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THE COMPTON CURRENT AND THE ENERGY DEPOSITION RATE FROM GAMMA QUANTA–A MONTE CARLO CALCULATION

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PREFACE

The Compton current generated by the prompt gamma rays from an atmospheric nuclear explosion is known to be an important source for the electromagnetic signals which accompany such explosions. Theoretical estimates for the current and for the energy deposition rate produced by the gamma quanta usually appeal to a variety of simplifying assumptions. The reason for this is that the multiple scattering of the gamma quanta introduces an essential complication into the physics of the problem.

In this work, the space-time dependence of the Compton current and the energy deposition rate are calculated using the methods of the Monte Carlo technique. Included are the effects of the energy spectrum of the gamma quanta and the time-spreading due to the multiple scattering of the quanta.

The purpose here is to provide accurate estimates for the current and the energy deposition rate, thereby eliminating one source of uncertainty in the theoretical estimates of the electromagnetic fields generated by this mechanism.

SUMMARY

The Compton current and the energy deposition rate generated by monoenergetic sources of gamma quanta are calculated for gamma quanta energies of 1, 1.5, 2, 3 and 4 Mev using standard Monte Carlo techniques. Histograms of the current and the energy deposition rate are presented which show the time-spreading of these quantities due to the multiple scattering of the gamma quanta.

It is found that if the secular time variation of the source of the gamma quanta is slow relative to the time-spreading induced by the multiple scattering, then the Compton current and the energy deposition rate are determined by the total flux of gamma quanta, direct beam plus scattered flux. If the secular time variation of the source is rapid relative to the time-spreading induced by the multiple scattering, then there are two cases to be considered. For a source that is increasing with time, the Compton current and the energy deposition rate are determined mainly by the flux of the direct beam. For a source that decreases rapidly with time, these quantities are mainly determined by the scattered flux.

The properties of the Maienschein prompt gamma-ray spectrum are investigated and it is found that with this spectrum, the main contribution to the Compton current in sea level density air comes from gamma quanta energies of 1.5 Mev at a distance of 500 meters from the source. At a distance of 1000 meters, the spectrum hardens and the important energies center around 2.5 Mev.

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CONTENTS

I. INTRODUCTION

The gamma quanta from nuclear explosions in the atmosphere are known to provide one of the important mechanisms for the generation of the electromagnetic fields which accompany such explosions.[1] The gamma quanta produce Compton recoil electrons which move preferentially away from the point of the explosion. Since the flux of gamma quanta varies with distance from the origin, a nonuniform charge distribution appears which gives rise to an electric field. The field does not persist since the recoil electrons deposit energy as they travel along their paths. Some of this energy appears in the form of large numbers of secondary electrons. Thus, the air becomes conducting and currents flow which act to reduce the field.

Theoretical estimates for the space-time behavior of the electromagnetic field depend somewhat sensitively on the assumptions for the Compton current and the energy deposition rate. These quantities are usually estimated under a variety of simplifying assumptions. For example, Kompaneets assumes the current and the energy deposition rate have the same radial dependence and the same time dependence.[1] The radial dependence is described by means of a relaxation distance while the time dependence is assumed to follow the secular time dependence of the source.

It is natural to appeal to simplifying assumptions since an accurate estimate of the space-time variation of the Compton current and the energy deposition rate is rather complex. Such an estimate would have to include the effect of the energy spectrum of the gamma quanta emitted by the source and the effect of time-spreading due to the multiple scattering of the gamma quanta. These effects, which have not been treated in any detail to date, will be discussed in this report.

In general, the Compton current $\vec{j}(\vec{r},t)$ and the energy deposition rate $\dot{s}(\vec{r},t)$ at the time t and at the position $\vec{r}$ from a source emitting $\dot{N}(t,E)$ gamma quanta of energy E per unit energy interval and per unit time interval are,

$$\vec{j}(\vec{r},t) = \int_0^\infty \int_{-\infty}^t \vec{J}(\vec{r},t-t',E)\, \dot{N}(t',E)dE\, dt', \qquad (1.1)$$

and

$$\dot{s}(\vec{r},t) = \int_0^\infty \int_{-\infty}^t \dot{S}(\vec{r},t-t',E)\, \dot{N}(t',E)dE\, dt', \qquad (1.2)$$

where $\vec{J}(\vec{r},t,E)$ and $\dot{S}(\vec{r},t,E)$ are the current and energy deposition rate, respectively, produced by a single gamma quantum of initial energy E emitted from the source at the origin of time. The functions $\vec{J}$ and $\dot{S}$ will be determined here for a number of initial gamma quanta energies E.

We will consider an isotropic, point source of monoenergetic gamma quanta in a uniform atmosphere of sea level density. Under these conditions the Compton current has only a radial component, the azimuthal and polar components vanishing from symmetry. The method of calculation, which is described in the next section is based on the standard Monte Carlo techniques. Section III presents the numerical results for gamma quanta of energies 1, 1.5, 2, 3 and 4 Mev, respectively.

The basic results of the calculations are histograms which show the quantities $J(r,t,E)$ and $\dot{S}(r,t,E)$ as a function of time at selected distances r from the source. It should be remarked that the time dependence in question here arises from the time spreading due to the multiple scattering of the gamma quanta. Once the energy spectrum and the secular time dependence of the source of gamma quanta are specified, the current and energy deposition rate may be determined according to Eqs. (1.1) and (1.2). The quantities J and $\dot{S}$ give the current and energy deposition rate generated by a source which is a delta function in time.

If the source function $\dot{N}(t,E)$ is slowly varying in time relative to J and $\dot{S}$, then the integral of J and $\dot{S}$ on time is important in determining the time dependence of the current and energy deposition rate. We will therefore also display the results for $\int_0^t J\,dt$ and $\int_0^t \dot{S}\,dt$ in the section on numerical results.

II. METHOD OF CALCULATION

Let us briefly describe the principal assumptions and the basic steps of the calculations. The details of the problem formulation are given in the Appendix.

We begin by considering an isotropic, point source of mono-energetic gamma quanta in an atmosphere of sea level density. Since we do not examine the effect of the air-ground interface and since we limit the region of interest to a distance of 1 km from the source, we may treat the medium as being homogeneous and essentially infinite in extent. Thus, the deposition of energy depends only on time and on the distance r from the source. The Compton current generated by the gamma quanta also depends only on r and t and, from symmetry, the current has only a radial component. Since the range of a Compton recoil electron is quite small relative to the mean free path of the gamma quantum producing the recoil, we treat energy deposition and Compton current as local quantities. It should be emphasized that the variation with time of the energy deposition rate and the Compton current comes from the multiple scattering of the quanta and not from any assumed secular time dependence of the source. The problem we solve is for a source which is a delta function in time.

A TYPICAL GAMMA QUANTUM HISTORY

We assume that a gamma quantum may be affected by two processes, (1) Compton scattering and (2) photoelectric absorption. Since we will be interested in gamma energies of 4 Mev and less, the effects of pair-production are negligible and pair-production attenuation is ignored. The steps in the calculations are as follows:

(1) Starting with a quantum of energy E traveling in the radial direction from an assumed source origin, the distance to the first collision is determined by a random choice and depends on the total mean free path of the quantum. The survival probability, which is the ratio of the total Compton (Klein-Nishina) scattering cross section to the total cross section (Compton plus photoelectric absorption), is computed as a weighting function. The use of this weighting function is described in the next step.

(2) The energy of the gamma quantum E' after the first collision is determined by a random choice and depends on the cross section for scattering from one energy to a smaller energy. The new energy uniquely determines the angle through which the quantum scatters and also determines the angle between the direction of the Compton recoil electron and the radial direction. The energy deposition is given by the energy of the Compton recoil electron, E-E', and the contribution of this event to the radial Compton current is obtained by projecting the range of the Compton recoil along the radial direction. These quantities are weighted by the survival probability computed in Step (1).

(3) As we continue to follow the gamma quantum, we now have the typical case of a quantum of known energy E', originating at a known radius and whose direction of motion is at a known angle with respect to the radial direction. As in Step (1), the distance to the next collision is determined by random choice and the survival

probability is computed. These quantities are functions of the gamma quantum energy E'. The radial distance to the collision point is easily found.

(4) Now the energy of the quantum E" after the collision is determined by random choice as dictated by the scattering cross section. The energy E" uniquely determines the angle between the initial direction of motion and the direction of motion of the scattered quantum. A random choice is then made for the azimuthal angle of the scattering relative to the original direction of motion, uniform in the interval 0 to 2π. The geometry of the collision now determines the angle between the direction of motion of the scattered quantum and the radius vector through the collision point. In addition, the angle between the direction of motion of the Compton recoil electron and the radius vector may be computed. Thus, as in Step (2), the energy deposition and the contribution to the Compton current may be calculated for this collision.

Steps (3) and (4) are now repeated until the gamma quantum degrades in energy to below some prescribed minimum value. Experience showed that a choice for the cutoff energy of 5 per cent of the initial energy provided accurate estimates for the total energy deposition. The accuracy of the Monte Carlo calculations reported here was judged by comparing the total energy deposition, or more precisely, the build up factors with results given by other workers. The comparison, which is quite favorable, is shown in Section III. Presumably, the results for the Compton current given here are of similar quality.

The method described above for calculating the Compton current does not account for the multiple scattering of the Compton recoil electron. The scattering reduces the projected range of the recoils to about 0.63 of the practical range for recoil electrons in the energy range of 0.5 to 2 or 3 Mev. The multiple scattering correction has not been included in the results for the Compton current presented in the following section.

PROCESSING THE DATA

We have described the procedure for following the history of a typical gamma quantum. Some 40,000 histories are followed for each problem and, since between 10 to 20 collisions take place before the quantum degrades in energy to below the cutoff, we have of the order of one million data points in energy deposition and Compton current to process.

To convert these data to energy deposition rate per unit volume at a given point in space and to determine the Compton current as a function of time, the medium was divided into a series of concentric shells or radial zones about the source. The radius of a given collision determined then the zone in which the collision occurred. The total energy deposition per unit volume per initial quantum was obtained by accumulating the energy deposition for all the collision events taking place in the zone and then dividing by the product of the zone volume and the number of histories. The total Compton current was processed in a similar fashion. The radial thickness of the zones was chosen so that the zone size was small compared to the scattering mean free path of the initial gamma quantum and yet not so small as to give poor statistics in a given zone. A zone thickness of 25 meters was found to satisfy these requirements.

The total energy and the total Compton current are accumulated in a zone over an extended period of time due to the multiple scattering of the gamma quanta. To determine the nature of this time spreading, the age t of a gamma quantum was computed for each history and the delay time τ of an event was measured relative to the transit time of light to the radius of the event. That is,

$$\tau = t - r/c, \tag{2.1}$$

where c is the speed of light. A set of time zones of width $\Delta\tau$ was carried with each space zone and the delay time τ of an event determined the time zone with which an event was associated. In this way histograms were constructed to show the rate of energy deposition per unit volume per initial quantum at each space zone, as a function of delay time τ. The area under a histogram gives, of course, the total energy deposited per unit volume. Histograms for the Compton current were determined in a similar fashion. A time zone size of 10^{-8} sec was found to be adequate for describing the general variation with time of the energy deposition and the Compton current.

The contribution of the direct beam to the energy deposition and the Compton current was computed separately and was not mixed with the contributions from the scattered flux.

SMOOTHING THE HISTOGRAMS

The histograms giving the energy deposition rate and the rate of arrival of the Compton current at a given space zone usually exhibit some fluctuations among adjacent time zones. The fluctuations are most pronounced at large delay times and for the space zones at large distances from the origin. In this space-time regime the statistics are naturally the poorest.

