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Note 57

Extrapolating Transient Multipole Fields

By

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ABSTRACT

An improved method of extrapolating transient EM fields has been developed and a computer code has been written to implement the technique. This paper contains a summary of the theory and a listing and description of the computer code.

CONTENTS

I. INTRODUCTION

The field extrapolation method reported here is a considerable improvement over the previously used method.[1] The spherical vector components are treated directly, using only E_θ as input rather than transforming to rectangular components, extrapolating, and then transforming back again. The new method is much simpler and is faster to perform numerically. In addition, the numerical method of performing the convolution integrals has been improved to make it faster and more accurate. Program M, the new computer method described in Sec. IV, requires computer time linearly proportional to the number of time points treated, whereas the old method used computer time proportional to the square of the number of time points.

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[1] D. D. Babb and K. D. Granzow, Extrapolating Electromagnetic Fields From Values in a Spherical Region, Sec. II, WL-TR-64-179, Air Force Weapons Laboratory, 1965.(EMP Theoretical Note 52).

II. ANALYSIS

The electromagnetic fields to be extrapolated are known within and on the surface of a sphere whose center lies in an infinitely conducting horizontal plane. The field possesses azimuthal symmetry about the vertical axis and is equal to zero at time equal to zero. The problem is to find the field as a function of time in the upper half space outside the sphere. The field components are given for any r by the following equations[2]:

$$E_r = -\sum_{\ell=1}^{\infty} \frac{\sqrt{\ell(\ell+1)}}{r} \Xi_\ell(r)\, a_\ell(t^*)\, \overline{P}_\ell(\cos\theta) \tag{1}$$

$$E_\theta = \sum_{\ell=1}^{\infty} \Lambda_\ell(r)\, a_\ell(t^*)\, \overline{P}_\ell^{\,1}(\cos\theta) \tag{2}$$

$$B_\phi = \sum_{\ell=1}^{\infty} \frac{1}{c}\frac{\partial}{\partial t^*} \Xi_\ell(r)\, a_\ell(t^*)\, \overline{P}_\ell^{\,1}(\cos\theta) \tag{3}$$

where only odd values of ℓ are needed in the summations for the assumed symmetry (the analysis will include both even and odd terms for the sake of generality). The functions $\overline{P}_\ell$ are normalized Legendre polynomials and $\overline{P}_\ell^{\,1}$ are the normalized associated Legendre polynomials of the first kind;

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2 K. D. Granzow, Transient Spherical Waves, EMP Theoretical Note XXIV, Dec. 1, 1966 (also published as "Time-Domain Treatment of a Spherical Boundary-Value Problem," Journal of Applied Physics, Vol. 39, No. 7, June 1968, P. 3435). The equations here are the ϕ-independent part (m=0) of Eqs. 2, 3, and 6 of the reference.

t^* is retarded time defined as $t^* = t - r/c$; Λ_ℓ and Ξ_ℓ are differential operators given by

$$\Xi_\ell(r) = \sum_{j=0}^{\ell} \frac{\mu_{\ell j}}{r^{j+1} c^{\ell-j}} \frac{\partial^{\ell-j}}{\partial t^{*\,\ell-j}}$$

$$\Lambda_\ell(r) = \sum_{j=0}^{\ell+1} \frac{\nu_{\ell j}}{r^{j+1} c^{\ell+1-j}} \frac{\partial^{\ell+1-j}}{\partial t^{*\,\ell+1-j}}$$

$$\mu_{\ell j} = \frac{\prod_{k=0}^{j} (\ell+k)(\ell-k+1)}{\ell(\ell+1)\, 2^j\, j!}, \quad j=0,\ 1,\ \ldots,\ \ell$$

$$\nu_{\ell j} = \mu_{\ell j} \frac{\ell(\ell+1) + j(j-1)}{\ell(\ell+1) - j(j-1)}, \quad j=0,\ 1,\ \ldots,\ \ell, \quad \nu_{\ell,\ell+1} = \nu_{\ell\ell}$$

The field component E_θ is given at $r = R$ by

$$E_\theta(R, t^*) = \sum_{\ell=1}^{\infty} A_\ell(t^*)\, \overline{P}_\ell^{\,1}(\cos\theta) \tag{4}$$

Upon equating the coefficients of Eqs. 2 and 4, one can express the unknown coefficients $a_\ell(t^*)$ in terms of the known coefficients $A_\ell(t^*)$

$$\Lambda_\ell(R)\, a_\ell(t^*) = A_\ell(t^*) \tag{5}$$

It is convenient to express the above coefficients in terms of a dimensionless time variable $\tau = ct^*/R$. The operator $\Lambda_\ell(R)$ now becomes

$$\Lambda_\ell(R) = \frac{1}{R^{\ell+2}} \sum_{j=0}^{\ell+1} \nu_{\ell j} \frac{\partial^{\ell+1-j}}{\partial \tau^{\ell+1-j}}$$

Redefine the coefficients

$$b_\ell(\tau) = a_\ell\left(\frac{\tau R}{c}\right)$$

$$B_\ell(\tau) = A_\ell\left(\frac{\tau R}{c}\right)$$

Equation 5, expressed in terms of τ, becomes

$$\Lambda_\ell(R)\ b_\ell(\tau) = B_\ell(\tau) \tag{6}$$

Equation 6 is a differential equation whose solution will be found with the aid of a Green's function.[3] The problem now becomes one of finding the Green's function that will satisfy the following equation:

$$\Lambda_\ell(R)\ G_\ell(\tau, \tau') = \delta(\tau-\tau'),\ \ \tau'>0 \tag{7}$$

with the initial condition

$$G_\ell^{(k)}(0, \tau') = 0,\ \ k=0,\ 1,\ 2,\ \ldots,\ \ell$$

The first step is to solve the homogeneous case. This involves finding the roots of the auxiliary equation. The roots of the resulting polynomials are given in the appendix of Ref. 2 as the roots of $\lambda_\ell(z) = 0$. If ℓ is even, there is one real root, $p_{\ell 0}$, and the rest are complex, $p_{\ell n} \pm iq_{\ell n}$, where $n = 1, 2, 3, \ldots, m$; $m = \ell/2$ for ℓ even; $m = (\ell+1)/2$ for ℓ odd. If ℓ is odd, there is no real root.

[3] The solution given here parallels the solution of the equation $\Xi_\ell(r_o)\ \alpha_E(\ell, m, t^*) = \beta_E(\ell, m, t^*)$ given by Granzow in Multipole Theory in the Time Domain, EMP Theoretical Note VII, March 1965 (also Journal of Mathematical Physics, Vol. 7, 1966, p. 634), and in Ref. 1.

Let $H_\ell(\tau,\tau')$ be defined as $G_\ell(\tau,\tau')/R^{\ell+2}$, then H_ℓ can be expressed as a linear combination of exponentials multiplying sines and cosines with coefficients α_ℓ, $\beta_{\ell n}$, and $\gamma_{\ell n}$ yet to be determined.

$$H_\ell(\tau,\tau') = \alpha_\ell e^{p_{\ell 0}(\tau-\tau')} + \sum_{n=1}^{m} e^{p_{\ell n}(\tau-\tau')} \Big[\beta_{\ell n} \sin q_{\ell n}(\tau-\tau') + \gamma_{\ell n} \cos q_{\ell n}(\tau-\tau')\Big] \tag{8}$$

In order to apply the boundary conditions, the first ℓ derivatives of Eq. 8 are needed.

$$\frac{d^k}{d\tau^k} H_\ell(\tau-\tau') = \alpha_\ell p_{\ell 0}^k e^{p_{\ell 0}(\tau-\tau')} + \sum_{n=1}^{m} e^{p_{\ell n}(\tau-\tau')} r_{\ell n}^k \times \Big[(\beta_{\ell n} \sin k\theta_{\ell n} + \gamma_{\ell n} \cos k\theta_{\ell n}) \cos q_{\ell n}(\tau-\tau') + (\beta_{\ell n} \cos k\theta_{\ell n} - \gamma_{\ell n} \sin k\theta_{\ell n}) \sin q_{\ell n}(\tau-\tau')\Big] \tag{9}$$

where

$$p_{\ell n} + iq_{\ell n} = r_{\ell n} e^{i\theta_{\ell n}}$$

Taking the limit as τ approaches τ' from above,

$$\left.\frac{d^k H_\ell(\tau,\tau')}{d\tau^k}\right|_{\tau\to\tau'} = \alpha_\ell p_{\ell 0}^k + \sum_{n=1}^{m} r_{\ell n}^k \Big[\beta_{\ell n} \sin k\theta_{\ell n} + \gamma_{\ell n} \cos k\theta_{\ell n}\Big] = \delta_{\ell k}, \quad k=0,\ 1,\ 2,\ \ldots,\ \ell \tag{10}$$

gives $\ell + 1$ linear equations in $\ell + 1$ unknowns, and the constants α_ℓ, $\beta_{\ell n}$, and $\gamma_{\ell n}$ can be determined. These constants are given in App. A.

The Green's function can now be found, and the solution of Eq. 6 can be written

$$b_\ell(\tau) = \int_0^\tau G_\ell(\tau-\tau')\, B_\ell(\tau')\, d\tau' \tag{11}$$

In order to evaluate the right-hand side of Eqs. 1, 2, and 3, the first $\ell + 1$ derivatives of $b_\ell(\tau)$ are needed. By direct differentiation of Eq. 11 and application of the boundary condition on H_ℓ (and, hence on G_ℓ) expressed in Eq. 10, one can write

$$\frac{d^k b_\ell}{d\tau^k} = R^{\ell+2}\left[\delta_{k,\,\ell+1}\, B_\ell(\tau) + \int_0^\tau H_\ell^{(k)}(\tau,\tau')\, B_\ell(\tau')\, d\tau'\right] \tag{12}$$

$$k=0,\ 1,\ 2,\ \ldots,\ \ell+1$$

where

$$H_\ell^{(k)}(\tau,\tau') = \frac{\partial^k}{\partial\tau^k} H_\ell(\tau,\tau')$$

For simplicity of notation, let

$$C_\ell^k(\tau) = \int_0^\tau H_\ell^{(k)}(\tau,\tau')\, B_\ell(\tau')\, d\tau' \tag{13}$$

Substituting the right-hand side of Eq. 9 for $H_\ell^{(k)}(\tau,\tau')$ and defining the symbols $I_{x\ell}(\tau)$, $I_{c\ell n}(\tau)$, and $I_{s\ell n}(\tau)$ for the following integrals:

$$I_{x\ell}(\tau) = \int_0^\tau e^{p_{\ell 0}(\tau-\tau')}\, B_\ell(\tau')\, d\tau' \tag{14}$$

$$I_{c\ell n}(\tau) = \int_0^\tau e^{p_{\ell n}(\tau-\tau')} \cos q_{\ell n}(\tau-\tau')\, B_\ell(\tau')\, d\tau' \tag{15}$$

$$I_{s\ell n}(\tau) = \int_0^\tau e^{p_{\ell n}(\tau-\tau')} \sin q_{\ell n}(\tau-\tau')\, B_\ell(\tau')\, d\tau' \tag{16}$$

$$n=1,\ 2,\ \ldots,\ m$$

one can write $C_\ell^k(\tau)$ as

$$C_\ell^k(\tau) = \alpha_\ell\, p_{\ell 0}^k\, I_{x\ell}(\tau) + \sum_{n=1}^{m} r_{\ell n}^k \Big[(\beta_{\ell n} \sin k\theta_{\ell n} + \gamma_{\ell n} \cos k\theta_{\ell n})\, I_{c\ell n}(\tau)$$

$$+ (\beta_{\ell n} \cos k\theta_{\ell n} - \gamma_{\ell n} \sin k\theta_{\ell n})\, I_{s\ell n}(\tau)\Big] \tag{17}$$

