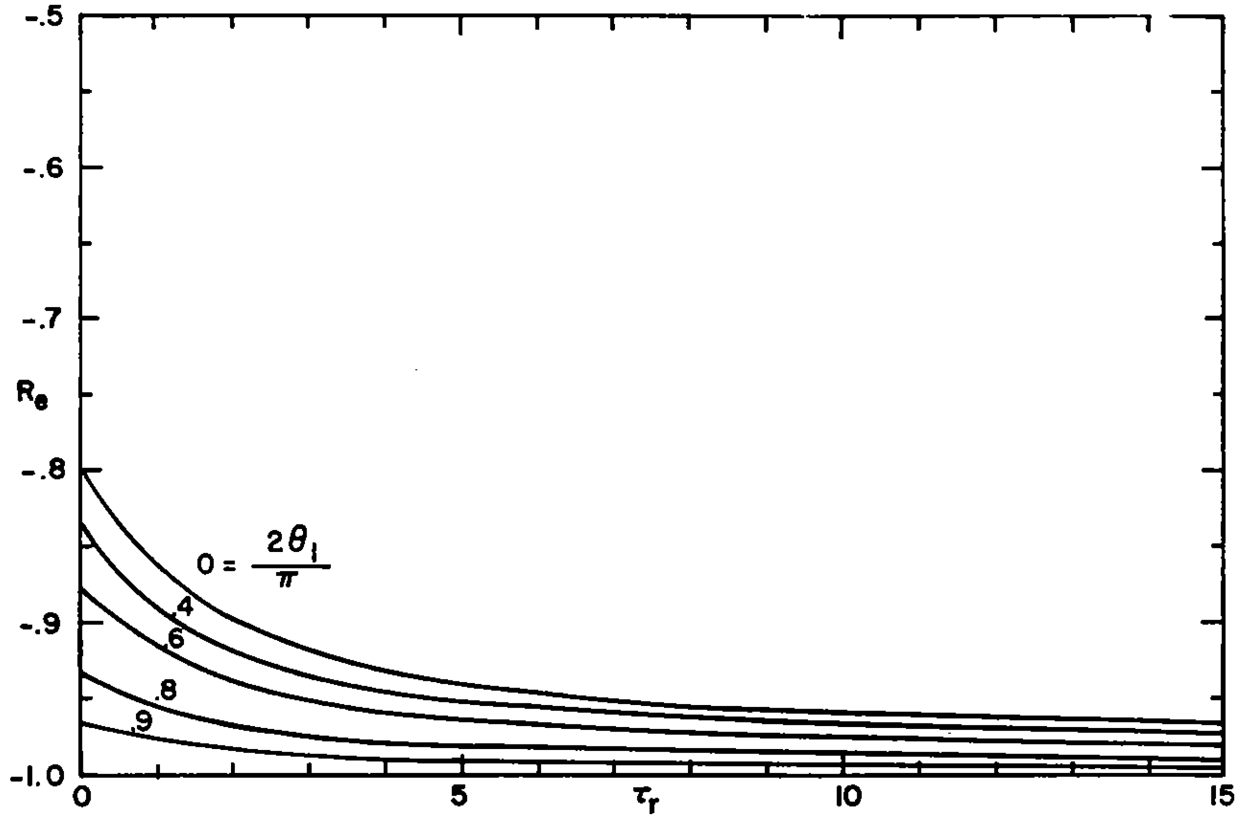


Figure 4. REFLECTION OF STEP FUNCTION WAVE WITH ELECTRIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 40$