In order to improve the statistics and "smooth out" the fluctuations, the following averaging procedure was followed at the suggestion of J. I. Marcum. The histograms for three adjacent space zones were averaged together, time zone by time zone, the result then giving a smoothed histogram for the middle zone of the three. This was done for all the space zones in the grid, the final result yielding a "first-smoothed" set of histograms for the grid.

This procedure may be iterated giving the second-smoothed histograms, etc. Experience dictated that four smoothings represented an optimal smoothing for the histograms and the results presented in Section III were smoothed four times. This method of smoothing conserves the area under the histograms and therefore does not change either the total energy deposition or the total integrated Compton current.

THE ENERGY DEPOSITION RATE BY THE ONCE SCATTERED BEAM

The general solution of the gamma ray multiple scattering problem may be expressed in integral form. However, the evaluation of these multiple integrals is quite laborious and, as is done here, one usually turns to the straightforward Monte Carlo method.

The energy deposition rate by the once scattered beam is rather simple to formulate, though, and we include it here in order to compare with the Monte Carlo results. One would expect that at a distance of about one mean free path from the source, the dominant contribution to the scattered flux would come from the once scattered beam.

Consider then the energy deposition rate at a distance r from the source and at a delay time τ where τ is measured from the time of arrival of the direct beam. Let 0 denote the source point, Q the deposition point and P the point in space where the gamma quantum first scatters and then subsequently scatters at the point Q. It is evident that the locus of points formed by the points P is an ellipsoid of revolution with the points 0 and Q as foci. Let $\ell = \overline{OP}$ and $\ell' = \overline{PQ}$. $\overline{OQ}$ is the distance r and

$$r + \frac{\tau}{c} = \ell + \ell' , \qquad (2.2)$$

is the equation of the ellipsoid. The gamma quantum has its initial energy E over the path $\overline{OP}$ and scatters to an energy E' over the path $\overline{PQ}$. Let α be the ratio of the gamma quantum energy to the rest energy of the electron, mc^2. The quantity α' is related to the delay time τ by,

$$\frac{1}{\alpha'} = \frac{1}{\alpha} + \frac{2r\rho + \rho^2}{2\ell(r+\rho-\ell)} , \qquad (2.3)$$

where, for convenience, we define

$$\rho = \tau/c. \qquad (2.4)$$

Let $k(\alpha)$ be the absorption coefficient (cm^{-1}) of a quantum with (dimensionless) energy α, $p(\alpha';\alpha)$ be the probability of scattering to energy α' within $d\alpha'$ from the energy α and let $\langle\alpha\rangle$ be the average energy after a collision of a quantum of initial energy α. The average energy is,

$$\langle\alpha\rangle = \int_{\frac{\alpha}{1+2\alpha}}^{\alpha} \alpha' \; p(\alpha';\alpha)d\alpha' . \qquad (2.5)$$

It may be shown that the energy deposition rate at the delay time τ is,

$$\dot{S}(\tau;r,\alpha) = \frac{mc^3 k(\alpha)}{4\pi r} \int_{\frac{\rho}{2}}^{r+\frac{\rho}{2}} p(\alpha';\alpha)k(\alpha')\alpha'^2(\alpha'-\langle\alpha'\rangle)e^{-k(\alpha)\ell}e^{-k(\alpha')\ell'} \frac{d\ell}{\ell\ell'}, \; , \qquad (2.6)$$

where ℓ' is defined by Eq.(2.2).

The above integral was evaluated numerically and the result will be compared with the Monte Carlo calculations which we now present in the following section.

III. NUMERICAL RESULTS

We present here the numerical results for a total of five problems corresponding to initial gamma quanta energies of 1, 1.5, 2, 3 and 4 Mev, respectively. For all problems the space around the source was divided into 50 spherical concentric shells, each shell or space zone having a width of 25 meters. We are thus able to process data out to a maximum radius of 1225 meters from the source. Collisions which take place at distances greater than 1225 meters are followed in case back scattering produces an event in the region of interest.

Associated with each space zone were 50 time zones, each with a time-width of 10^{-8} sec. All events occurring with a time delay greater than $5x10^{-7}$ sec were accumulated in the last time zone. We remind the reader that the time delay of an event is measured relative to the arrival time of the direct beam at the radius of the event.

The limitation of a 50x50 space-time grid was dictated by the desire to keep the problem within the fast memory of the IBM-7090 computer at The RAND Corporation and thereby avoid the use of magnetic tapes in the data processing. This resulted in a considerable saving in machine time. For example, 40,000 histories for a gamma quantum with an initial energy of 3 Mev degrading to a cutoff energy of 0.15 Mev took 26 min of computer time. The same number of histories for a 1 Mev gamma degrading to 0.05 Mev took 58 min. The reason for this increase in running time is the relatively small fractional energy loss per collision of very low energy gamma quanta.

The energy deposition rate and the radial Compton current have been calculated at twelve stations in the interval from 0 to 1012.5 meters from the source for the five gamma quantum energies considered. Rather than display all the results in this report, for illustrative purposes we will show the data for a 1.5 Mev source at a distance of 512.5 meters from the source, and a 3 Mev source at a distance of 1012.5 meters. Results for other energies and distances are available on request.

Figures 1 and 2 show the energy deposition rate $\dot{S}(r,t,E)$ and the radial Compton current $J(r,t,E)$, respectively, as a function of time for $E = 1.5$ Mev. Figures 3 and 4 show the data for $E = 3$ Mev. The data in the figures are for the contributions from the scattered flux. The contribution from the direct beam is as indicated on the figures.

The data have several interesting features. It may be noted that the energy deposition rate decays more slowly with time than the current. This is a reasonable result since one would expect that at long delay times, after a gamma quantum has suffered many collisions, the quantum would be relatively ineffective at producing an outward radial current. On the other hand, the energy deposition continues until the quantum finally dies. The time spreading resulting from the multiple scattering is shown in Figs. 5 and 6 where the ratio of the accumulated energy deposition to the total energy deposition is given as a function of time for $E = 1.5$ and $E = 3$ Mev, respectively. The analogous quantity for the current is also shown. It may be noted that for $E = 1.5$ Mev, about 0.5 of the total energy is deposited within about 5 shakes and about 0.7 within 17 shakes. There is a rather long tail on the energy deposition rate since about 0.14 of the total energy is deposited after 50 shakes. The integrated Compton current accumulates more rapidly with about 0.75 of the total arriving in 5 shakes and only 0.013 accumulating after 50 shakes.

It is of some interest to examine the radial dependence of the Compton current and the energy deposition rate since the convenient approximation is often made that these quantities have the same radial dependence. However, as Figs. 7 and 8 indicate, the ratio of $J(r,\tau,E)$ to $\dot{S}(r,\tau,E)$ for fixed delay times τ, varies rather strongly with radius.

A convenient approximation that is often invoked in estimating the energy deposition is to assume an exponential variation with radius. It is also usually assumed that the Compton current varies in the same manner. In order to provide an estimate for the appropriate relaxation distance, in Fig. 9 we show the total energy deposition times the square of the radius as a function of the radius for the five gamma quanta energies considered. Figure 10 shows the total current and, as may be seen in Fig. 11 which shows the ratio of the total current to the total energy deposition as a function of distance, the radial dependence of the two quantities is somewhat different.

We have been examining the current and energy deposition rate generated by monoenergetic sources of gamma quanta. For a source with a prescribed energy spectrum of emitted gamma quanta, the current and energy deposition rate must be determined by folding the results given here with the spectrum as indicated by Eqs. (1.1) and (1.2). In order to get some feeling for the important gamma quanta energies involved, we have examined the integrated Compton current per unit energy interval for the Maienschein prompt gamma ray spectrum.[2] Figure 12 shows the results at selected distances from the origin. In the figure, the points indicated by triangles were obtained by plotting $r^2 \int_0^\infty J(r,t,E)dt$ as a function of energy and interpolating. It appears that for the Maienschein spectrum the important energy group centers around 1.5 Mev for distances out to about 500 meters from the source and at 1000 meters from the source the spectrum hardens and the important energy interval peaks at about 2.5 Mev.

We complete the discussion of numerical results by showing the build up factors for energy deposition as a function of distance from the source measured in units of the Compton scattering mean free path. The build up factor is defined as the ratio of the total energy deposition to the energy deposition by the direct beam S_d where

$$S_d = \frac{E}{4\pi r^2} k_t e^{-k_s r}, \tag{3.1}$$

where k_s is the Compton scattering absorption coefficient and the values for the energy absorption coefficient $k_t (cm^{-1})$ are shown in Table 1. Figures 13 and 14 show the build up factors. Also shown are the build up factors reported in Ref. (3) which are in substantial

Table 1

Energy Absorption Coefficient $k_t(cm^{-1})$ as a Function of Gamma Quantum Energy E(Mev).

E (Mev)	$k_t(cm^{-1})$
1	$3.43x10^{-5}$
1.5	$3.14x10^{-5}$
2	$2.86x10^{-5}$
3	$2.51x10^{-5}$
4	$2.29x10^{-5}$

agreement with the present Monte Carlo results. The build up factors computed here are slightly smaller than those of Ref. (3) since, in the interest of shortening the machine time, we followed the gamma quanta down in energy to 5 per cent of the initial energy.

Finally, in Fig. 15 we show the comparison of the analytic results for the once scattered beam with the Monte Carlo calculation of the energy deposition rate for E = 3 Mev at a distance of 287.5 meters from the source. The Monte Carlo calculations for the energy deposition rate are somewhat larger than the results given by the once scattered beam. This is reasonable since the Monte Carlo results include contributions to the energy deposition rate by higher order scatterings. However, as indicated by Fig. 15, the general shape of the curves are quite similar.

IV. DISCUSSION

We have determined the Compton current and the energy deposition rate generated by monoenergetic sources of gamma quanta, emitted instantaneously in time. These quantities, which we denoted by $J(r,t)$ and $\dot{S}(r,t)$, respectively, are essentially the "Green's Functions" of the problem. Once the energy spectrum of a source is specified together with the variation of the source strength with time, then the actual current and energy deposition rate at any space-time point may be determined as indicated by Eqs. (1.1) and (1.2) of the text.

The work presented here brings out a very important coupling between the secular time dependence of a source and the relative importance of the contributions made by the direct beam and the scattered flux. For a source whose intensity is increasing with time, we may state this coupling as follows: (1) If the intensity of the gamma quantum source varies rapidly with respect to the time variations of J and $\dot{S}$, then the actual current and energy deposition rate, which we denote by j and $\dot{s}$, respectively, are determined by the direct or unscattered beam and the contributions by the scattered flux may be essentially ignored. (2) If, on the other hand, the source varies slowly with respect to J and $\dot{S}$, then j and $\dot{s}$ depend on the total integrated contributions of the direct beam plus the scattered flux.