The terms in the expansion of E_r, Eq. 1, can be written as

$$\frac{1}{r}\Xi_\ell(r)\, a_\ell(t^*) = \sum_{j=0}^{\ell} \mu_{\ell j} \left(\frac{R}{r}\right)^{j+2} C_\ell^{(\ell-j)}(\tau) \tag{18}$$

Similarly, terms in the expansion of E_θ, Eq. 2, can be written as

$$\Lambda_\ell(r)\, a_\ell(t^*) = \frac{R}{r} B_\ell(\tau) + \sum_{j=0}^{\ell+1} \nu_{\ell j} \left(\frac{R}{r}\right)^{j+1} C_\ell^{(\ell+1-j)}(\tau) \tag{19}$$

The terms in the expansion of B_ϕ are

$$\frac{1}{c}\frac{\partial}{\partial t^*}\Xi_\ell(r)\, a_\ell(t^*) = \frac{R}{r} B_\ell(\tau) + \sum_{j=0}^{\ell} \mu_{\ell j} \left(\frac{R}{r}\right)^{j+1} C_\ell^{(\ell+1-j)}(\tau) \tag{20}$$

Finally, Eqs. 1, 2, and 3 can be written using Eqs. 18, 19, and 20 as

$$E_r = -\sum_{\ell=1}^{\infty} \sqrt{\ell(\ell+1)} \sum_{j=0}^{\ell} \mu_{\ell j} \left(\frac{R}{r}\right)^{j+2} C_\ell^{\ell-j}(\tau)\, \overline{P}_\ell(\cos\theta) \tag{21}$$

$$E_\theta = \sum_{\ell=1}^{\infty} \left[\frac{R}{r} B_\ell(\tau) + \sum_{j=0}^{\ell+1} \nu_{\ell j} \left(\frac{R}{r}\right)^{j+1} C_\ell^{\ell+1-j}(\tau) \right] \bar{P}_\ell^{\,1}(\cos\theta) \quad (22)$$

$$B_\phi = \sum_{\ell=1}^{\infty} \left[\frac{R}{r} B_\ell(\tau) + \sum_{j=0}^{\ell} \mu_{\ell j} \left(\frac{R}{r}\right)^{j+1} C_\ell^{\ell+1-j}(\tau) \right] \bar{P}_\ell^{\,1}(\cos\theta) \quad (23)$$

Given the coefficients of an expansion of E_θ at $r = R$, $B_\ell(\tau)$, one first calculates the convolution integrals $I_{x\ell}(\tau)$, $I_{c\ell n}(\tau)$, and $I_{s\ell n}(\tau)$, Eqs. 14, 15, and 16, for each value of τ. The values of $C_\ell^k(\tau)$ are found for the same τ values from Eq. 17. The fields E_r, E_θ, and B_ϕ can then be determined at any value of r and a time history can be calculated using Eqs. 21, 22, and 23.

III. THE NUMERICAL INTEGRATION

In performing the numerical integration indicated in Eqs. 14, 15, and 16, one need not integrate from the origin for each new τ value chosen. In evaluating $I_{x\ell}(\tau+\Delta\tau)$, $I_{c\ell n}(\tau+\Delta\tau)$, and $I_{s\ell n}(\tau+\Delta\tau)$, the properties of the exponential and trigonometric functions can be used to evaluate the integral from zero to τ using $I_{x\ell}(\tau)$, $I_{c\ell n}(\tau)$, and $I_{s\ell n}(\tau)$. Thus $I_{x\ell}(\tau+\Delta\tau)$ can be written

$$I_{x\ell}(\tau+\Delta\tau) = e^{p_{\ell 0}\Delta\tau}\, I_{x\ell}(\tau) + \int_{\tau}^{\tau+\Delta\tau} e^{p_{\ell 0}(\tau+\Delta\tau-\tau')}\, B_{\ell}(\tau')\, d\tau' \tag{24}$$

Applying the trapezoidal rule to the above integral, one obtains

$$I_{x\ell}(\tau+\Delta\tau) = e^{p_{\ell 0}\Delta\tau}\left[I_{x\ell}(\tau) + \frac{\Delta\tau}{2}\, B_{\ell}(\tau)\right] + \frac{\Delta\tau}{2}\, B_{\ell}(\tau+\Delta\tau) \tag{25}$$

The trapezoidal rule will probably work well in most cases. However, it may happen that even though the data points are close enough together to describe B_{ℓ} very well, they may be far apart with regard to the behavior of the kernal of the integral (in this case an exponential, in the other cases, the product of an exponential and a trigonometric function). In this event it is assumed that $B_{\ell}(\tau)$ follows a straight line over the interval $(\tau, \tau+\Delta\tau)$, since the data are chosen to accurately describe $B_{\ell}(\tau)$. The integral is then evaluated exactly. Thus, for large $\Delta\tau$ let

$$B_{\ell}(\tau') = C\tau' + D \tag{26}$$

$$C = \frac{B_\ell(\tau+\Delta\tau) - B_\ell(\tau)}{\Delta\tau}$$

$$D = B_\ell(\tau) - C\tau$$

The integral in Eq. 24 can be evaluated exactly, resulting in

$$I_{x\ell}(\tau+\Delta\tau) = e^{p_{\ell 0}\Delta\tau} I_{x\ell}(\tau) - \frac{1}{p_{\ell 0}}\left[\left(1 - e^{p_{\ell 0}\Delta\tau}\right)\left(B_\ell(\tau) + \frac{C}{p_{\ell 0}}\right) + B_\ell(\tau+\Delta\tau) - B_\ell(\tau)\right] \qquad (27)$$

Similarly, for $I_{c\ell n}(\tau+\Delta\tau)$ the integral is split as follows:

$$I_{c\ell n}(\tau+\Delta\tau) = e^{p_{\ell n}\Delta\tau}\left[I_{c\ell n}(\tau)\cos q_{\ell n}\Delta\tau - I_{s\ell n}(\tau)\sin q_{\ell n}\Delta\tau\right] + \int_\tau^{\tau+\Delta\tau} e^{p_{\ell n}(\tau+\Delta\tau-\tau')} \cos q_{\ell n}(\tau+\Delta\tau-\tau')\, B_\ell(\tau')\, d\tau' \qquad (28)$$

For $\Delta\tau$ small, the trapozoidal rule is used:

$$I_{c\ell n}(\tau+\Delta\tau) = e^{p_{\ell n}\Delta\tau}\left[\left(I_{c\ell n}(\tau) + \frac{\Delta\tau}{2} B_\ell(\tau)\right)\cos q_{\ell n}\Delta\tau - I_{s\ell n}(\tau)\sin q_{\ell n}\Delta\tau\right] + \frac{\Delta\tau}{2} B_\ell(\tau+\Delta\tau) \qquad (29)$$

When $\Delta\tau$ is large, Eq. 26 is used for $B_\ell(\tau')$ and the integral in Eq. 28 can be evaluated exactly:

$$I_{c\ell n}(\tau+\Delta\tau) = e^{p_{\ell n}\Delta\tau}\left[I_{c\ell n}(\tau)\cos q_{\ell n}\Delta\tau - I_{s\ell n}(\tau)\sin q_{\ell n}\Delta\tau\right]$$

$$- \frac{1}{r_{\ell n}^2}\left[\left(1 - e^{p_{\ell n}\Delta\tau}\cos q_{\ell n}\Delta\tau\right)\left(p_{\ell n} B_\ell(\tau)\right.\right.$$

$$\left. + \frac{C}{r_{\ell n}^2}(p_{\ell n}^2 - q_{\ell n}^2)\right) - e^{p_{\ell n}\Delta\tau}\sin q_{\ell n}\Delta\tau\left(q_{\ell n} B_\ell(\tau)\right.$$

$$\left.\left. + \frac{2Cp_{\ell n} q_{\ell n}}{r_{\ell n}^2}\right) + C\Delta\tau\, p_{\ell n}\right] \tag{30}$$

Similarly, when the trapezoidal rule is used to approximate $I_{s\ell n}(\tau+\Delta\tau)$ for small $\Delta\tau$, one obtains

$$I_{s\ell n}(\tau+\Delta\tau) = e^{p_{\ell n}\Delta\tau}\left[\left(I_{c\ell n}(\tau) + \frac{\Delta\tau}{2} B_\ell(\tau)\right)\sin q_{\ell n}\Delta\tau\right.$$

$$\left. + I_{s\ell n}(\tau)\cos q_{\ell n}\Delta\tau\right] \tag{31}$$

For large $\Delta\tau$, again let $B_\ell(\tau') = C\tau' + D$, and the integral $I_{s\ell n}(\tau+\Delta\tau)$ can be expressed as

$$I_{s\ell n}(\tau+\Delta\tau) = e^{p_{\ell n}\Delta\tau}\left[I_{c\ell n}(\tau)\sin q_{\ell n}\Delta\tau + I_{s\ell n}(\tau)\cos q_{\ell n}\Delta\tau\right]$$

$$+ \frac{1}{r_{\ell n}^2}\left[\left(1 - e^{p_{\ell n}\Delta\tau}\cos q_{\ell n}\Delta\tau\right)\left(q_{\ell n} B_\ell(\tau) + \frac{2Cp_{\ell n} q_{\ell n}}{r_{\ell n}^2}\right)\right.$$

$$+ e^{p_{\ell n}\Delta\tau}\sin q_{\ell n}\Delta\tau\left(p_{\ell n} B_\ell(\tau) + \frac{C}{r_{\ell n}^2}(p_{\ell n}^2 - q_{\ell n}^2)\right)$$

$$\left. + C\Delta\tau\, q_{\ell n}\right] \tag{32}$$

The criterion used to determine whether $\Delta\tau$ is small or large depends upon the polynomial roots and $\Delta\tau$. At this time, if $p_{\ell 0}\Delta\tau < .1$ then $\Delta\tau$ is considered small, and the trapezoidal method is used for the evaluation of $I_{x\ell}(\tau+\Delta\tau)$ by Eq. 25. If $r_{\ell n}\Delta\tau < .1$, then $\Delta\tau$ is again considered small, and the trapezoidal method is used to integrate $I_{s\ell n}(\tau+\Delta\tau)$ and $I_{c\ell n}(\tau+\Delta\tau)$ by Eqs. 29 and 31. Otherwise, the other method is used.

Since the roots and coefficients appearing in the function H_ℓ are not dependent on any physical parameters in the problem, they are mathematical constants and once determined can be used over and over for any values of the physical parameters.

IV. DESCRIPTION OF PROGRAM M

Program M has been developed to extrapolate transient multipole fields through a computerized system according to the method explained in Secs. I through III.

The program requires 165 K octal locations on the CDC 6600. Four tapes labeled 1, 2, 4, and 8 are needed. Tape 2 is the input tape, while Tape 1 contains the coefficients of the normalized Legendre polynomials that were picked from the input tape. Tape 4 contains the calculated quantities referred to as $C_\ell^k(\tau)$. Tape 8 enables the operator to dump and restart the program periodically. Tape 8 is a half-inch tape with 556 bits per inch. If the number of time points is less than 2000, then Tapes 1 and 4 can be half-inch tapes; otherwise, they must be one-inch tapes with 800 bits per inch.