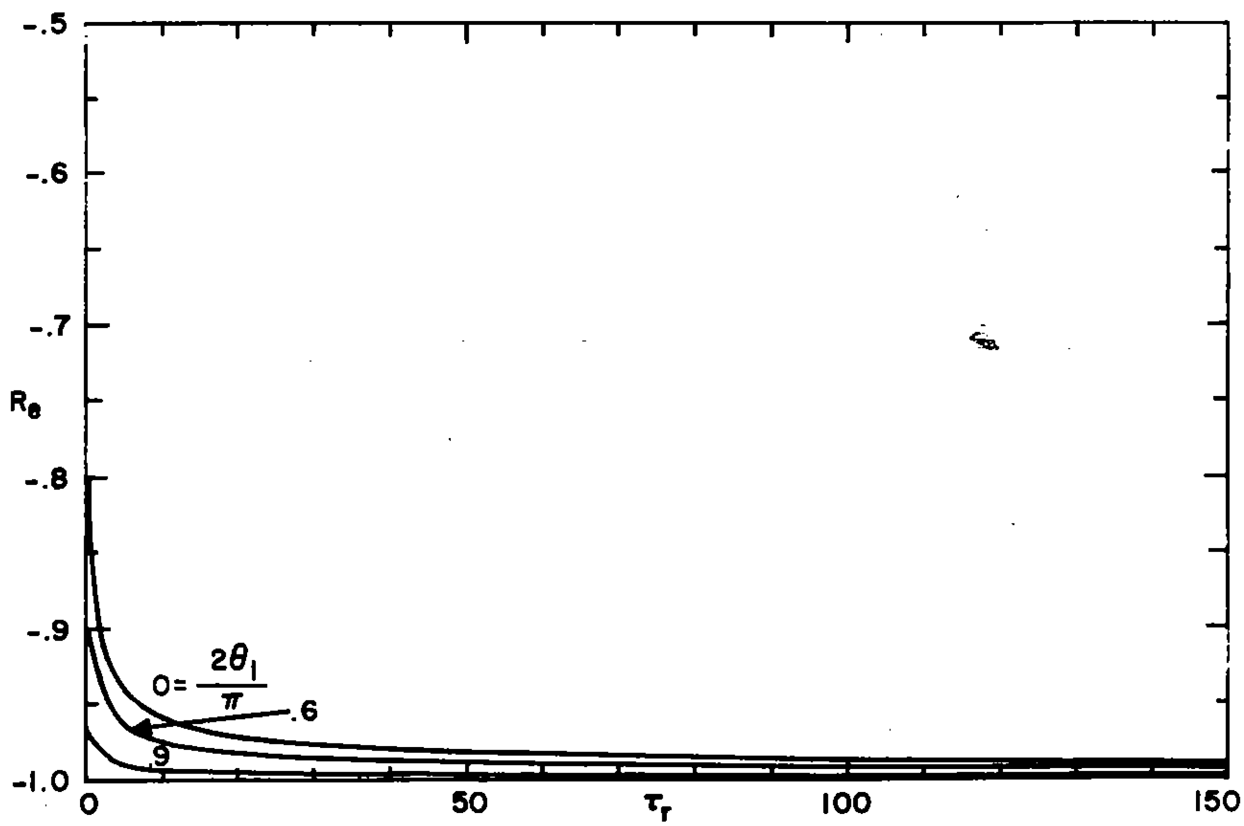


Figure 5. REFLECTION OF STEP FUNCTION WAVE WITH ELECTRIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 80$