This result may be established as follows: Let us examine the energy deposition rate for a monoenergetic source and write

$$\dot{S}(r,t) = S_1(r)\delta(t) + \dot{S}_2(r,t), \qquad (4.1)$$

where $S_1(r)$ is the energy deposited by the direct beam, $\dot{S}_2(r,t)$ is the energy deposition rate by the scattered flux, $\delta(t)$ is the delta-function and the time t is measured from the arrival time of the direct beam at the position r. According to the results shown in Figs. (1) and (3), the function $\dot{S}_2(r,t)$ may be crudely represented by,

$$\dot{S}_2(r,t) = S_2(r)\lambda\, e^{-\lambda t} \qquad (4.2)$$

where λ is a constant and $S_2(r)$ is the total energy deposited by the scattered flux. If we imagine that the number of gamma quanta emitted by the source per unit time is

$$\dot{N}(t) = \alpha e^{\alpha t},$$

where α is a constant then, according to Eq. (1.2), $\dot{s}$ is

$$\dot{s}(r,t) = \alpha e^{\alpha t}[S_1(r) + \frac{\lambda}{\alpha+\lambda} S_2(r)]. \qquad (4.3)$$

It is immediately obvious that if the source varies rapidly with respect to $\dot{S}_2$, that is, $\alpha \gg \lambda$, then the direct beam contribution $S_1(r)$ dominates. On the other hand, if $\alpha \ll \lambda$, the actual energy deposition rate $\dot{s}$ depends on the total energy deposited, direct beam plus scattered flux since in this limit Eq. (4.3) becomes

$$\dot{s}(r,t) = \alpha e^{\alpha t}(S_1(r) + S_2(r)). \qquad (4.4)$$

The relative importance of the direct beam and the scattered flux may be shown by writing Eq. (4.3) in terms of the build up factor which we denote by B(r). Recalling that $S_2(r) = (B(r)-1)S_1(r)$, we have,

$$\dot{s}(r,t) = \alpha e^{\alpha t} S_1(r)[1 + \frac{\lambda(B(r)-1)}{\alpha+\lambda}]. \qquad (4.5)$$

If one ignored the effect of multiple scattering, the energy deposition rate would simply be,

$$\dot{s}(r,t) = \alpha e^{\alpha t} S_1(r)B(r) \qquad (4.6)$$

and it is evident that the multiple scattering reduces the energy deposition rate by the factor

$$f = \frac{1}{B}[1 + \frac{\lambda(B-1)}{\alpha+\lambda}]. \qquad (4.7)$$

As a numerical example consider a 3 Mev source at a distance of 1000 meters. The build up factor B is about 4 in this instance and from Fig. (3) we find $\lambda \approx 0.35\text{x}10^8 \text{ sec}^{-1}$. Thus the above factor is,

$$f = \tfrac{1}{4}[1 + \frac{1.05\text{x}10^8}{\alpha + .35\text{x}10^8}]. \qquad (4.8)$$

The correction factor depends on α, ranging from 0.445 for $\alpha = 1\text{x}10^8 \text{ sec}^{-1}$ to 0.31 for $\alpha = 4\text{x}10^8 \text{ sec}^{-1}$.

For a source whose intensity is decreasing with time we take

$$\dot{N}(t) = \alpha e^{-\alpha t}, \quad 0 \le t, \qquad (4.9)$$

and find

$$\dot{s}(r,t) = \alpha e^{-\alpha t} S_1(r) + \frac{\alpha\lambda}{\lambda-\alpha} S_2(r)(e^{-\alpha t} - e^{-\lambda t}). \qquad (4.10)$$

Again, if α is small compared to λ, then the energy deposition rate depends on the total flux and $\dot{s}$ is given by Eq. (4.4). On the other hand, if the source strength decreases rapidly ($\alpha \gg \lambda$), we have

$$\dot{s}(r,t) \approx \alpha e^{-\alpha t} S_1(r) + \lambda e^{-\lambda t} S_2(r), \qquad (4.11)$$

and it is clear that for large times the scattered flux dominates.

In general, there does not appear to be any particularly simple relationship between the Compton current generated by monoenergetic gamma quanta sources and the energy deposition rate generated by such sources. For example, the assumption that the current and the energy deposition rate have the same radial dependence is clearly not valid. However, on the basis of the results obtained in this work, it appears that the following prescription provides fairly accurate estimates for the Compton current and the energy deposition rate. (1) For a source whose intensity increases with time and if the source varies rapidly in the sense described above, compute the energy deposition rate from the direct beam only. The Compton current is then proportional to the energy deposition rate and the constant of proportionality for a given initial gamma quantum energy may be read from the r = 0 intercepts of Fig. (11). (These values should be reduced by a nominal factor of 0.63 to account for the multiple scattering of the Compton recoils.) (2) If the source is slowly varying with time, the current and energy deposition rate do not have the same radial dependence. Figure (11) shows the relationship between the two quantities. It may be noted that for distances greater than 200 or 300 meters the ratio of the current to the energy deposition rate charges rather slowly with distance from the source.

It may finally be remarked that in cases (1) and (2) above, the current and the energy deposition rate follow the secular time dependence of the source.

APPENDIX

We present the equations used in the calculations more or less in the order in which they appear in the numerical work.

Consider a gamma quantum which has had a collision at a distance r from the origin of coordinates. The quantum has an energy E after the collision and the direction of motion is at an angle θ with respect to the radius vector to the collision point. If x is a random number, uniformly distributed in the interval 0 to 1, the distance the quantum travels to the next collision ℓ is

$$\ell = - \lambda(\alpha) \; \ell n \; x, \qquad (I.1)$$

where α is the energy of the quantum in units of the rest energy of the electron and $\lambda(\alpha)$ is the total mean free path of the quantum given by

$$\lambda = [\lambda_{kn}^{-1} + \lambda_{pe}^{-1}]^{-1}. \qquad (I.2)$$

λ_{kn} and λ_{pe} are the Compton scattering (Klein-Nishina) mean free path and the photoelectric absorption mean free path, respectively. λ_{kn} is determined from the total Compton scattering cross section $\sigma(\alpha)$ which is,[4]

$$\sigma(\alpha) = 2\pi r_o^2 \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ell n(1+2\alpha)}{\alpha} \right] + \frac{\ell n(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \qquad (I.3)$$

where r_o is the classical electron radius. The photoelectric absorption coefficient was calculated by the following interpolation formula, fitting the data given in Ref. (5).

$$\lambda_{pe}^{-1} = 1.608\text{x}10^{-8}/\alpha^{3.222}(\text{cm}^{-1}). \qquad \text{(I.4)}$$

The probability $p(\alpha',\alpha)$ of a gamma quantum scattering from energy (dimensionless) α to energy α' is,

$$p(\alpha',\alpha) = \frac{1}{\sigma(\alpha)} \frac{\pi r_o^2}{\alpha^2} \left[\frac{2}{\alpha} - \frac{2}{\alpha'} + \frac{1}{\alpha'^2} + \frac{1}{\alpha^2} - \frac{2}{\alpha\alpha'} + \frac{\alpha}{\alpha'} + \frac{\alpha'}{\alpha}\right]$$

for $\alpha \geq \alpha' \geq \frac{\alpha}{1+2\alpha}$,

$$= 0 \text{ for } \alpha' < \frac{\alpha}{1+2\alpha} \text{ or } \alpha' > \alpha. \qquad \text{(I.5)}$$

To determine the energy α' after a collision we first compute a provisional value of α' by

$$\alpha' = \frac{\alpha}{1+2\alpha}\left[2\alpha x+1\right], \qquad \text{(I.6)}$$

where x is a random number between 0 and 1. Choosing another random number we find the quantity $xp_{max}(\alpha)$ where $p_{max}(\alpha)$ is $p(\alpha',\alpha)$ evaluated at $\alpha' = \frac{\alpha}{1+2\alpha}$. Then, provided $xp_{max}(\alpha)$ is less than $p(\alpha',\alpha)$, α' is taken as the energy after the collision. This procedure is repeated, if necessary, until a value of α' is found which satisfies these requirements.

We may now compute the angle ψ through which the gamma quantum scatters from the relation

$$\cos\psi = 1 + \frac{1}{\alpha} - \frac{1}{\alpha'}, \qquad \text{(I.7)}$$

and the angle φ between the direction of the recoil Compton electron and the original direction of the gamma quantum is determined from,

$$\sin\beta = \frac{\alpha' \sin\psi}{\{(1+\alpha-\alpha')^2-1\}^{\frac{1}{2}}}, \tag{I.8}$$

and

$$\cos\beta = \frac{\alpha-\alpha' \cos\psi}{\{(1+\alpha-\alpha')^2-1\}^{\frac{1}{2}}} . \tag{I.9}$$

The directions of the recoil Compton and the scattered gamma quantum lie in a plane and there remains to determine the angle θ_1 between the direction of motion of the scattered gamma quantum and the radius vector and also the angle φ between the direction of motion of the Compton recoil and the radius vector. These are determined by

$$\begin{aligned}\cos\theta_1 &= \cos\theta(\cos\theta \cos\psi + \sin\theta \sin\psi \cos\chi)\\ &+ \sin\theta(\sin\delta \cos\psi - \cos\delta \sin\psi \cos\chi),\end{aligned} \tag{I.10}$$

and

$$\begin{aligned}\cos\varphi_1 &= \cos\theta(\cos\delta \cos\beta - \sin\delta \sin\beta \cos\chi)\\ &+ \sin\theta(\sin\delta \cos\beta + \cos\delta \sin\beta \cos\chi),\end{aligned} \tag{I.11}$$

where χ is the azimuthal angle of scattering of the gamma quantum, chosen at random in the interval 0 to 2π, and the angle δ is defined by