The central processer (CP) time can be approximated as follows:

(number of time points) x (.6 sec) = time taken to calculate all the $C_\ell^k(\tau)$;

(number of time points) x (.04 sec) = time to extrapolate.

The peripheral processer (PP) time is approximately twice the CP time. These times should always overestimate the time required. The program is set up to dump itself on Tape 8 every 15 minutes. The ability to restart after dumping is included only in the portion of the program that calculates the quantities $C_\ell^k(\tau)$. The extrapolation procedure takes so little time that there is no need for a restart capability during that section.

INPUT

The tape input consists of the coefficients of the Legendre expansion of E_θ, B_ϕ, and E_r (given on Tape 2) in blocks of 496 words, each terminated by an end-of-file mark, as shown below.

Word	Description
1	observer number (floating point)
2	time (retarded time in seconds)
3-22	coefficients of E_θ
23-42	coefficients of E_r
43-62	coefficients of B_ϕ
63-124	same as words 1-62
125-186	" " " "
187-248	" " " "
249-310	" " " "
311-372	" " " "
373-434	" " " "
435-496	" " " "

In addition, the origin observer number, the corresponding radius, the radii to which the extrapolations are performed, and the observer numbers and angles are input on cards.

Cards	Columns	Format	Description
1	1-10	I10	observer number of origin of extrapolation
	11-20	E10.3	radius, in meters, of origin
2, . . . , n[4]	1-10	I10	observer number of extrapolation
	11-20	E10.3	radius of extrapolation (meters)
	21-30	E10.3	angle of extrapolation (degrees)
last card	1-80		blank

The polynomial roots ($p_{\ell n} \pm iq_{\ell n}$) and the Green's function coefficients ($\beta_{\ell n}$, $\gamma_{\ell n}$) are also on input cards and are read in prior to the above data. However, these cards are the same for each set of calculations and should remain with the source deck. It is important to remember that these cards, as well as data card 1, are not used with the restart program.

OUTPUT

All output is on microfilm. The message below is printed each time a restart group is dumped on Tape 8.

RESTART GROUP NO______IS COMPLETED

LAST TIME POINT WAS______

Also, at that time, the extrapolated fields are printed and plotted. Samples of input and output are shown on the following pages.

4 These cards start the data cards necessary for the restart program.

RUN NUMBER 3400
(14976..90.)

TIME	ERADIAL	ETHETA	BPHI
0.	0.	0.	0.
1.0006882E-09	-1.7725442E-14	2.9561382E-02	9.8530588E-11
2.0013803E-09	-7.1332043E-14	6.3609247E-02	2.1200174E-10
3.0020725E-09	-1.5397828E-13	1.0242935E-01	3.4137125E-10
4.0027646E-09	-9.0513321E-14	1.4625592E-01	4.8751608E-10
5.0034568E-09	-3.4398399E-13	1.9557643E-01	6.5179472E-10
6.0041489E-09	-9.7588775E-13	2.5069481E-01	8.3519229E-10
7.0048411E-09	-1.1894905E-13	3.1124492E-01	1.0376415E-09
8.0055332E-09	-1.8109012E-12	3.7900290E-01	1.2624414E-09
9.0062254E-09	-1.2854060E-12	4.5252319E-01	1.5078824E-09
1.0006918E-08	-2.3706373E-12	5.3351856E-01	1.7773188E-09
1.1007610E-08	-4.7950199E-12	6.2225587E-01	2.0718328E-09
1.2008302E-09	-6.3652238E-12	7.1680717E-01	2.3864419E-09
1.3008994E-08	1.0760466E-12	8.1381586E-01	2.7137588E-09
1.4009686E-08	-7.1647338E-12	9.2329428E-01	3.0740753E-09
1.5010378E-08	-5.9038515E-12	1.0354243E+00	3.4488686E-09
1.6011070E-08	-8.8765915E-12	1.1556631E+00	3.8485119E-09
1.7011763E-08	-6.4697238E-12	1.2800036E+00	4.2642331E-09
1.8012455E-08	-2.6925373E-12	1.4101130E+00	4.6998756E-09
1.9013147E-08	-5.6939536E-12	1.5496928E+00	5.1640664E-09
2.0013839E-08	-1.6413419E-11	1.6981895E+00	5.6520619E-09
2.1014531E-08	-1.9204368E-11	1.8508751E+00	6.1619452E-09
2.2015223E-08	-6.5922642E-12	2.0033696E+00	6.6775986E-09
2.3015915E-08	-3.1971062E-11	2.1764884E+00	7.2390666E-09
2.4016608E-08	-3.4421143E-11	2.3473180E+00	7.8081792E-09
2.5017300E-08	-4.5600619E-11	2.5279938E+00	8.4037093E-09
2.6017992E-08	-4.8167243E-12	2.6955284E+00	8.9881683E-09
2.7018684E-08	-1.1549070E-11	2.8870553E+00	9.6211620E-09
2.8019376E-08	-9.0344092E-12	3.0813344E+00	1.0271537E-08
2.9020068E-08	7.9075706E-12	3.2774247E+00	1.0937352E-08
3.0020760E-08	-7.8389277E-11	3.5185944E+00	1.1690663E-08
3.1021453E-08	-4.0504430E-11	3.7185576E+00	1.2381371E-08
3.2022145E-08	-2.1364277E-11	3.9310209E+00	1.3096350E-08
3.3022837E-08	-1.0194083E-10	4.1884246E+00	1.3909864E-08
3.4023529E-08	-8.5429840E-11	4.4164974E+00	1.4684533E-08
3.5024221E-08	-3.7698652E-11	4.6377855E+00	1.5445393E-08
3.6024913E-08	-4.0273819E-11	4.8852286E+00	1.6270821E-08
3.7025605E-08	-5.1281697E-11	5.1417310E+00	1.7117980E-08
3.8026298E-08	-9.7854093E-11	5.4205628E+00	1.8024903E-08
3.9026990E-08	-6.6648639E-11	5.6708952E+00	1.8874563E-08

Fig. 1--Sample Output

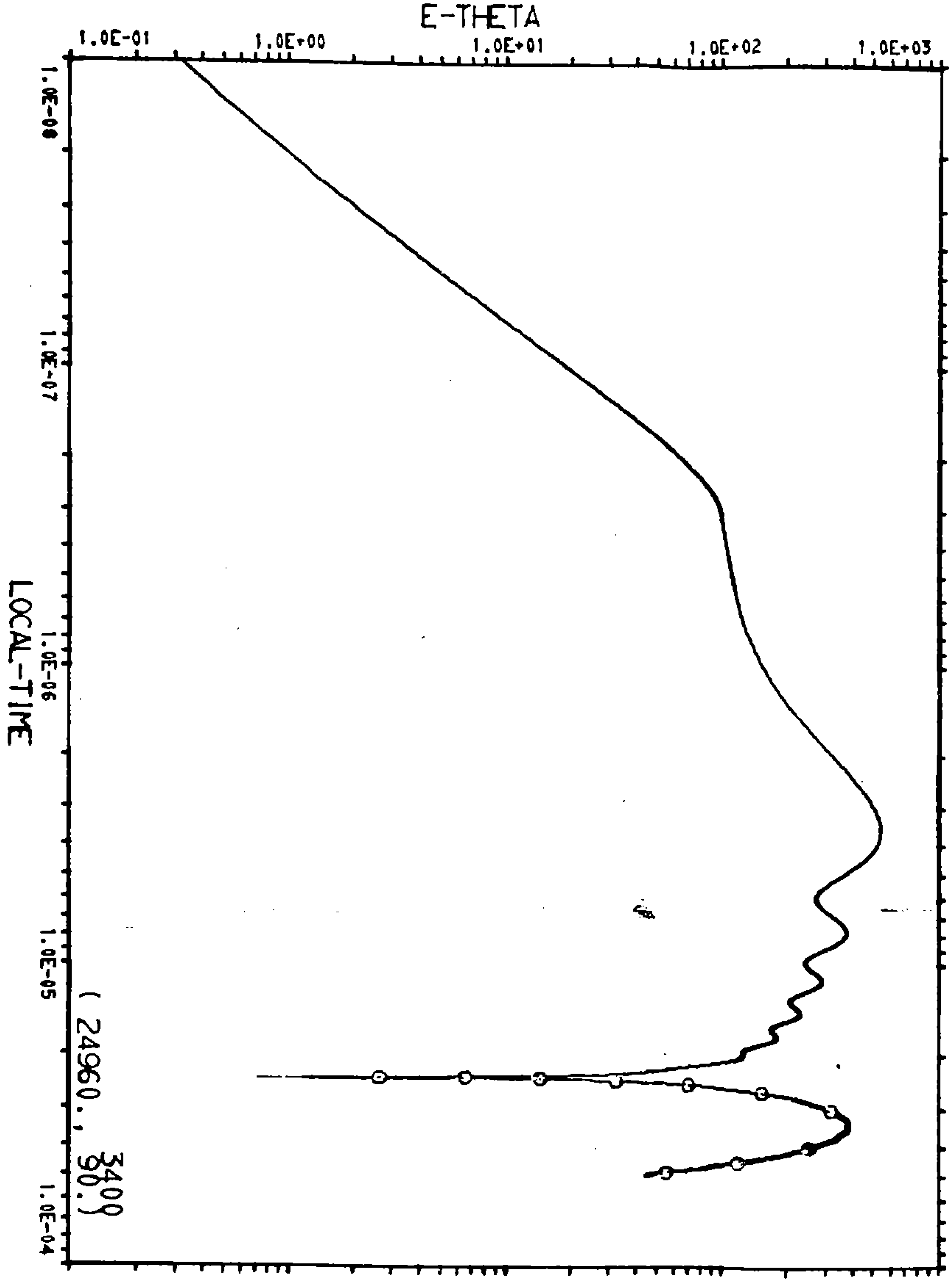


Fig. 2--E_θ Graph (Log-log)