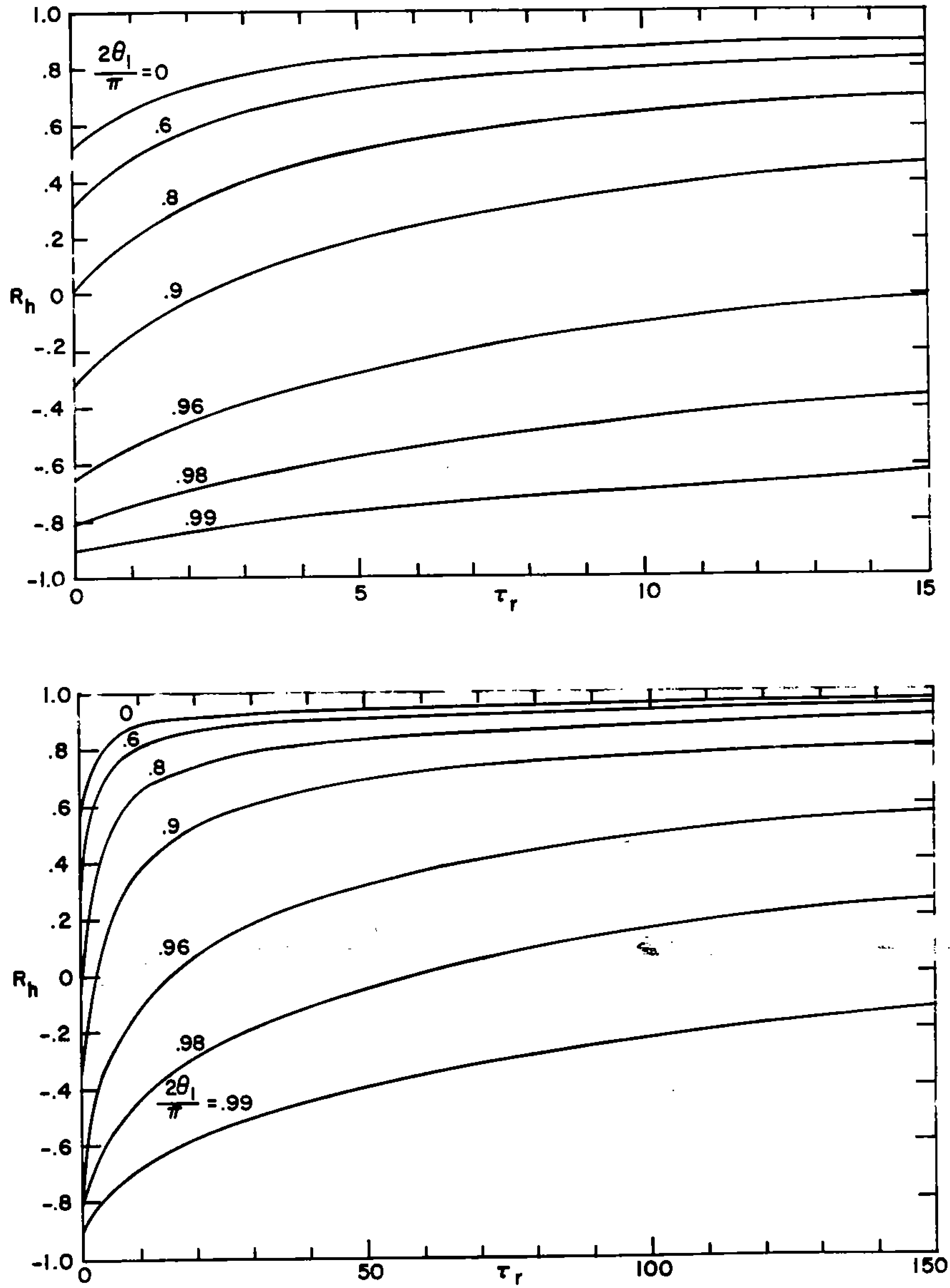


Figure 6. REFLECTION OF STEP FUNCTION WAVE WITH MAGNETIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 10$