$$\sin\delta = \frac{\ell \sin\theta}{r_2} , \tag{I.12}$$

$$\cos\delta = \frac{r + \ell \cos\theta}{r_2} , \tag{I.13}$$

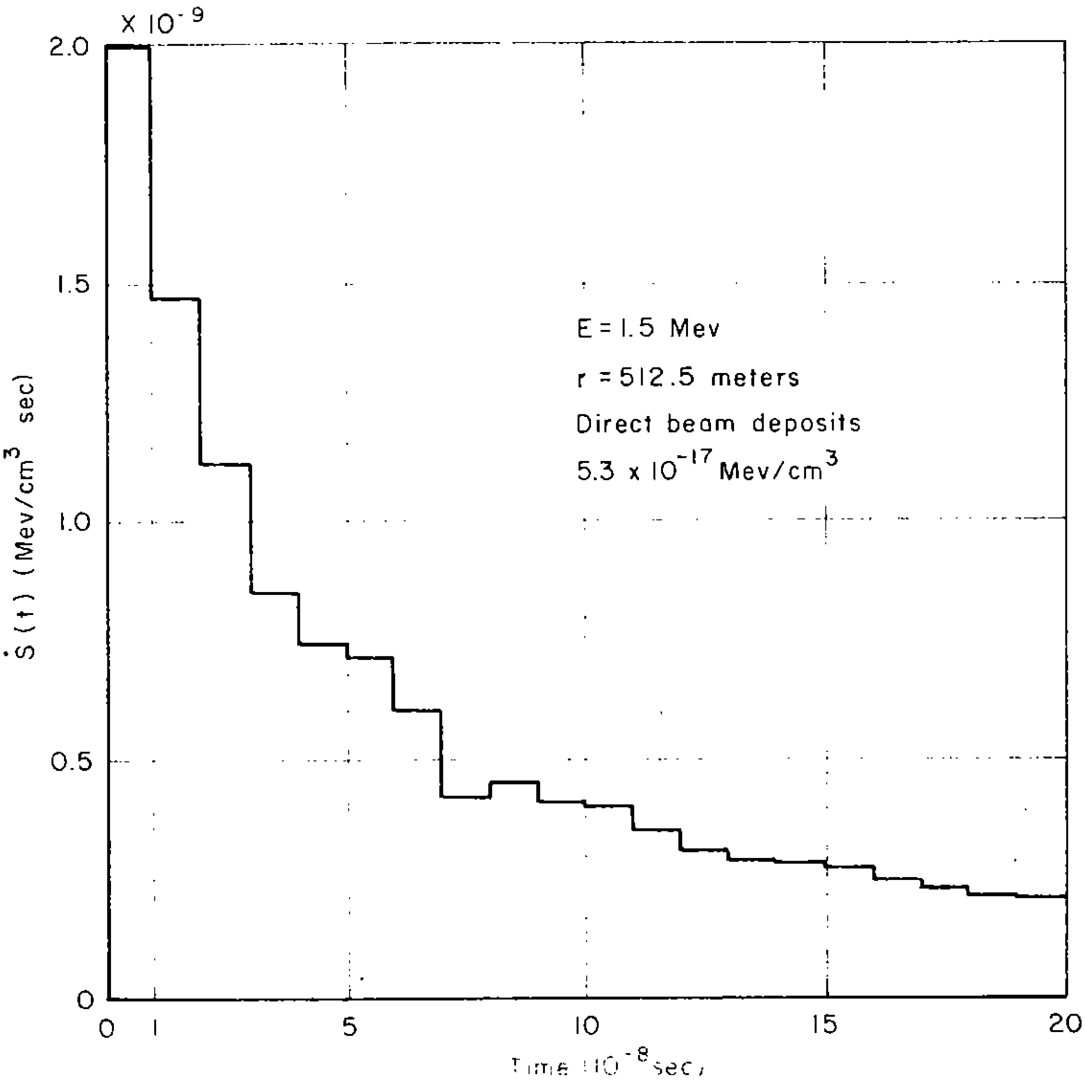
where

$$r_2^2 = r^2 + \ell^2 + 2r\ell \cos\theta. \tag{I.14}$$

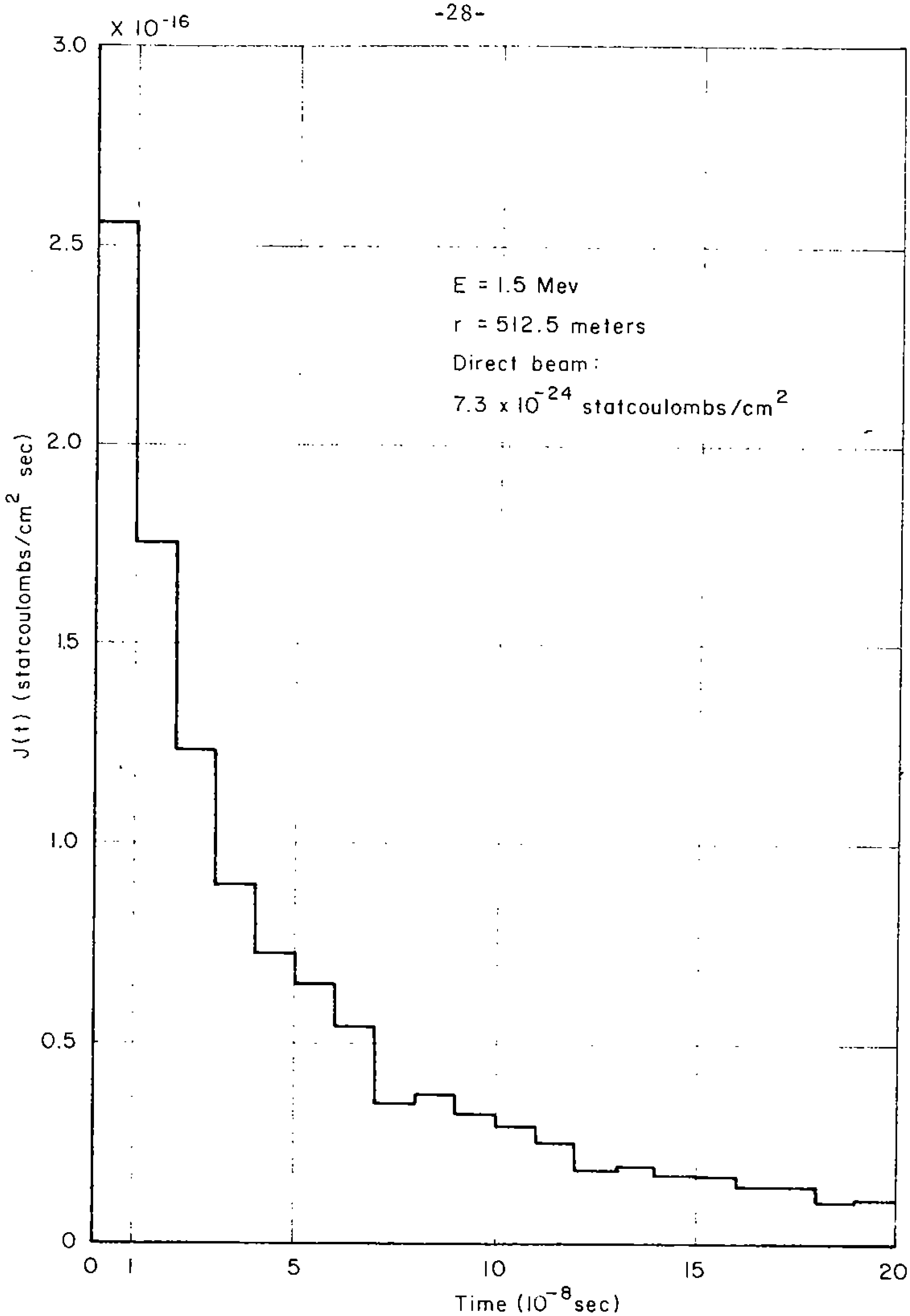
The energy deposited at the collision point r_2 is, of course, $\alpha-\alpha'$. The range of the Compton recoil R_e is[5]

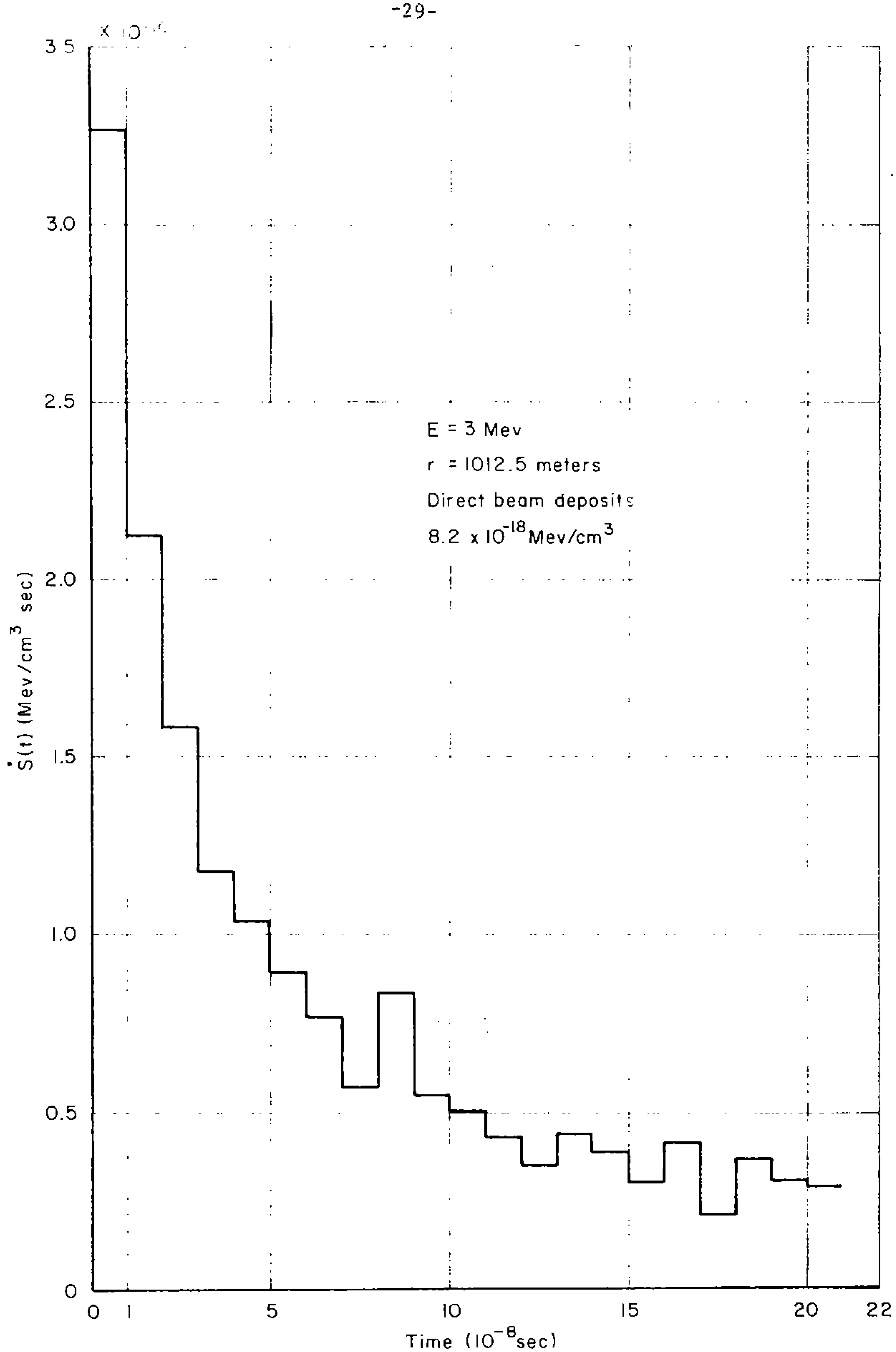
$$R_e(\text{cm}) = 131.5\ (\alpha-\alpha')^{1.38}, \quad \alpha-\alpha' < 1.565,$$
$$= 226\ (\alpha-\alpha') - 108.5, \quad \alpha-\alpha' > 1.565. \qquad \text{(I.15)}$$

($\alpha-\alpha'$ is the energy of recoil electron.)