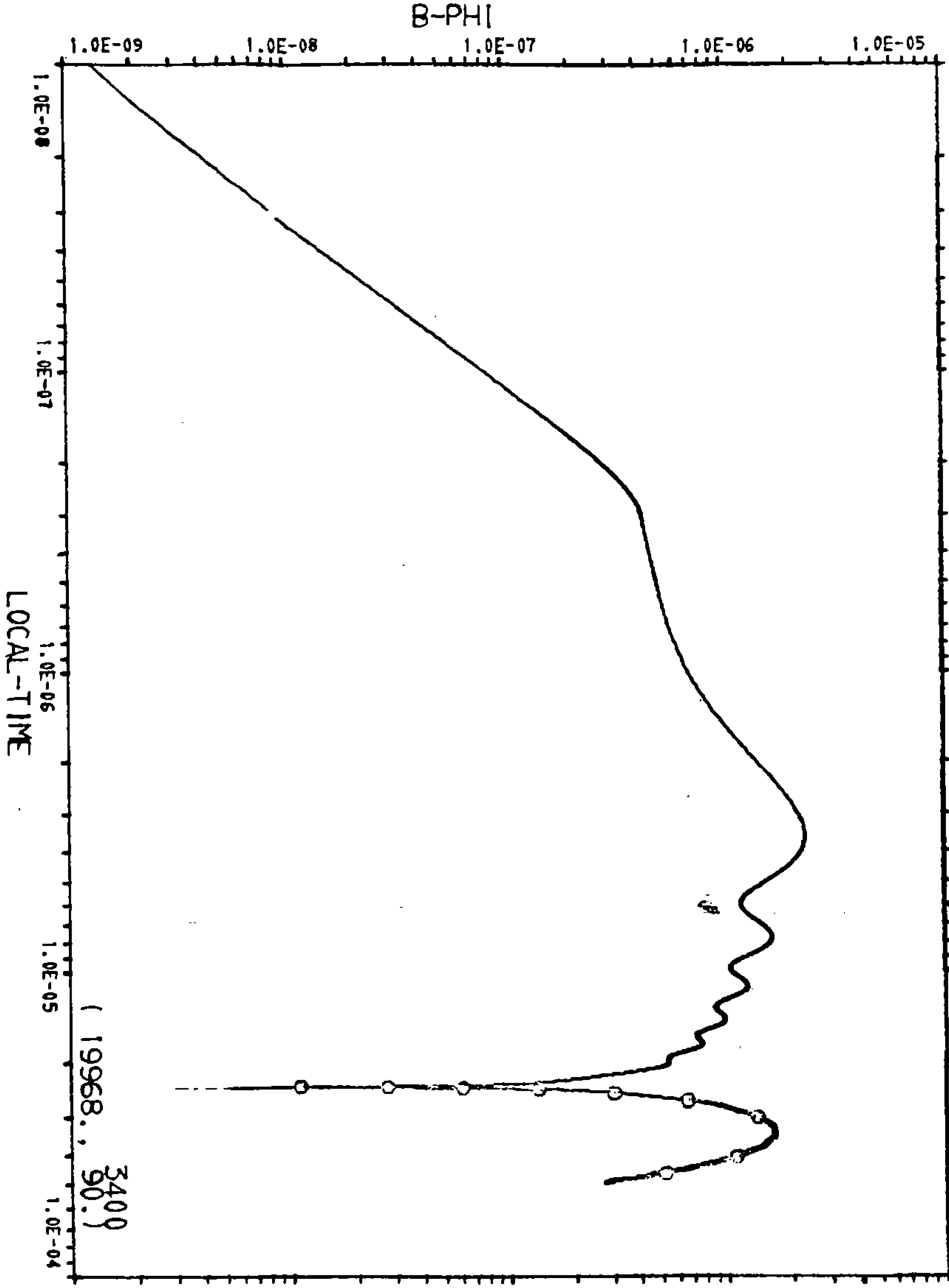


Fig. 3--B_ϕ Graph (Log-log)

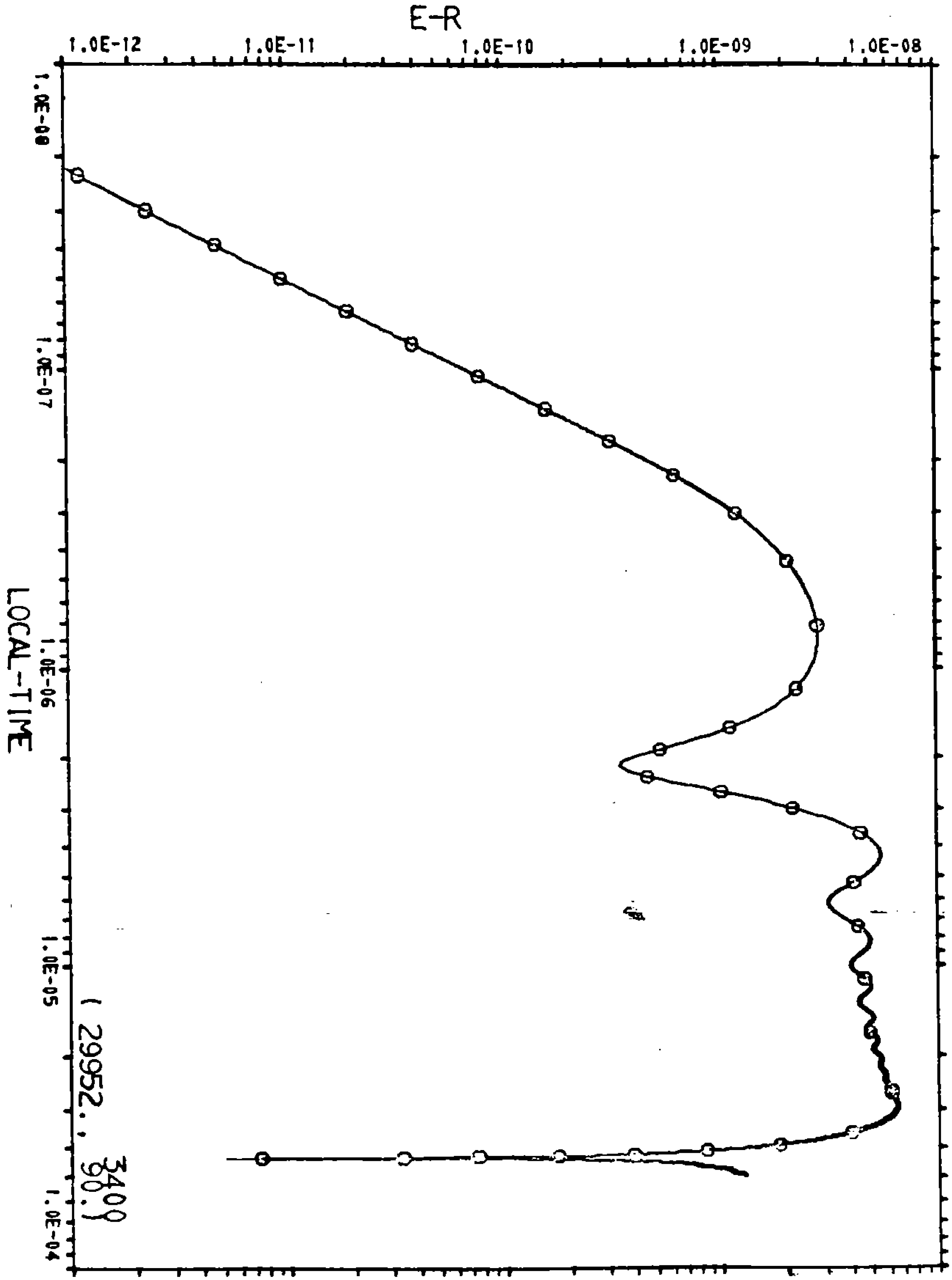


Fig. 4--E_r Graph (Log-log)

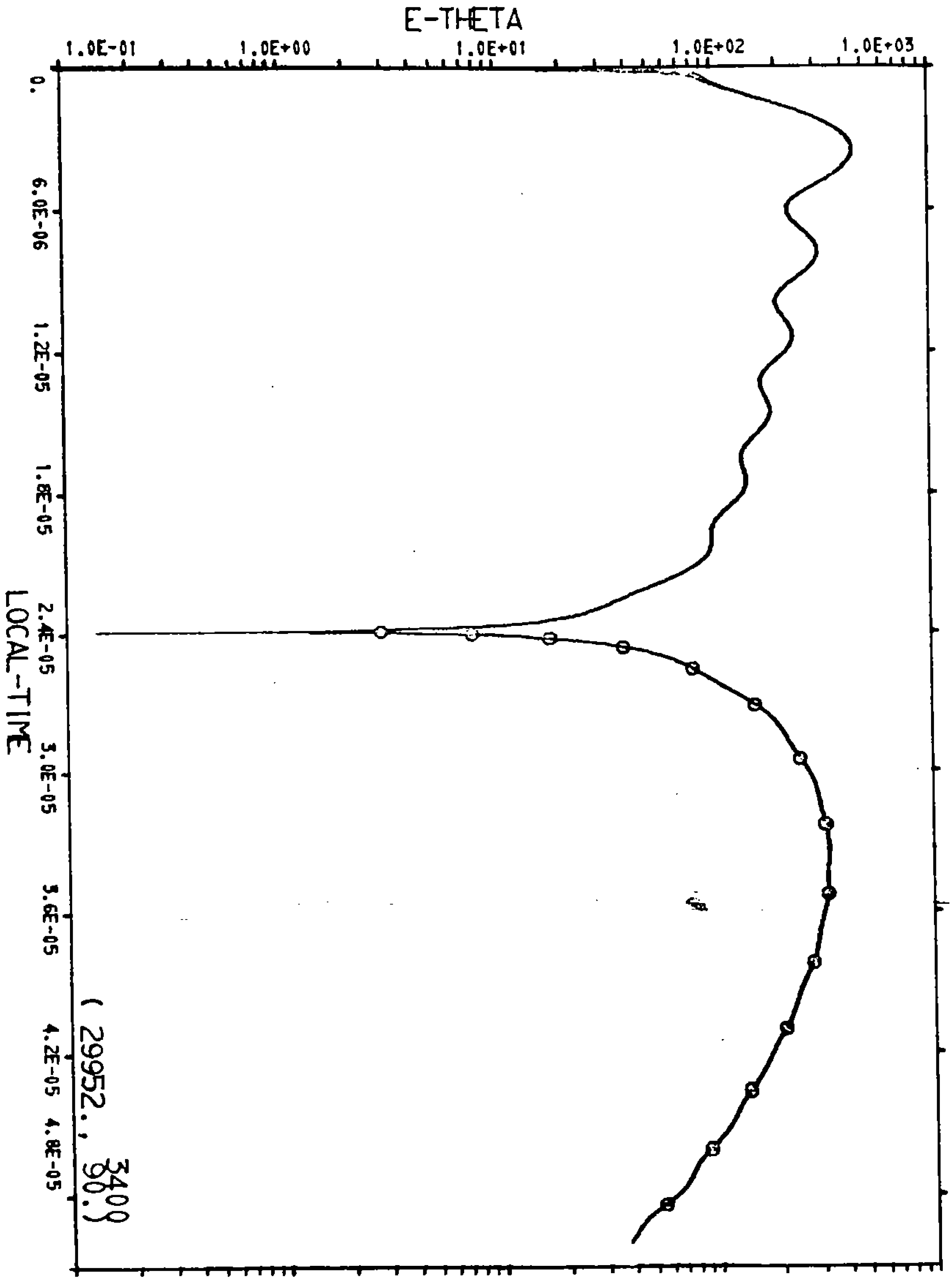


Fig. 5--E_{θ} Graph (Log-linear)