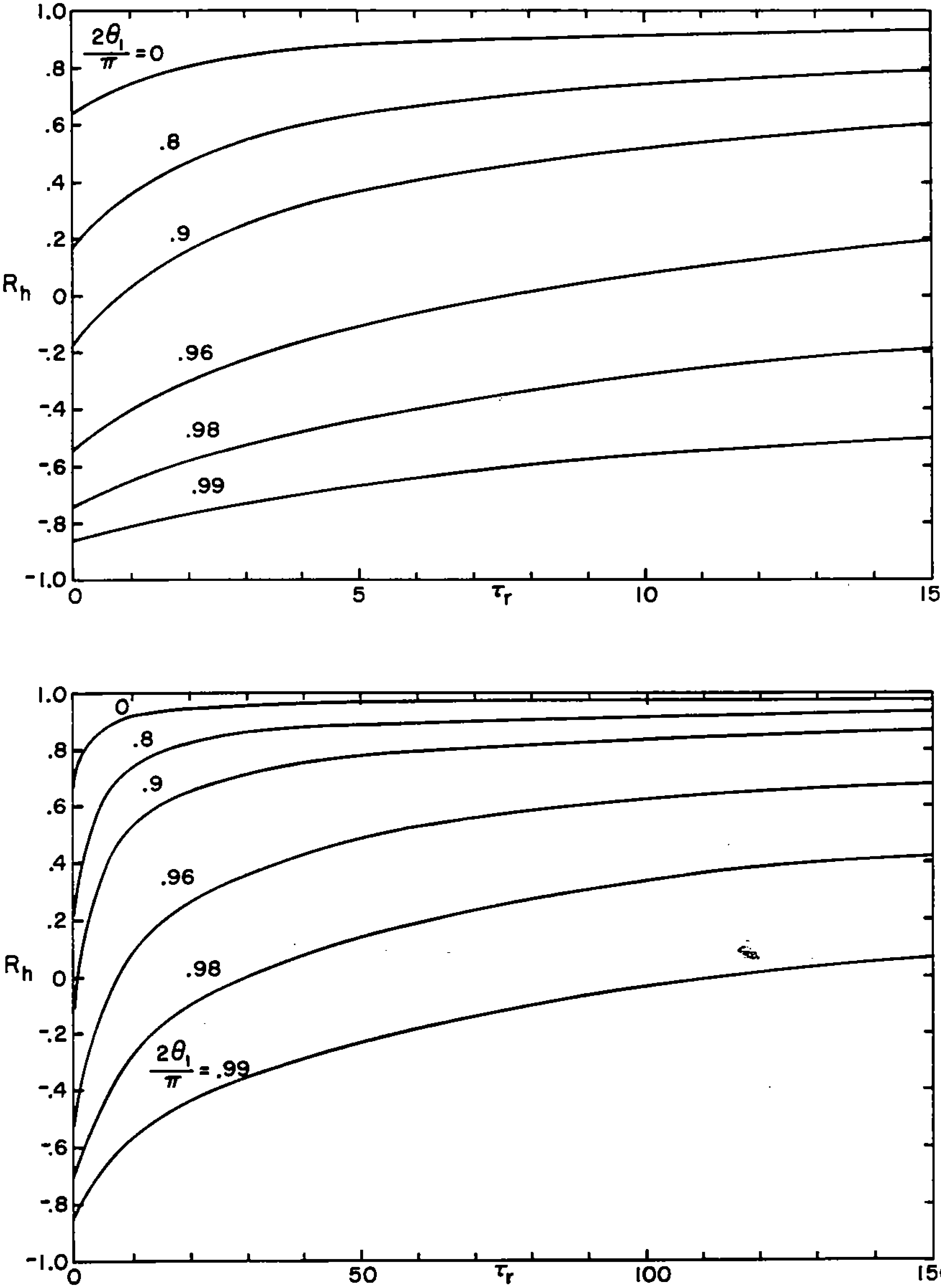


Figure 7. REFLECTION OF STEP FUNCTION WAVE WITH MAGNETIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 20$