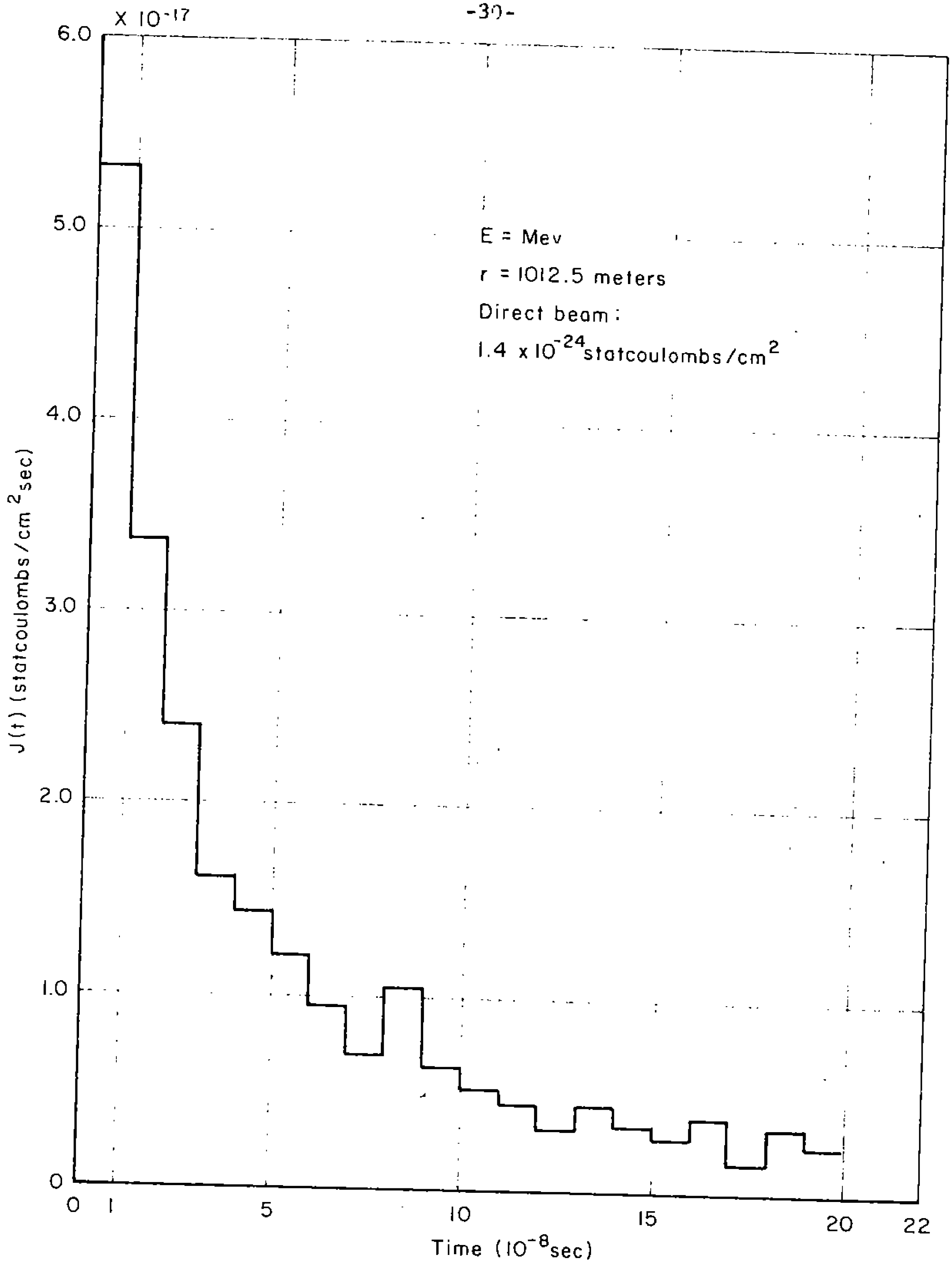


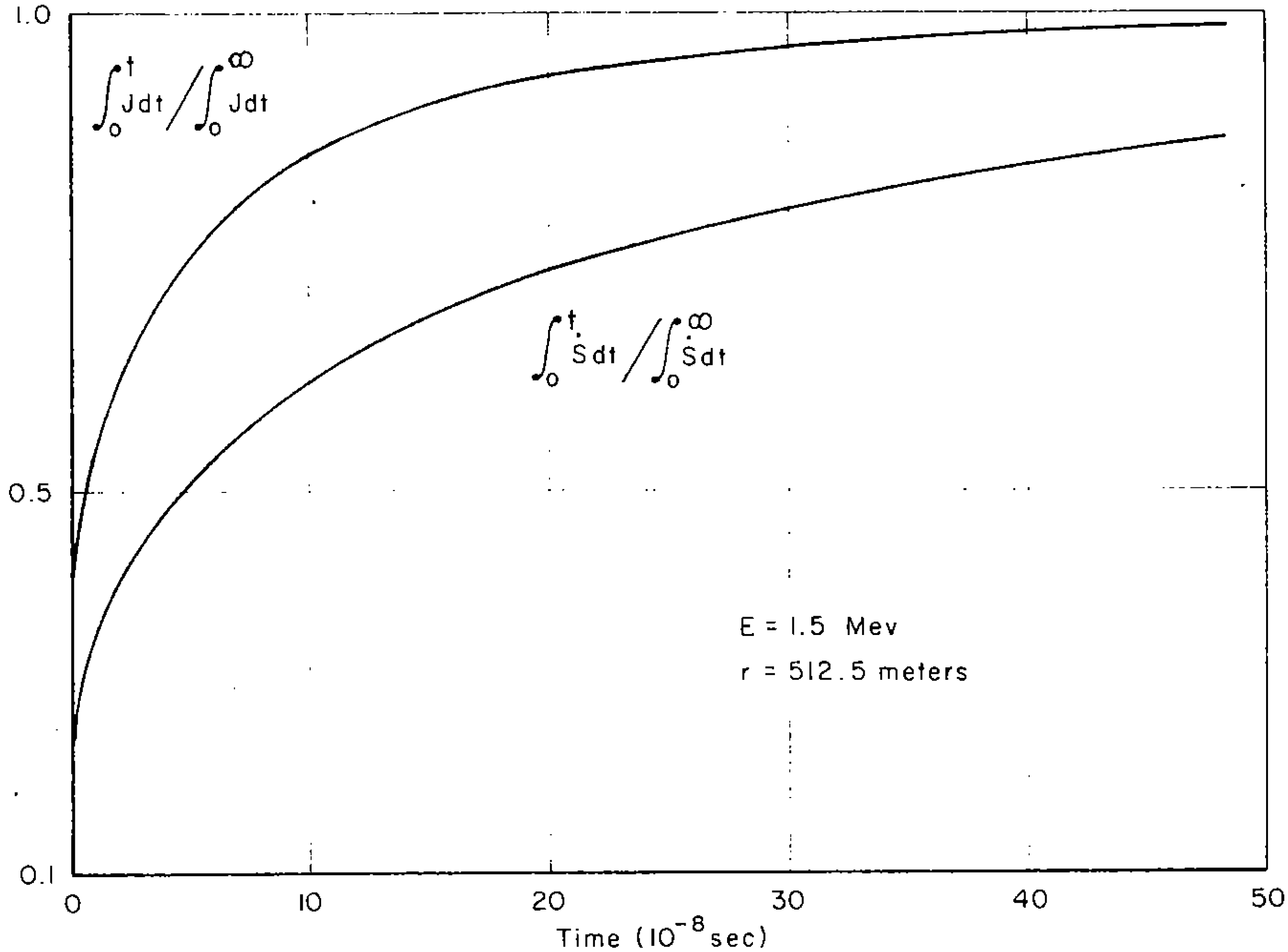
[illegible] function of time

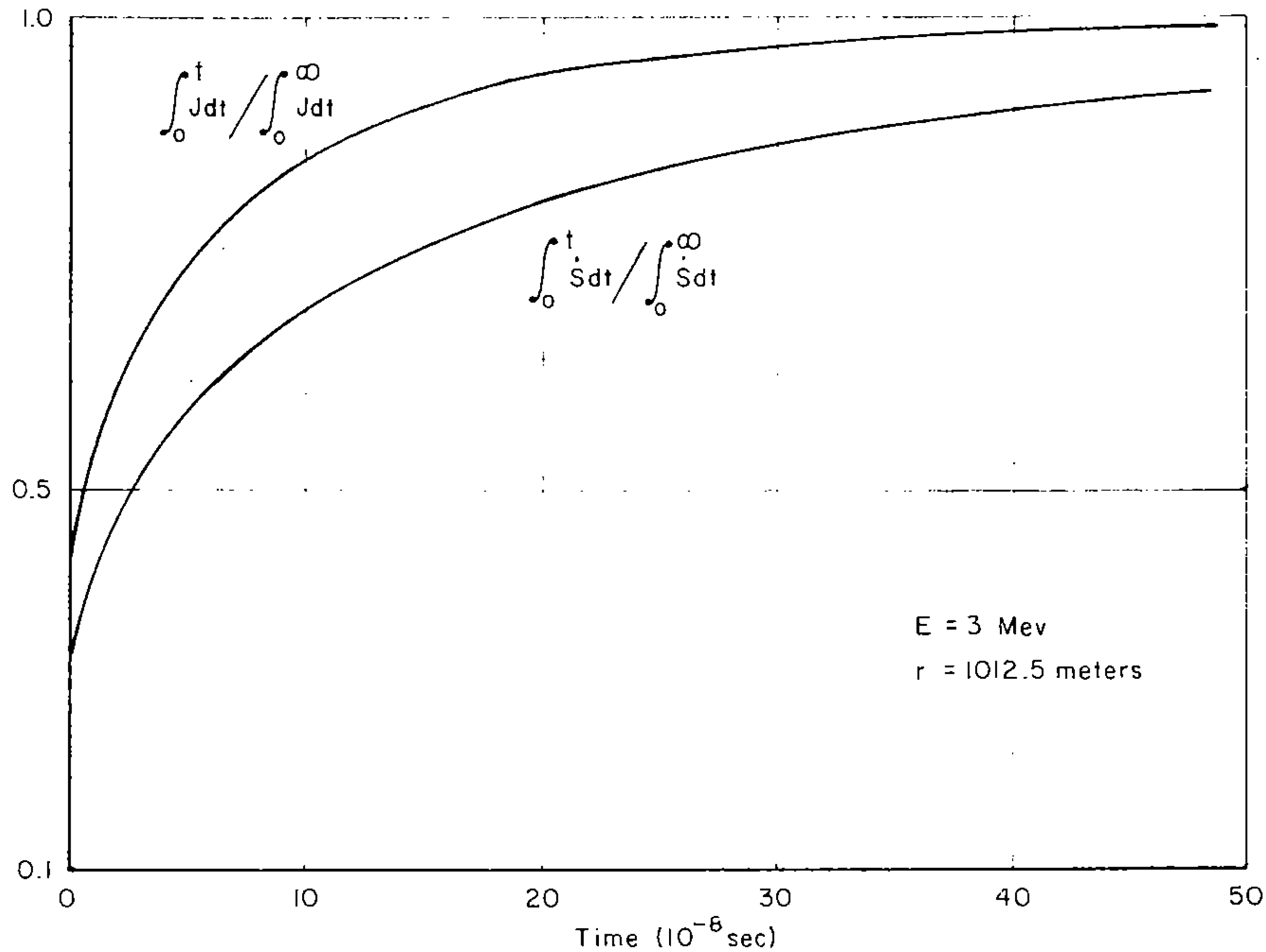

X 10^-16
3.0
2.5
2.0
1.5
1.0
0.5
0
J(t) (statcoulombs/cm^2 sec)
E = 1.5 Mev
r = 512.5 meters
Direct beam:
7.3 x 10^-24 statcoulombs/cm^2
0
1
5
10
15
20
Time (10^-8 sec)


X 10
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
$\dot{S}(t)$ (Mev/cm^3 sec)
E = 3 Mev
r = 1012.5 meters
Direct beam deposits
8.2 x 10^{-18} Mev/cm^3
0
1
5
10
15
20
22
Time (10^{-8}sec)