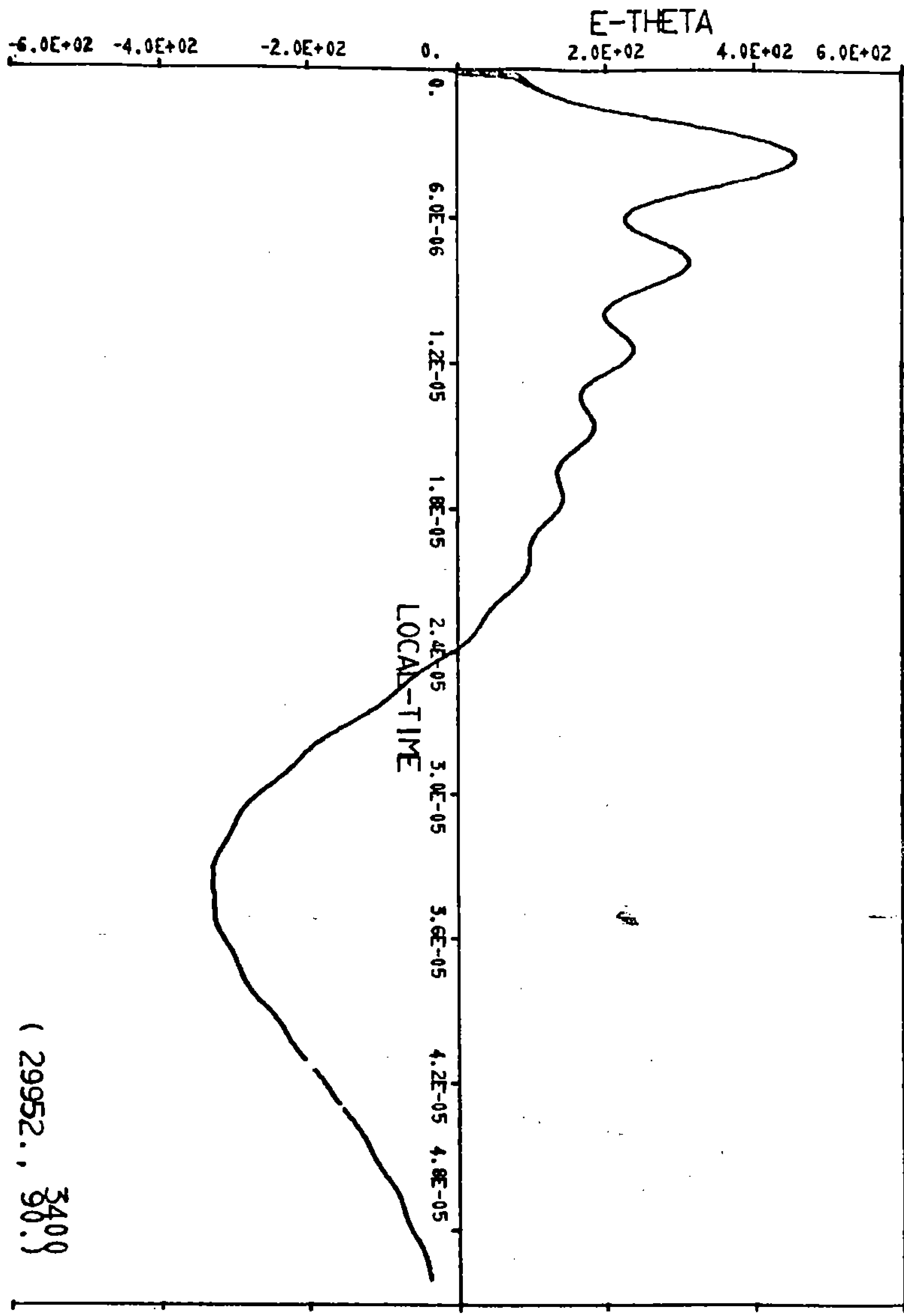


Fig. 6--E_θ Graph (Linear-linear)

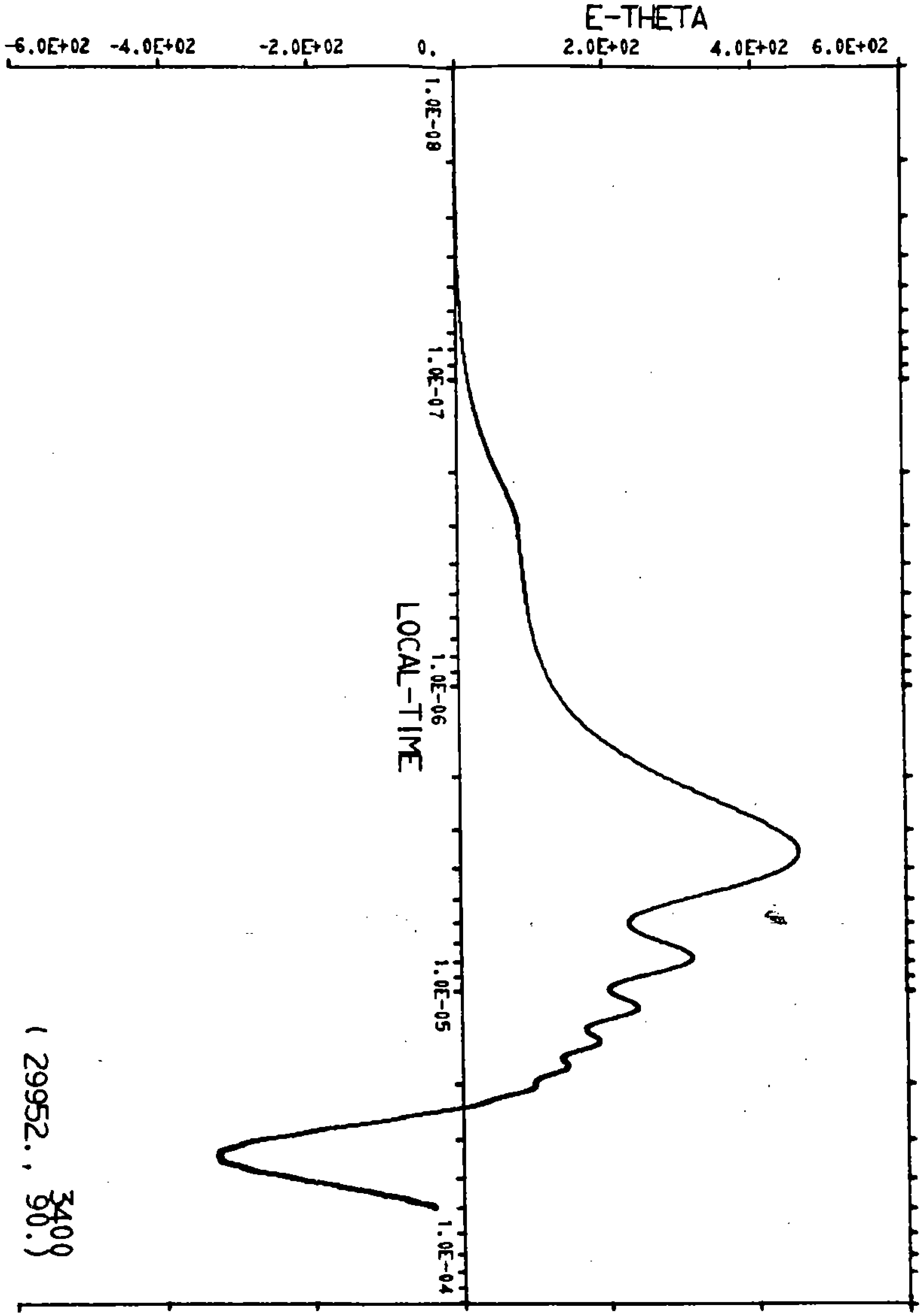


Fig. 7--E_θ Graph (Linear-log)

APPENDIX A

GREEN'S FUNCTION COEFFICIENTS
(COMPUTER LISTING)

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
1		1.154700538E+00	0.
2	2.459477531E-01	1.216726137E-01	-2.459477531E-01
		2.459477531E-01	0.
3		1.250481260E-01	3.336973334E-02
		-2.357121774E-02	-3.336973334E-02
4	1.750733791E-02	1.264723817E-02	-1.795235275E-02
		-4.381193554E-03	4.450148329E-04
5		5.854831311E-03	1.955108009E-03
		-1.422342446E-03	-2.293544435E-03
		-1.522683925E-04	3.384364262E-04
6	5.610986921E-04	5.032619124E-04	-6.136818310E-04
		-2.559632734E-04	3.167221423E-05
		1.483961877E-05	2.091092460E-05
7		1.396937552E-04	5.137458704E-05
		-3.745467255E-05	-7.025416450E-05
		-6.489338731E-06	1.900164063E-05
		1.495148318E-06	-1.220631667E-07
8	1.014423883E-05	1.027253811E-05	-1.149788654E-05
		-6.496285499E-06	4.820788491E-07
		9.203881178E-07	9.397943057E-07
		3.362813415E-08	-6.822543642E-08
9		2.011344840E-06	7.809576674E-07
		-5.617771988E-07	-1.176465636E-06
		-1.572020373E-07	4.201240051E-07
		7.100580499E-08	-2.175298317E-08
		-1.837343304E-09	-2.863053277E-09
10	1.174971877E-07	1.285288278E-07	-1.361649484E-07
		-9.275785790E-08	3.134787774E-09
		1.847942247E-08	1.914324475E-08
		6.911866394E-10	-3.613798074E-09
		-1.360198879E-10	3.526160692E-12

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
11		1.935537968E-08	7.782037385E-09
		-5.508427439E-09	-1.252151805E-09
		-2.167540414E-09	5.262871471E-09
		1.316976252E-09	-4.253537204E-10
		-1.259618143E-10	-1.023060727E-10
		-2.447876133E-12	4.268992532E-12
12	9.455415045E-10	1.090996795E-09	-1.112377194E-09
		-8.608565013E-10	2.569915833E-12
		2.075000265E-10	2.249641843E-10
		9.661797697E-12	-6.329009602E-11
		-5.961571389E-12	2.449322986E-12
		7.814045132E-14	1.423628390E-13
13		1.332203948E-10	5.488253674E-11
		-3.824698667E-11	-9.261737099E-11
		-1.930072471E-11	4.339519566E-11
		1.397421099E-11	-4.338124900E-12
		-2.164655096E-12	-1.553682975E-12
		-2.642040693E-14	2.312121472E-13
		4.921821812E-15	2.343181201E-14
14	5.591806104E-12	6.709128913E-12	-6.650125906E-12
		-5.646908368E-12	-1.177488958E-13
		1.539699500E-12	1.747979616E-12
		9.444466211E-14	-6.219893675E-13
		-8.973979921E-14	4.570981276E-14
		6.260333255E-15	4.482946642E-15
		7.792272016E-17	-1.143097157E-16
15		6.882519198E-13	2.986767378E-13
		-1.983366404E-13	-5.050411823E-13
		-1.208735098E-13	2.558490667E-13
		9.859865150E-14	-2.881846973E-14
		-2.014073647E-14	-1.404752564E-14
		-3.938680015E-17	3.484394083E-15
		2.128277876E-16	-9.972202772E-17

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
		-1.475875121E-18	-3.298933076E-18
16	2.532249605E-14	3.130306067E-14	-3.036195305E-14
		-2.766911750E-14	-1.099317716E-15
		8.226601011E-15	9.732490644E-15
		6.554518440E-16	-4.058807770E-15
		-7.706312548E-16	4.186323765E-16
		9.760710785E-17	5.310861987E-17
		5.058570337E-19	-6.637533367E-18
		-8.749930853E-20	-1.162194363E-20
17		2.766960881E-15	1.176691097E-15
		-7.982457263E-16	-2.117556238E-15
		-5.640508052E-16	1.137921305E-15
		5.022401228E-16	-1.377007933E-16
		-1.233359933E-16	-8.629634139E-17
		8.184798724E-19	2.884178407E-17
		2.745034803E-18	-1.804119693E-18
		-1.458029221E-19	-9.826048277E-20
		-1.281460758E-21	1.566660576E-21
18	9.061513704E-17	1.146860834E-16	-1.093421916E-16
		-1.053458407E-16	-5.795706014E-18
		3.339753762E-17	4.095405168E-17
		3.338334137E-18	-1.917573825E-17
		-4.443057363E-18	2.453631919E-18
		7.937378860E-19	3.789795671E-19
		-7.842450010E-21	-9.013065951E-20
		-3.900685114E-21	1.924947921E-21
		1.420964995E-23	4.136166887E-23
19		8.902080304E-18	3.827386986E-18
		-2.567289781E-18	-7.045826507E-18
		-2.043579730E-18	3.965367883E-18
		1.946986463E-18	-5.015429409E-19
		-5.460800900E-19	-3.878592947E-19
		6.984869630E-21	1.583644725E-19
		1.991594317E-20	-1.506003481E-20