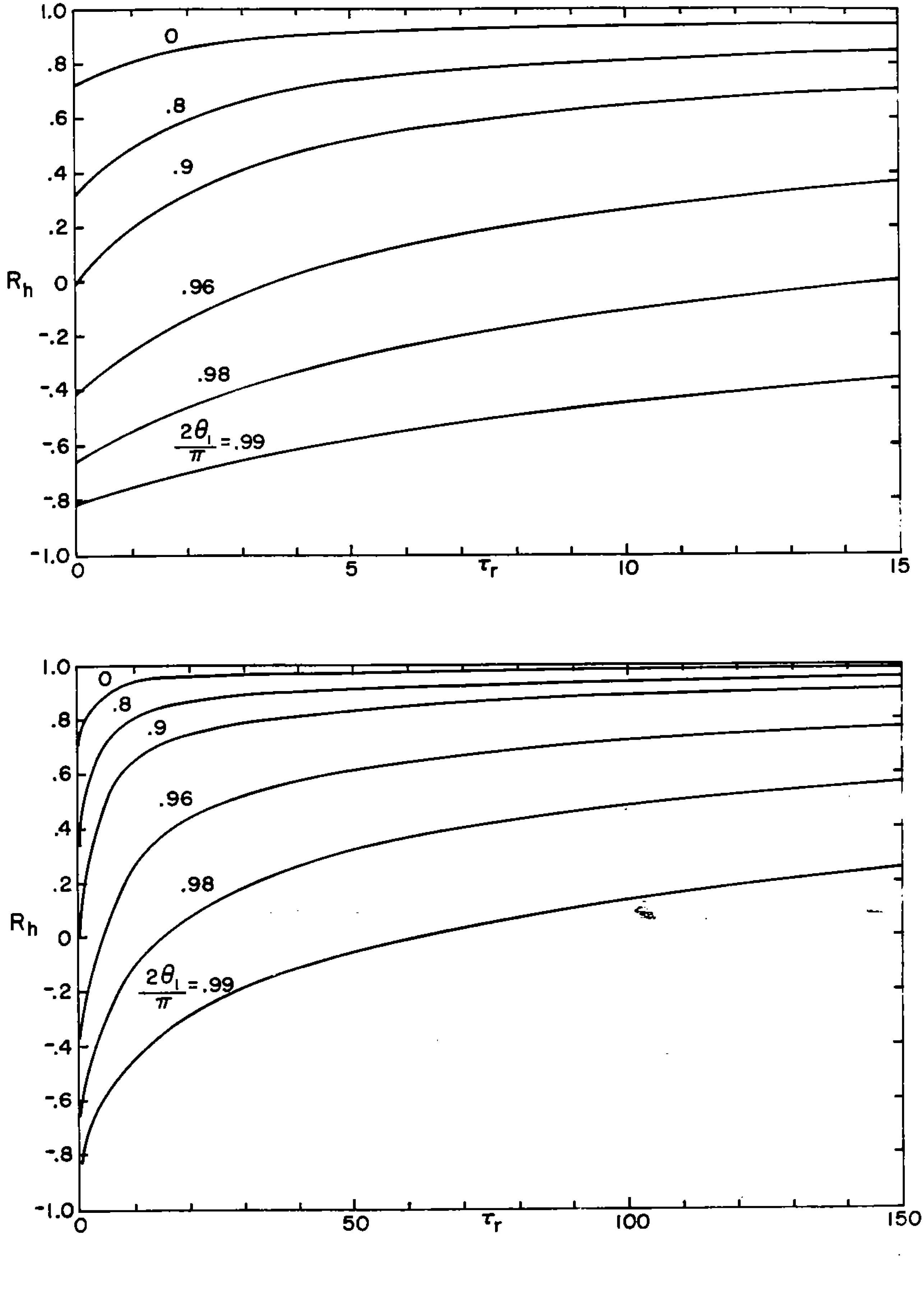


Figure 8. REFLECTION OF STEP FUNCTION WAVE WITH MAGNETIC FIELD PARALLEL TO INTERFACE: ϵ_r = 40