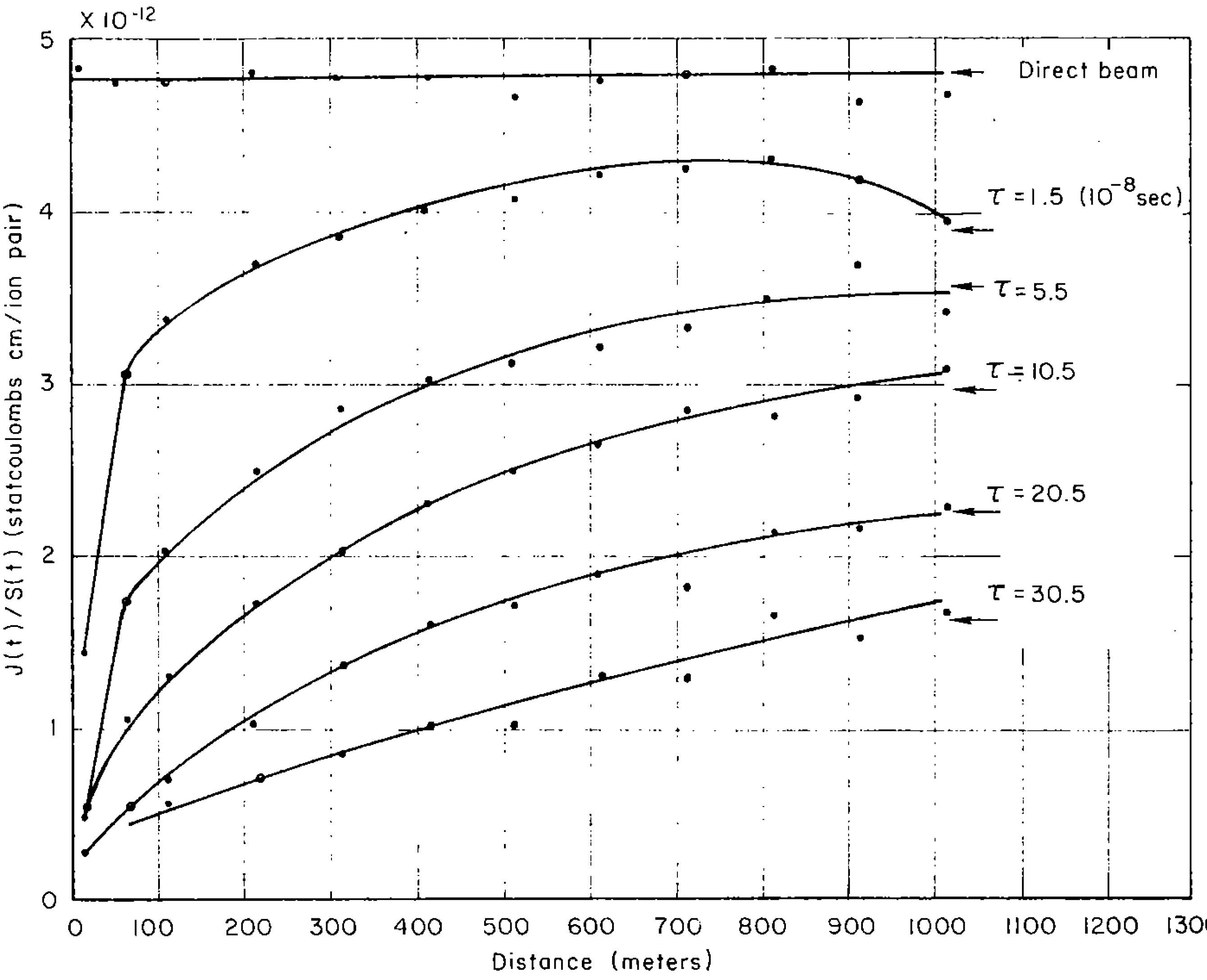
[illegible]


X 10-17
6.0
5.0
4.0
3.0
2.0
1.0
0
J(t) (statcoulombs/cm 2 sec)
E = Mev
r = 1012.5 meters
Direct beam:
1.4 x 10-24 statcoulombs/cm2
0 1 5 10 15 20 22
Time (10-8 sec)

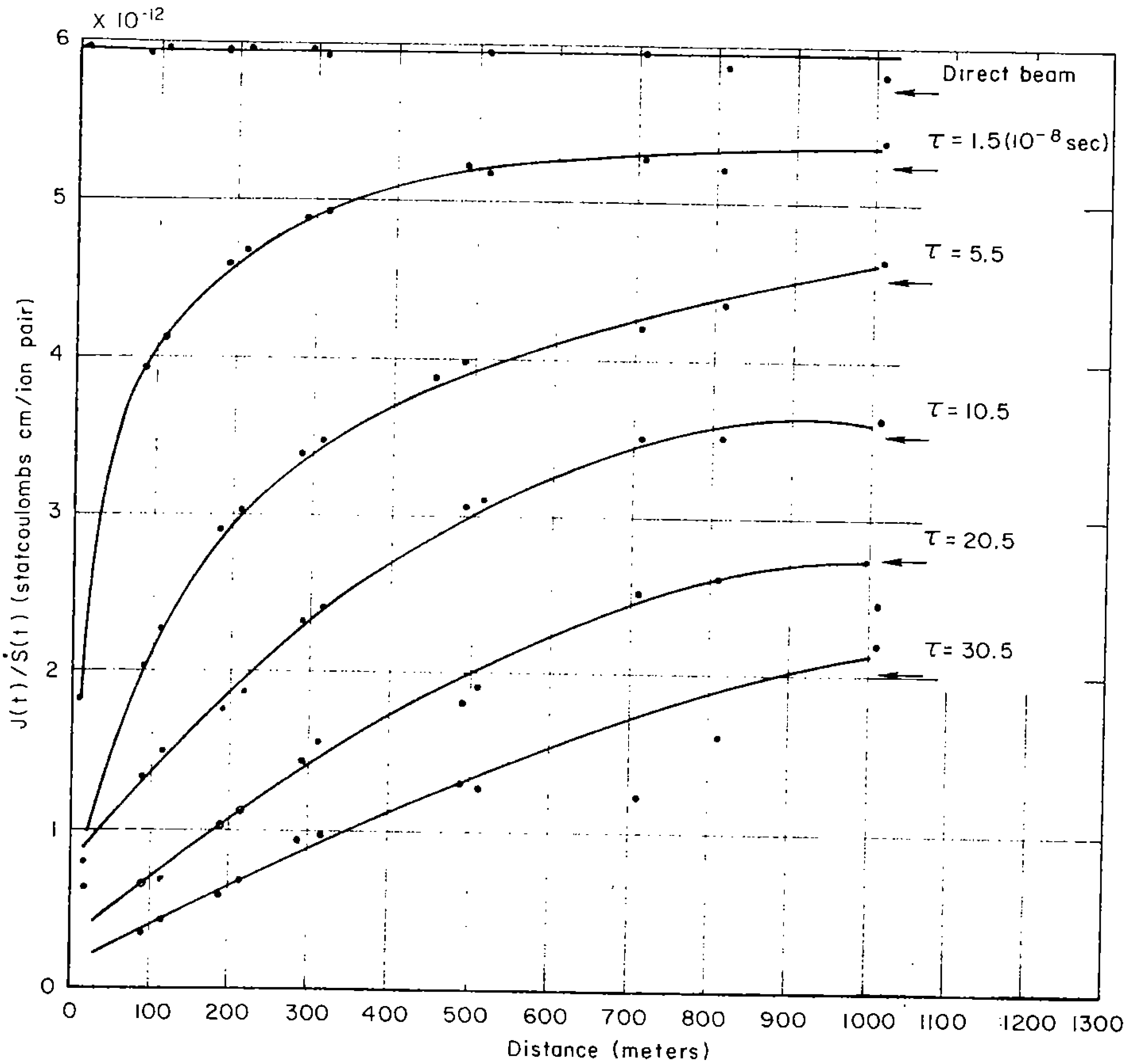

1.0
0.5
0.1
0
10
20
30
40
50
Time (10^{-8}sec)
$\int_0^t J dt \Big/ \int_0^\infty J dt$
$\int_0^t \dot{S} dt \Big/ \int_0^\infty \dot{S} dt$
E = 1.5 Mev
r = 512.5 meters



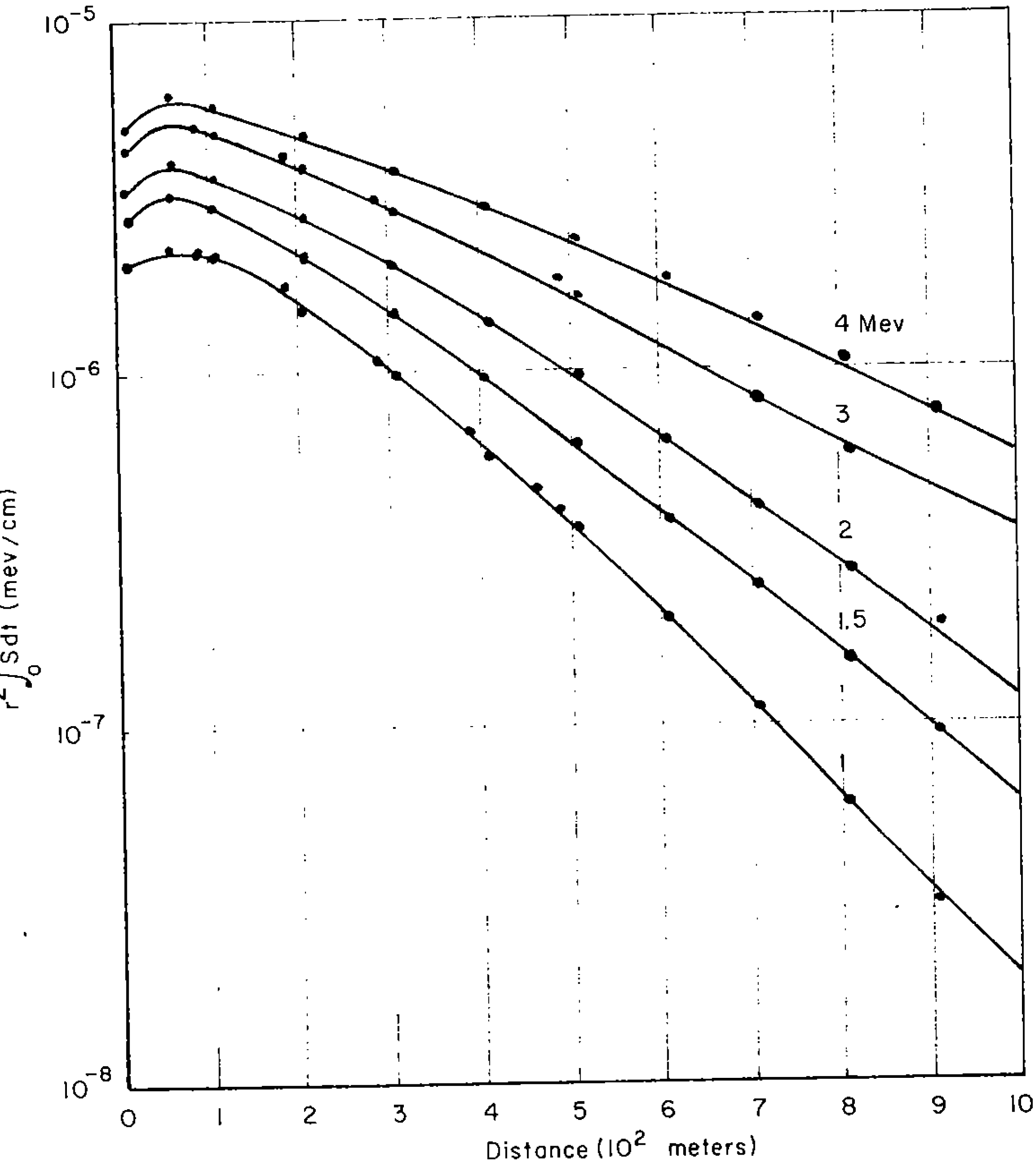
1.0
0.5
0.1
0
10
20
30
40
50
Time (10^{-8} sec)
$\int_0^t J dt \Big/ \int_0^\infty J dt$
$\int_0^t \dot{S} dt \Big/ \int_0^\infty \dot{S} dt$
E = 3 Mev
r = 1012.5 meters