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
		-2.137597346E-21	-9.317465793E-22
		-6.377146399E-24	1.009846812E-22
		8.711115867E-25	1.984685752E-25
20	2.626713742E-19	3.388441064E-19	-3.185758675E-19
		-3.210520846E-19	-2.171310909E-20
		1.069424222E-19	1.352796693E-19
		1.295073245E-20	-6.922083957E-20
		-1.870136934E-20	1.029423371E-20
		4.207384067E-21	1.882710831E-21
		-1.094486476E-22	-6.530646114E-22
		-4.464702403E-23	3.457568078E-23
		1.849806260E-24	1.229342789E-24
		1.216713085E-26	-1.230897579E-26
21		2.3423051E-20	1.0160007E-20
		-6.7481639E-21	-1.9055984E-20
		-5.9230187E-21	1.1132139E-20
		5.9572332E-21	-1.4473264E-21
		-1.8487103E-21	-1.3380253E-21
		3.1012924E-23	6.3201861E-22
		9.7259075E-23	-7.8902799E-23
		-1.5689975E-23	-5.1784384E-24
		2.0009426E-25	1.2726940E-24
		4.1326785E-26	-2.0449082E-26
		-7.4387430E-29	-3.0974797E-28
22	6.2927690E-22	8.2463349E-22	-7.6640316E-22
		-8.0156692E-22	-6.2679358E-23
		2.7769934E-22	3.6088742E-22
		3.9535967E-23	-1.9815465E-22
		-6.0548529E-23	3.2932150E-23
		1.6109640E-23	6.9929579E-24
		-6.6384889E-26	-3.1296348E-24
		-2.8063795E-25	2.6873725E-25
		2.6149045E-26	9.5507194E-27
		6.1609512E-29	-9.0773922E-28
		-5.3088760E-30	-1.7681020E-30

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
23		5,0930867E-23	2.2241778E-23
		-1,4676576E-23	-4.2383854E-23
		-1,3929387E-23	2.5551975E-23
		1,4669249E-23	-3.3965325E-24
		-4,9390785E-24	-3.6316163E-24
		9,7987070E-26	1.9226551E-24
		3,4545199E-25	-2.9246964E-25
		-7,3571241E-26	-1.9775494E-26
		2,3435406E-27	8.2206720E-27
		4,2517965E-28	-3.7159768E-28
		-1,4059579E-29	-9.4004453E-30
		-7,1108083E-32	5.9971371E-32
24	1,3314010E-24	1,7716307E-24	-1.6231000E-24
		-1,7548774E-24	-1.6115201E-25
		6,2204478E-25	8.3982214E-25
		1,0712720E-25	-4.8483543E-25
		-1,6529793E-25	8.5947483E-26
		4,9361786E-26	2.2053054E-26
		-2,5235337E-27	-1.1493455E-26
		-1,2720268E-27	1.3223388E-27
		1,8494371E-28	4.6191497E-29
		-2,0138213E-30	-1.1452644E-29
		-2,9273460E-31	1.2767719E-31
		1,4551254E-34	1.5613068E-33
25		8,0421360E-26	3.4874602E-26
		-2,4085912E-26	-6.7781091E-26
		-2,2489131E-26	4.2739134E-26
		2,5535734E-26	-6.5453013E-27
		-9,5652898E-27	-6.5275031E-27
		4,1296588E-28	4.0119770E-27
		7,7665402E-28	-7.6459869E-28
		-2,1887878E-28	-3.3869572E-29
		1,3711667E-29	2.9260769E-29
		1,7983372E-30	-2.5814010E-30
		-1,7683279E-31	-3.3467438E-32

GREENS FUNCTION COEFFICIENTS

ORDER	ALPHA	BETA	GAMMA
		2,4209175E-34	4.6242906E-33
		2,0343775E-35	5.5387262E-36
26	2.0780419E-27	2,7888654E-27	-2.5486191E-27
		-2,8216855E-27	-2.6442188E-28
		1,0383213E-27	1.4051110E-27
		1,8801048E-28	-8.5838111E-28
		-3,1506805E-28	1.6707534E-28
		1,0583406E-28	4.4865899E-29
		-7,2192198E-30	-2.7653745E-29
		-3,4861888E-30	4.1451423E-30
		7,2476444E-31	9.5246695E-32
		-2,3665123E-32	-6.1225321E-32
		-2,4863615E-33	2.3826213E-33
		6,6527081E-35	4.5057985E-35
		2,5966362E-37	-1.8752383E-37
27		5,2830400E-29	2.0403048E-29
		-2,1272595E-29	-4.2659276E-29
		-9,4599457E-30	3.2319552E-29
		1,7166902E-29	-9.9667505E-30
		-9,3702814E-30	-2.4106139E-30
		1,9066259E-30	3.3448310E-30
		3,7708844E-31	-1.1698772E-30
		-2,8398872E-31	1.1773219E-31
		5,0618082E-32	2.8867922E-32
		-6,5913894E-34	-7.9485023E-33
		-6,0162594E-34	4.1579149E-34
		3,3435882E-35	2.0962670E-35
		3,3098079E-37	-8.9641428E-37
		-2,9675447E-39	-2.7182993E-39
28	2.9180249E-30	3,8981702E-30	-3.6565422E-30
		-4,0917065E-30	-2.7458589E-31
		1,6463323E-30	2.0479444E-30
		2,2169287E-31	-1.3803127E-30
		-5,1828091E-31	3.2611637E-31
		2,0800595E-31	6.5794238E-32

GREENS FUNCTION COEFFICIENTS

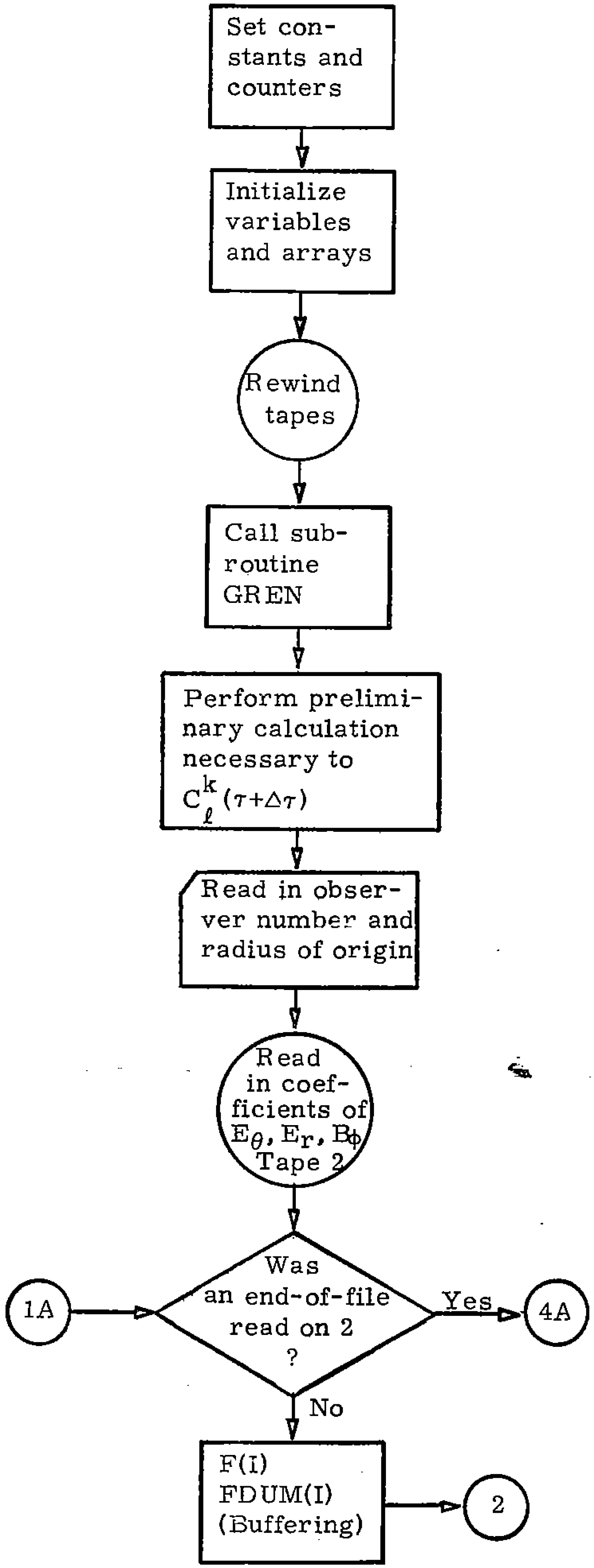
ORDER	ALPHA	BETA	GAMMA
		-2,2648532E-32	-5.7899222E-32
		-7,2291111E-33	1.1808777E-32
		2,3276008E-33	-1.3786837E-34
		-1,6296881E-34	-2.3122671E-34
		-1,0525636E-35	2.0361196E-35
		1,0807367E-36	1.1988876E-37
		-1,3028324E-39	-2.2863136E-38
		-7,4893804E-41	-2.9813439E-41
29		7,5753441E-33	-1.3395731E-33
		-1,2042670E-32	-1.8221419E-33
		9,1326069E-33	9.9409797E-33
		3,6425968E-34	-1.1038637E-32
		-5,2755589E-33	4.5978810E-33
		3,4644725E-33	4.7050294E-34
		-7,2824744E-34	-1.1726077E-33
		-1,4827622E-34	4.0046755E-34
		9,7979848E-35	-2.7146691E-35
		-1,3602044E-35	-1.1921527E-35
		-4,4686338E-37	2.2519684E-36
		1,8708971E-37	-4.7710591E-38
		-5,1662968E-39	-7.9036241E-39
		-1,5523285E-40	1.3210138E-40
		1,5954713E-43	7.1611665E-43
30	-4,6562108E-34	-7,8419428E-34	4.1170547E-34
		6,3076686E-34	4.1172939E-34
		1,8752187E-35	-5.5832686E-34
		-3,0208323E-34	1.8202266E-34
		1,6895392E-34	7.7947019E-35
		-1,3894387E-35	-7.9604181E-35
		-2,0312298E-35	1.9931698E-35
		7,8172020E-36	1.6149770E-36
		-6,2171894E-37	-1.6266746E-36
		-1,7775071E-37	2.2386559E-37
		3,3046735E-38	6.2132525E-39
		-5,9228805E-40	-2.5587219E-39
		-1,0292373E-40	6.0303407E-41