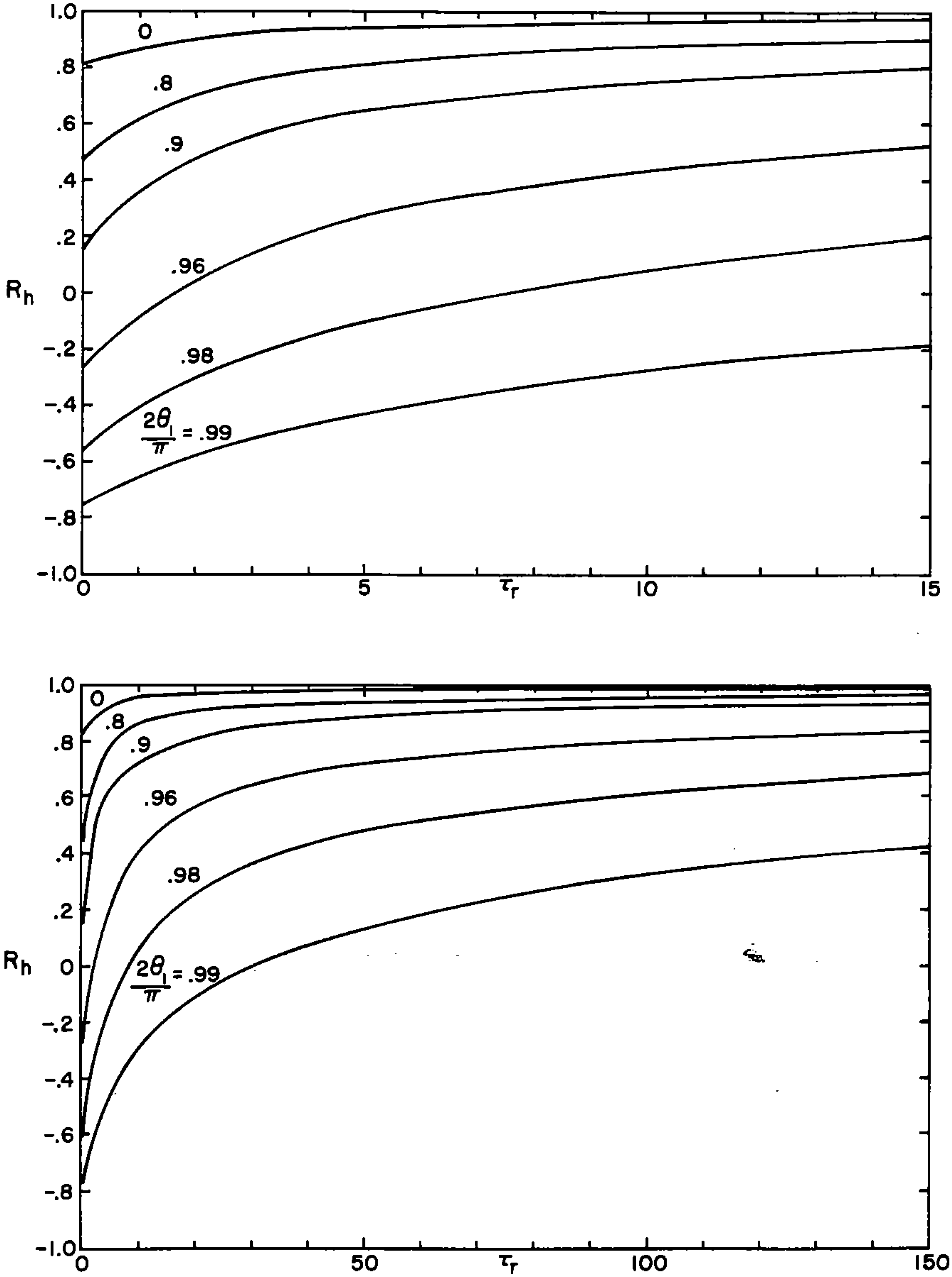


Figure 9. REFLECTION OF STEP FUNCTION WAVE WITH MAGNETIC FIELD PARALLEL TO INTERFACE: $\epsilon_r = 80$