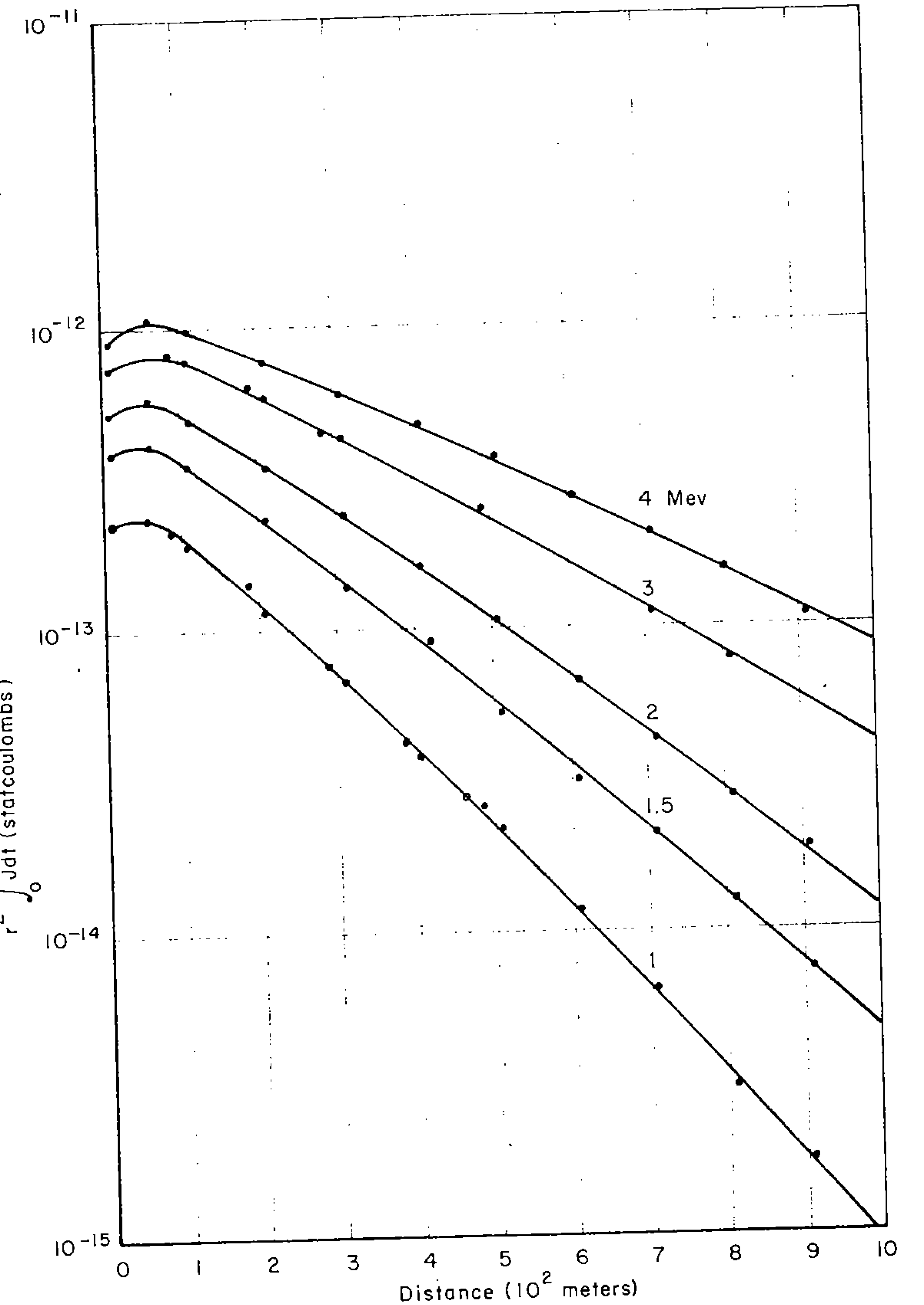
F[illegible] rate as a [illegible] τ. (E [illegible] MeV)



[illegible] rate as a
[illegible] τ. (E [illegible] Mev)



10^{-5}
10^{-6}
10^{-7}
10^{-8}
$r^2 \int_0^\infty \dot{S} dt$ (mev/cm)
4 Mev
3
2
1.5
1
0
1
2
3
4
5
6
7
8
9
10
Distance (10^2 meters)



[illegible] versus distance

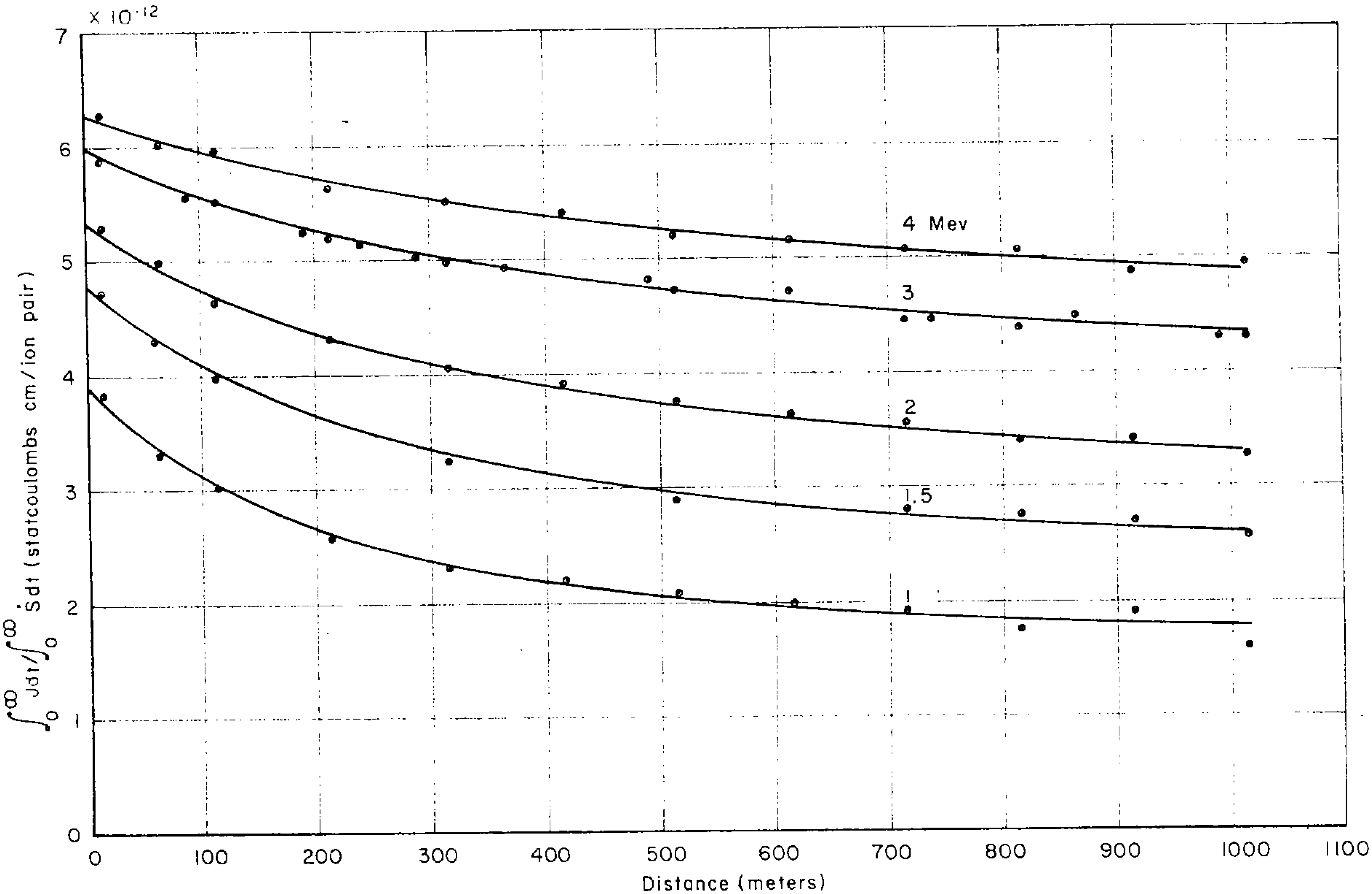


Fig. 11 — Ratio of integrated radial Compton current to total energy deposition $\int_0^\infty Jdt / \int_0^\infty \dot{S}dt$ as a function of distance

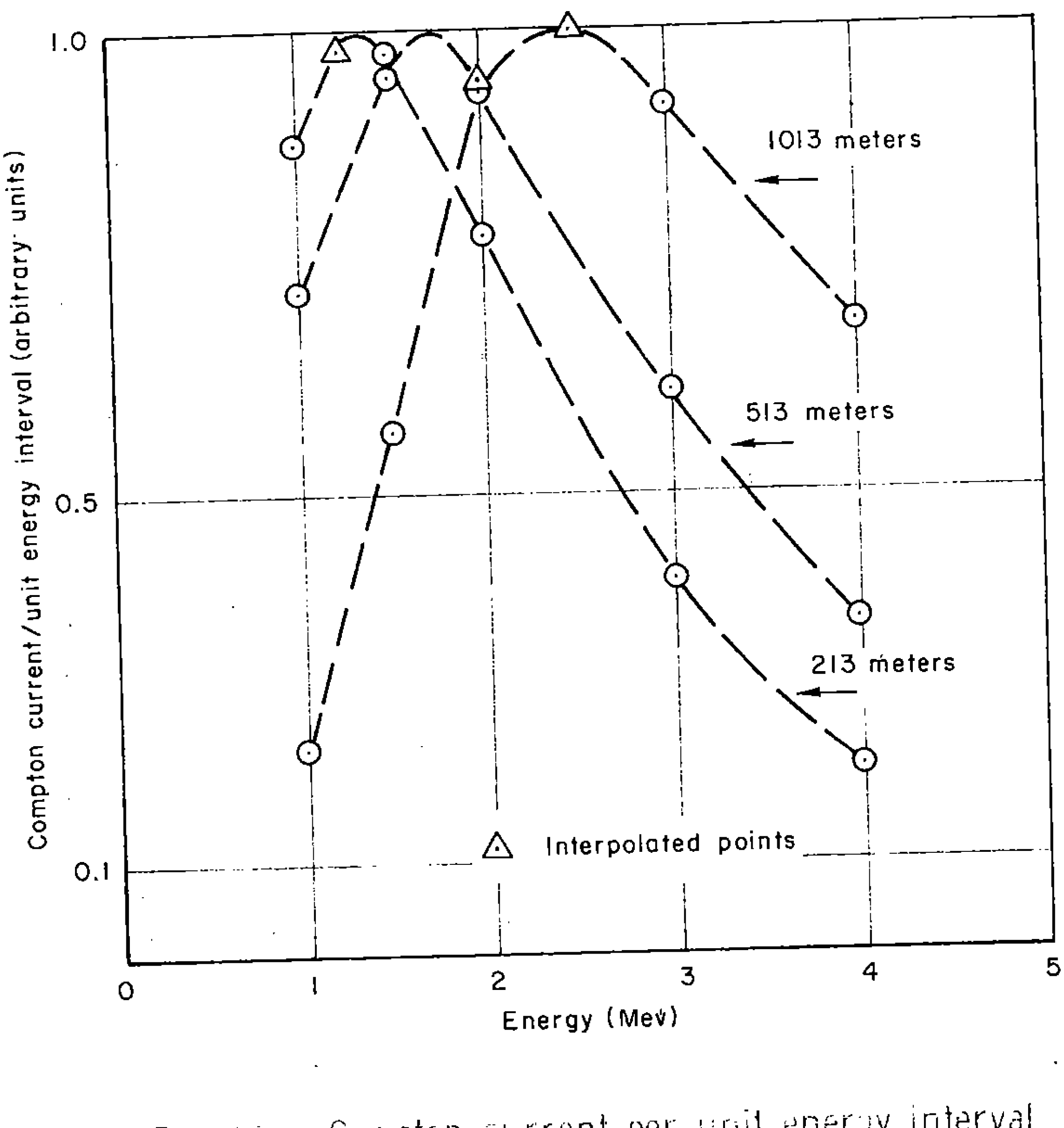


Fig. [illegible] — Compton current per unit energy interval for the Maienschein spectrum