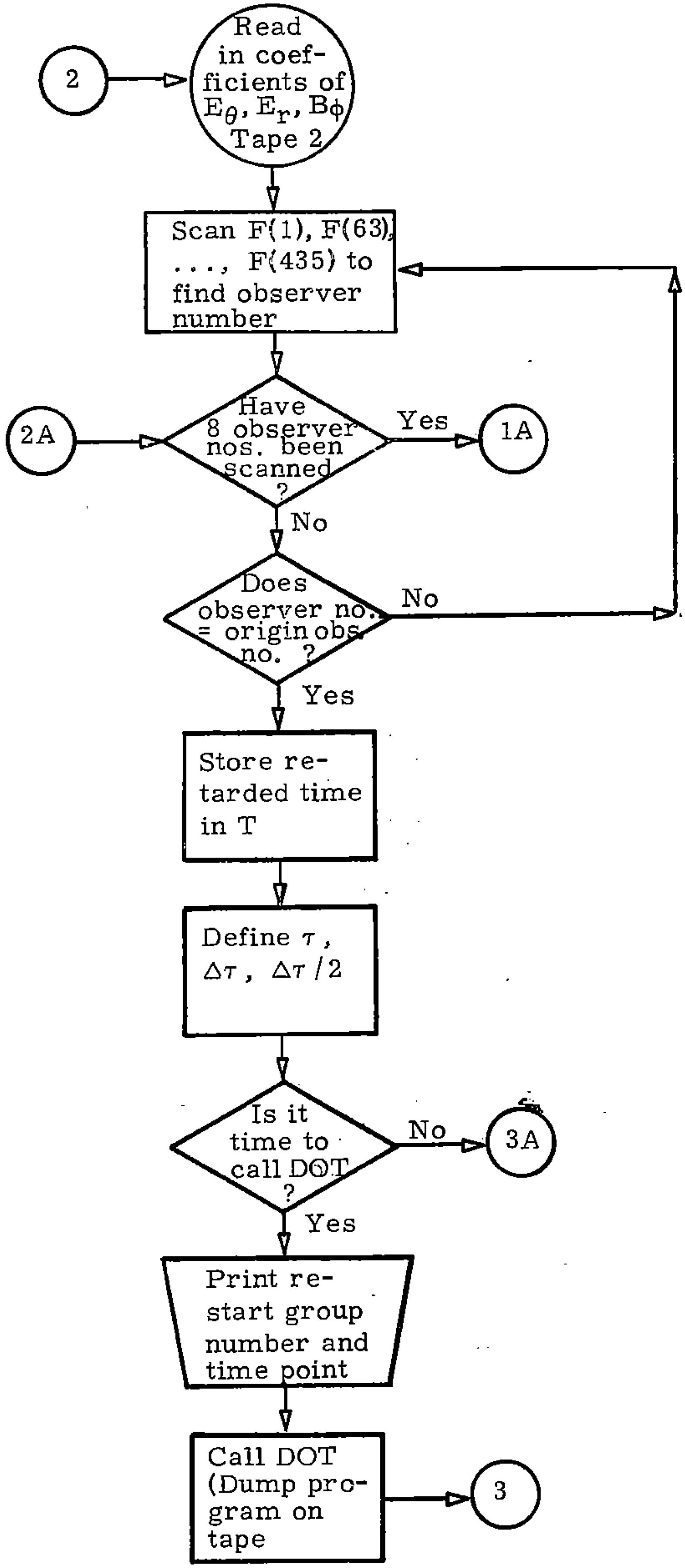
GREENS FUNCTION COEFFICIENTS

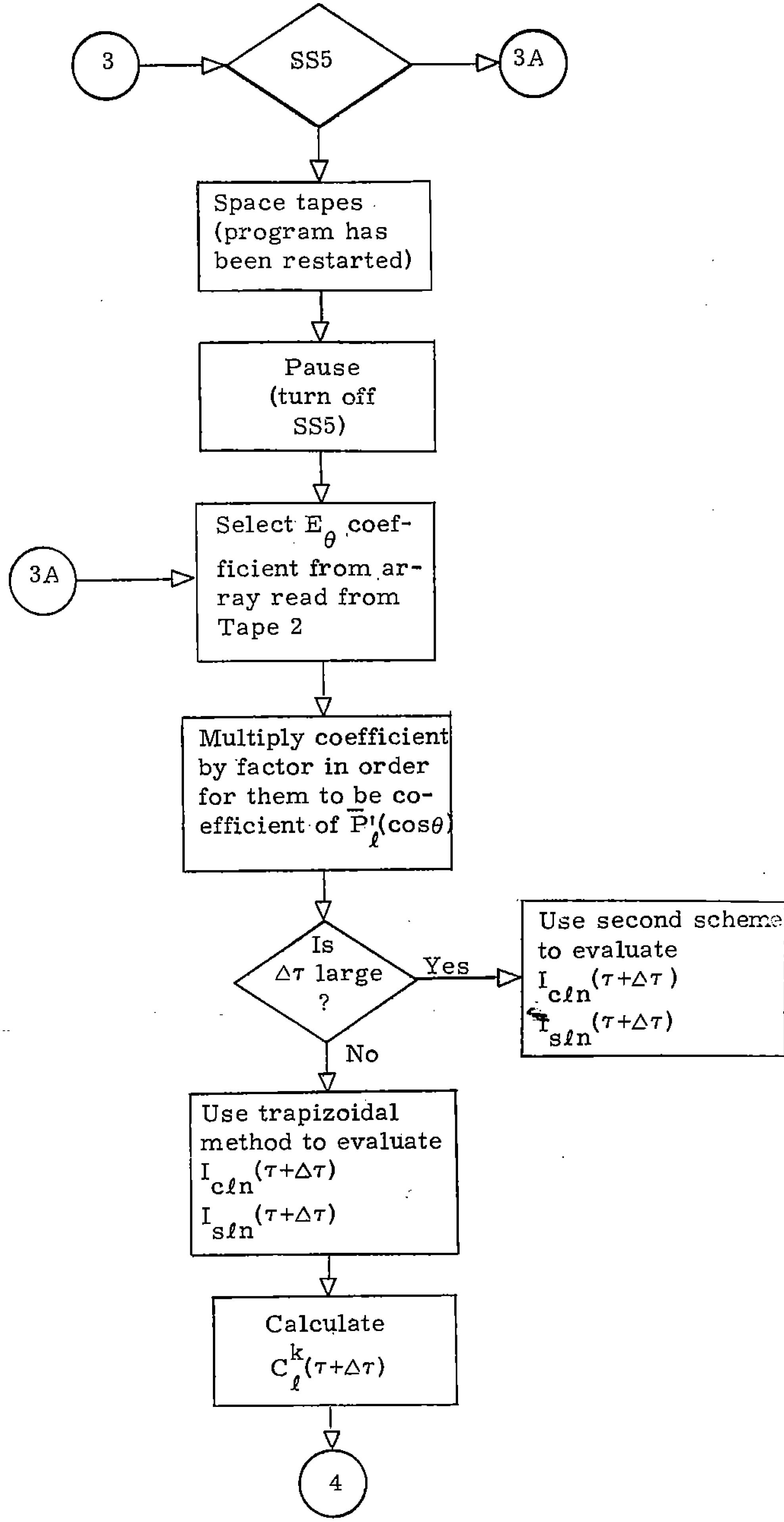
ORDER	ALPHA	BETA	GAMMA
		1.4137792E-42	1.9189850E-42
		7.7403594E-45	-1.0538474E-45

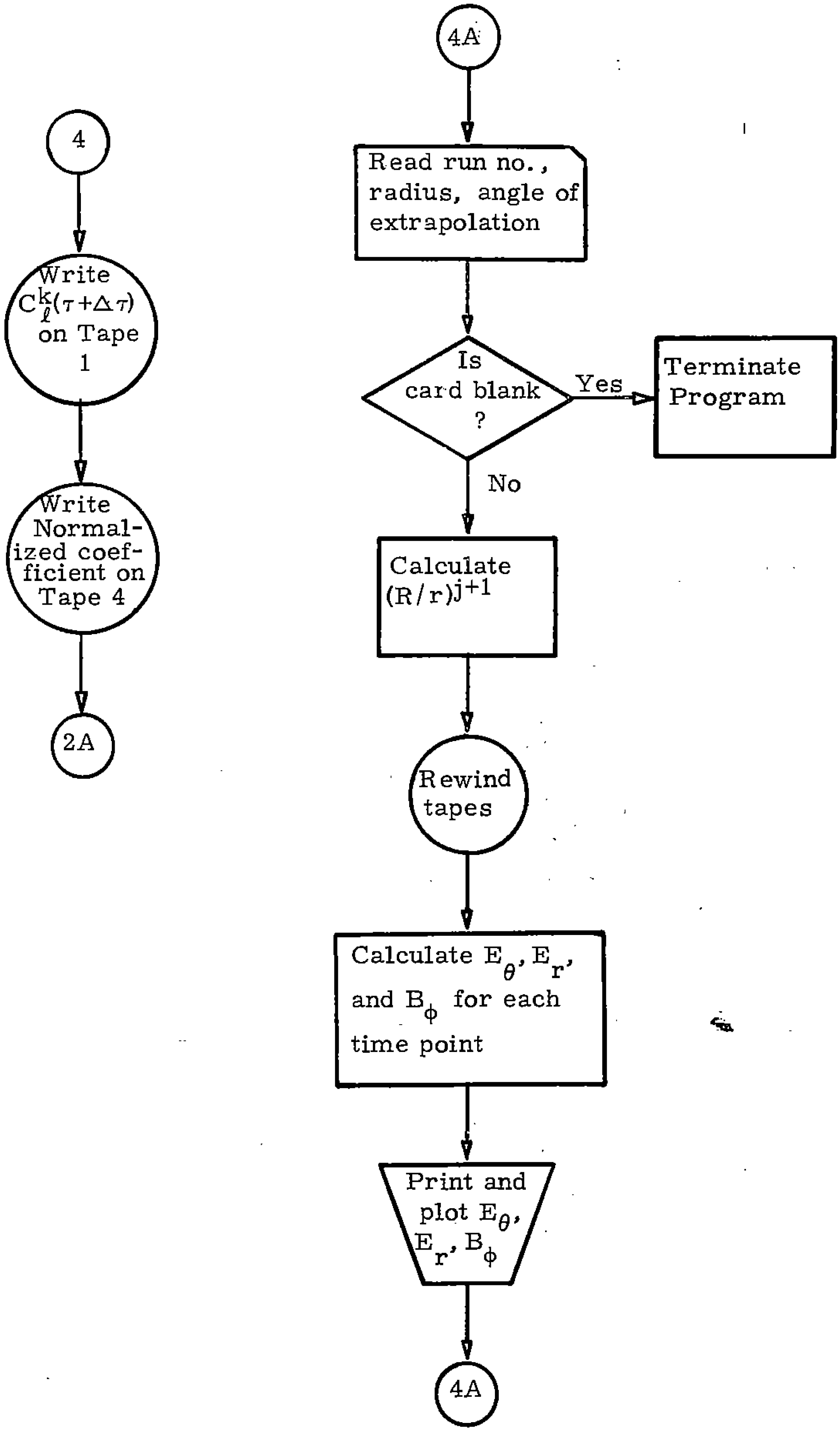
APPENDIX B

FLOW CHARTS FOR PROGRAM M

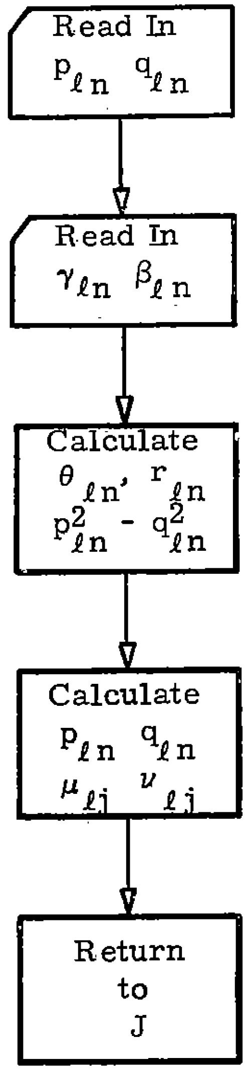

Set constants and counters
Initialize variables and arrays
Rewind tapes
Call subroutine GREN
Perform preliminary calculation necessary to $C_\ell^k(\tau+\Delta\tau)$
Read in observer number and radius of origin
Read in coefficients of E_θ, E_r, B_ϕ Tape 2
Was an end-of-file read on 2 ?
1A
Yes
4A
No
F(I) FDUM(I) (Buffering)
2