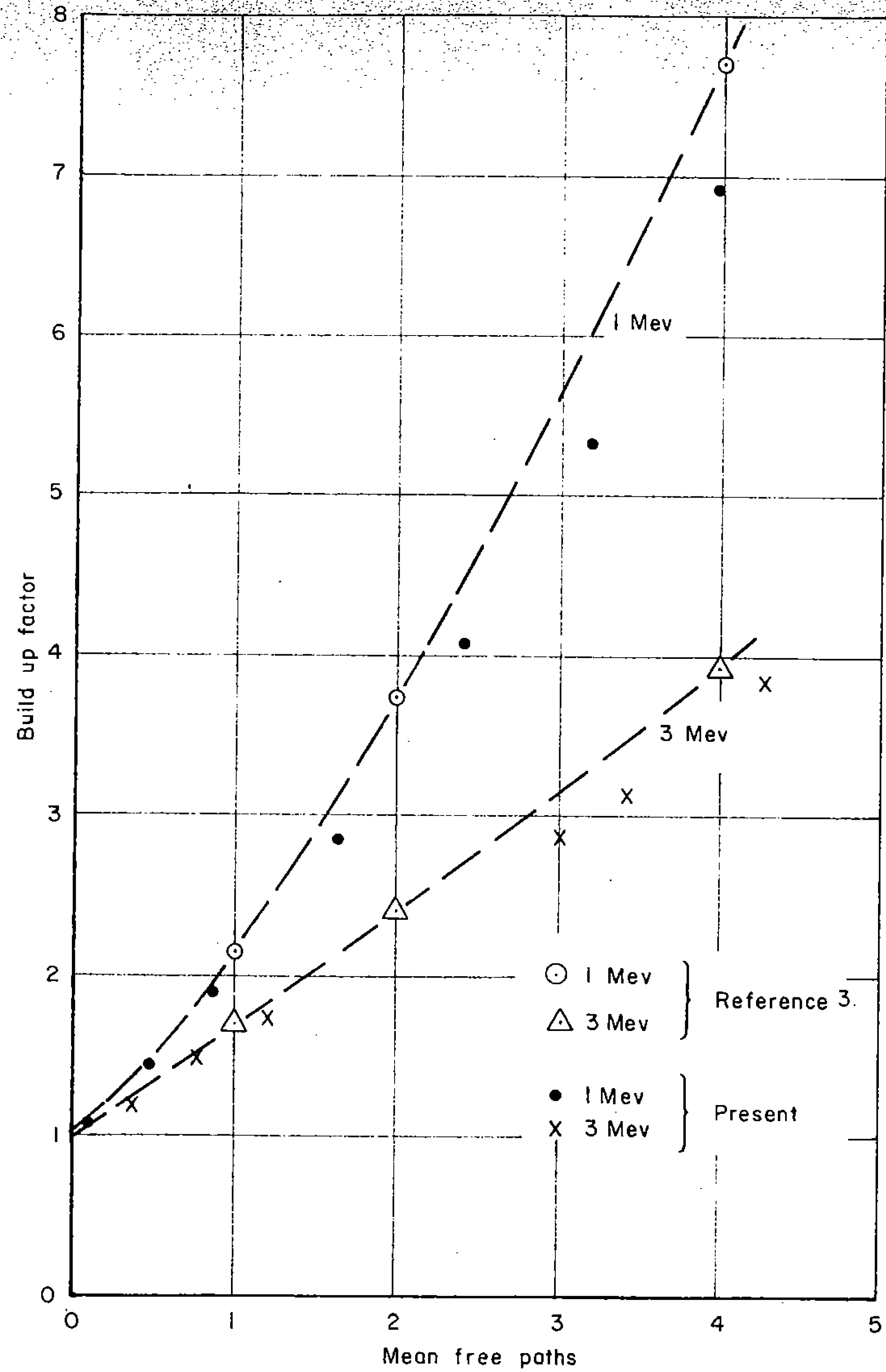


Fig. 13 - Build up factor for energy deposition

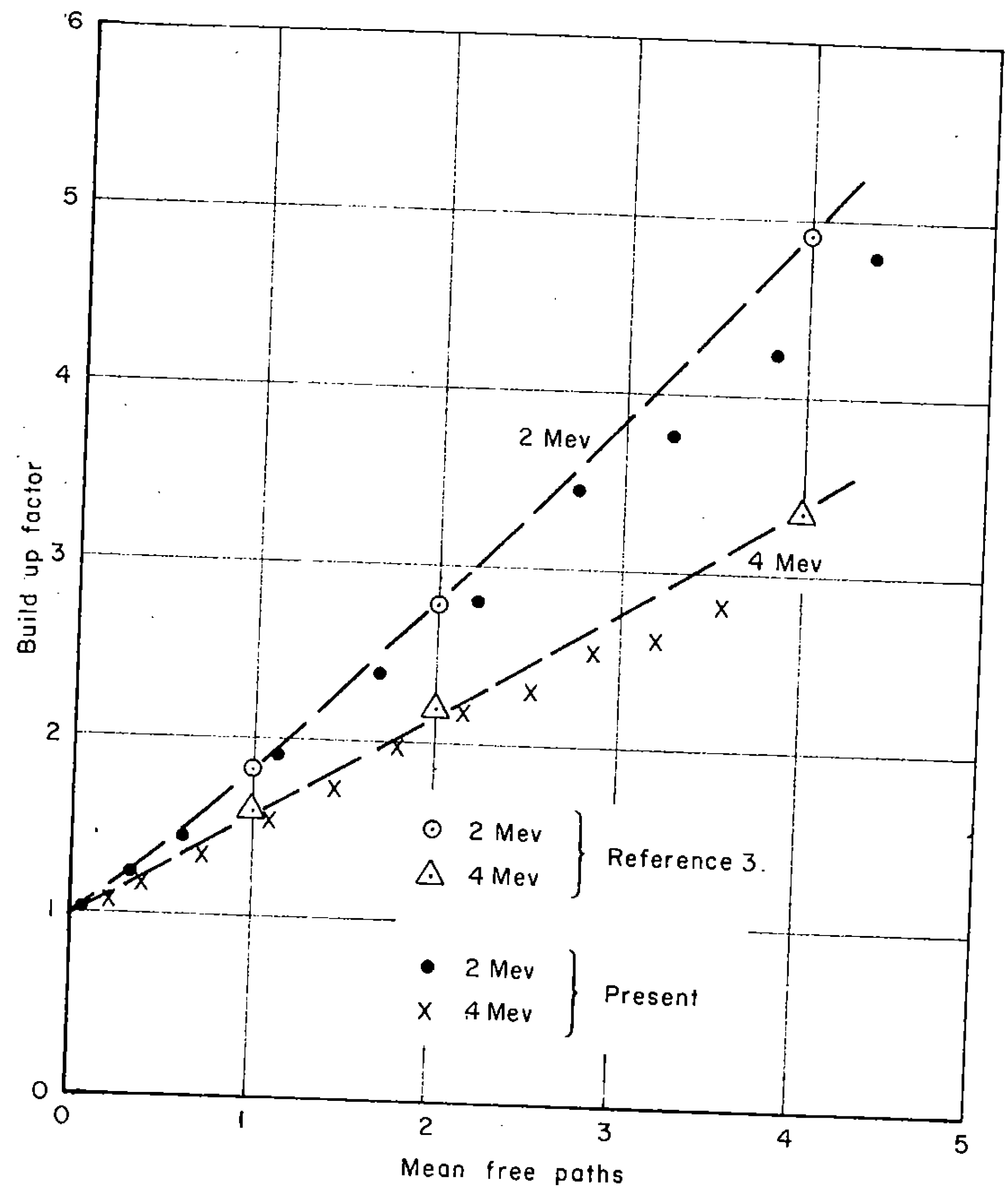


Fig. 14 Build up factor for energy deposition

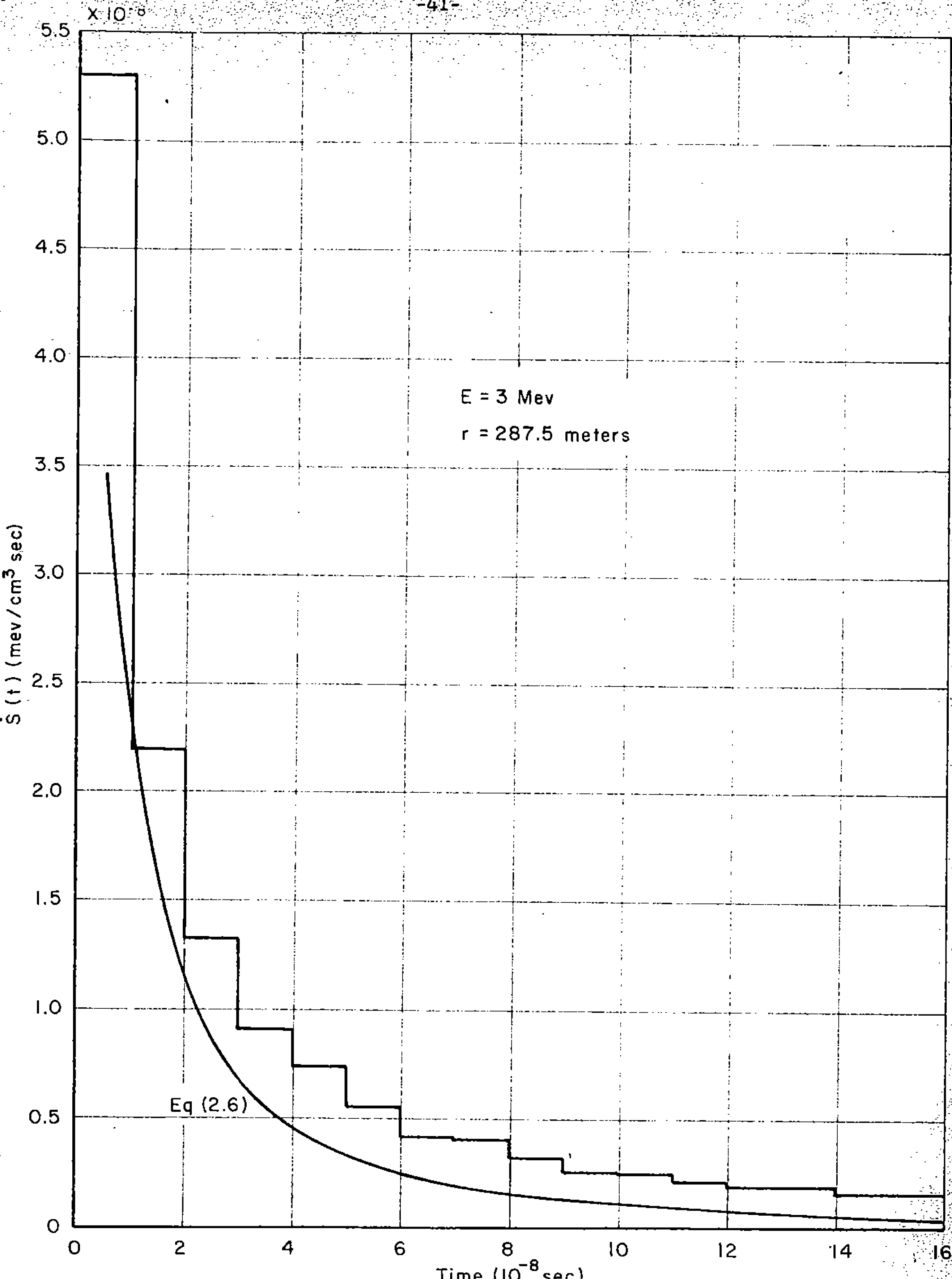


Fig. 15 -- Comparison of analytic, once scattered beam with Monte Carlo results

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