2
Read in coef- ficients of E_θ, E_r, B_ϕ Tape 2
Scan F(1), F(63), . . . , F(435) to find observer number
2A
Have 8 observer nos. been scanned ?
Yes
1A
No
Does observer no. = origin obs. no. ?
No
Yes
Store re- tarded time in T
Define τ, $\Delta\tau$, $\Delta\tau/2$
Is it time to call DOT ?
No
3A
Yes
Print re- start group number and time point
Call DOT (Dump pro- gram on tape
3

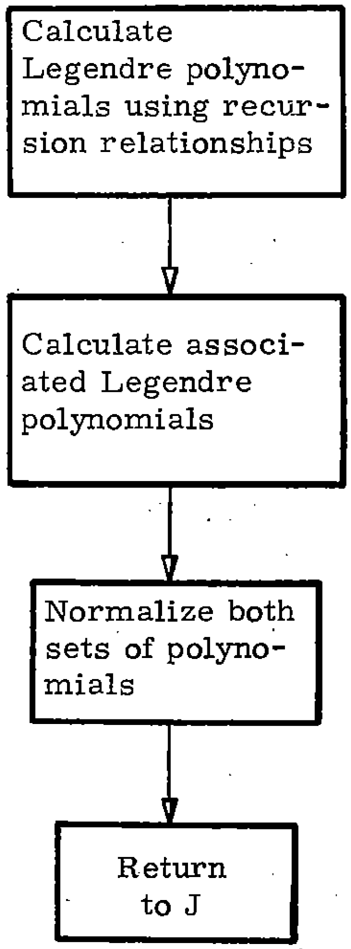

3
SS5
3A
Space tapes (program has been restarted)
Pause (turn off SS5)
3A
Select E_θ coefficient from array read from Tape 2
Multiply coefficient by factor in order for them to be coefficient of $\bar{P}'_\ell(\cos\theta)$
Is $\Delta\tau$ large ?
Yes
Use second scheme to evaluate $I_{c\ell n}(\tau+\Delta\tau)$ $I_{s\ell n}(\tau+\Delta\tau)$
No
Use trapizoidal method to evaluate $I_{c\ell n}(\tau+\Delta\tau)$ $I_{s\ell n}(\tau+\Delta\tau)$
Calculate $C_\ell^k(\tau+\Delta\tau)$
4


4
Write $C_\ell^k(\tau+\Delta\tau)$ on Tape 1
Write Normalized coefficient on Tape 4
2A
4A
Read run no., radius, angle of extrapolation
Is card blank ?
Yes
Terminate Program
No
Calculate $(R/r)^{j+1}$
Rewind tapes
Calculate E_θ, E_r, and B_ϕ for each time point
Print and plot E_θ, E_r, B_ϕ
4A

Subroutine GREN



Subroutine LEG



APPENDIX C

GLOSSARY FOR PROGRAM M

Variable	Description
ANG	angle, in degrees, of the extrapolated fields
B	E_θ coefficients of normalized Legendre polynomials, $B_\ell(\tau+\Delta\tau)$
BETA	$\beta_{\ell m}$
BLAST	$B_\ell(\tau)$
CEN	$I_{c\ell n}(\tau+\Delta\tau)$
CK	$C_\ell^k(\tau+\Delta\tau)$
DEL	$\Delta\tau/2$
DT	$\Delta\tau$
E	E_θ field values
ER	E_r field values
EB	B_ϕ field values
F	input coefficient array
GAMMA	$\gamma_{\ell n}$
LIMIT	number of time points, including time=0
LLL	number of Legendre polynomials used
NOBS	observer number of origin of extrapolation
NNJ	run number of extrapolated fields
P	$p_{\ell n}$
PI	π
PCEN	$I_{c\ell n}(\tau)$
PMQ	$p_{\ell n}^2 - q_{\ell n}^2$
PN	$\overline{P}_\ell$

Variable	Description
PQ	$p_{\ell n} q_{\ell n}$
P1	$\bar{P}_{\ell}^{1}$
Q	$q_{\ell n}$
R	radius of origin of extrapolation
RDUN	1, if radiation term is to be plotted; 0, otherwise
ROC	radius (R)/speed of light
ROR	(radius of origin/radius of extrapolation)$^{j+1}$
RL	radius of extrapolation (r)
RR	$(p_{\ell n}^2 + q_{\ell n}^2)^{1/2} = r_{\ell n}$
SEN	$I_{s\ell n}(\tau + \Delta\tau)$
SM	criterion for determining which integration scheme to use
SUM1, SUMR	partial sums for E_r
SUM2, SUME	partial sums for E_θ
SUME3, SUMB	partial sums for B_ϕ
T	retarded time in seconds
TAU	dimensionless time (τ)
THE	$\pi - \sin^{-1}(q_{\ell n}/r_{\ell n}) = \theta_{\ell n}$
THETA	ANG expressed in radians
U	$\mu_{\ell j}$
V	$\gamma_{\ell j}$
XT, XDOT	time increments for restarting

APPENDIX D

LISTING FOR PROGRAM M

```
      PROGRAM M(INPUT,FILMPR,OUTPUT=FILMPR,TAPE1,TAPE2,TAPE4,TAPE8,FILMP
     1L)
      DIMENSION CEN(220), SEN(220), PCEN(220), PN(45), P1(45), P(220), Q
     1(220), BETA(220), GAMMA(220), F(496), CK(450), B(20), B_AST(20), T
     2(5000), THE(220), RR(220), U(500), V(500), ROR(45), E(5000), ER(50
     300), EB(5000), PQ(220), PMQ(220)
      DIMENSION XC(6000), XS(6000), RK(6000), ZZ(20)
      DOUBLE PRECISION TH,XSIN,XCOS,SUM,SUME,SUMR,SUMB,SUM1,SUM2,SUM3,PN
     1,P1
      DIMENSION FDUM(496), CKDM(450), BDUM(20)
      DOUBLE PRECISION ROR,RZ
C     SET CONSTANTS AND COUNTERS
      PI=3.14159265358979З
      C=300000000.
      LLL=20
      SM=.1
      XT=900.
      XDOT=900.
      IRCF=0
      IRCC=0
      ISTRT=0
      PG=1.
C     INITIALIZE VARIABLES AND ARRAYS
      T(1)=0.
      TPRM=0.
      E(1)=0.
      ER(1)=0.
      EB(1)=0.
      DO 10 I=1,220
      CEN(I)=0.
10    SEN(I)=0.
      DO 20 I=1,20
20    BLAST(I)=0.
C     REWIND TAPES
      REWIND 1
      REWIND 2
      REWIND 4
      REWIND 8
C     PERFORM PRELIMINARY CALCULATIONS NECESSARY TO C
      CALL GREN (P,Q,BETA,GAMMA,PMQ,RR,THE,PQ,U,V)
      IN=1
      KN=0
      KP=0
      DO 50 I=1,LLL
      L=I*2-1
      LMT=L+2
      X=L
      ZZ(I)=SQRT(X*(X+1.))
      DO 40 K=1,LMT
      XK=K-1
      DO 30 N=1,I
      TH=THE(KP+N)*XK
      XCOS=DCOS(TH)
      XSIN=DSIN(TH)
      RZ=RR(KP+N)**(K-1)
      KN=KN+1
      XC(KN)=XCOS
      XS(KN)=XSIN
      RK(KN)=RZ
30    CONTINUE
40    CONTINUE
```

```
      KP=KP+I
50    CONTINUE
C     READ IN OBSERVER NUMBER AND RADIUS OF ORGIN
      READ 400, NOBS,R,RDUM
      ROC=R/C
C     READ IN COEFF OF ETHETA,ERADIAL,BPHI FROM TAPE 2
      READ (2) FDUM
C     THIS IS POINT 1A
60    CONTINUE
C     WAS AN END-OF-FILE READ ON TAPE 2
C     IF YES GO TO 250 (4A)    IF NO CONTINUE WITH 70
      IF (EOF,2) 250,70
70    CONTINUE
C     BUFFER IN THE INPUT COEFFICIENTS
      DO 80 I=1,496
80    F(I)=FDUM(I)
C     THIS IS POINT 2
C     READ IN COEFFICIENTS OF ETHETA,ERADIAL,BPHI FROM TAPE 2
      READ (2) FDUM
      IRCF=IRCF+1
C     SCAN F(1),F(63),....,F(435) TO FIND OBSERVER NO.
      DO 240 II=1,8
      J=62*(II-1)+1
      N=F(J)
C     DOES OBSERVER NO. EQUAL ORGIN OBSERVER NO.
C     IF THE OBSERVER NO IS CORRECT  THEN CALCULATIONS CAN BEGIN  90
C     IF NOT GO TO END OF DO-LOOP AND CONTINUE WITH LOOP
      IF (N-NOBS) 240,90,240
90    CONTINUE
      CALL SECOND(CPTM)
      IF(CPTM-180.)1722,1722,250
1722  CONTINUE
C     STORE RETARDED TIME IN T
      IN=IN+1
      T(IN)=F(J+1)
C     DEFINE TAU,DELTA TAU, AND ONE-HALF DETLA TAU
      TAU=F(J+1)/ROC
      DT=TAJ-TPRM
      DEL=DT*.5
C     IS IT TIME TO CALL DOT
C     IF IT IS TIME GO TO 100
      IF (CPTM-XT) 150,150,100
100   CONTINUE
      XT=XT+XDOT
      ISTRT=ISTRT+1
C     PRINT RESTART GROUP NO. AND TIME POINT
      PRINT 410, ISTRT,T(IN)
C     CALL DOT(DUMP PROGRAM ON TAPE)
      CALL DOT (8)
C     THIS IS POINT 3
C     IS SENSE SWITCH 5 ON OR OFF
C     IF THE SS IS OFF CONTINUE AT 140
C     IF SS 5 IS ON GO TO 110  TO SPACE TAPES AND CONTINUE WITH CALC.
      IF (SENSE SWITCH 5)110,140
C     SPACE TAPES  (PROGRAM HAS BEEN RESTARTED)
110   REWIND 1
      REWIND 2
      REWIND 4
      DO 120 IJL=1,IRCF
      READ (2) F
120   CONTINUE
      READ (2) FDUM
```

```
      DO 130 IJL=1,IRCC
      READ (1) CK
      READ (4) B
  130 CONTINUE
      CALL SECOND(CPTM)
      XT=CPTM+XDOT
C     PAUSE IN ORDER FOR SS 5 TO BE TURNED OFF
      PAUSE 5
  140 CONTINUE
C     THIS IS POINT 3A
  150 CONTINUE
      J=J+22
      IP=0
      KN=0
      KZ=0
      DO 210 KK=1,LLL
      KP=KZ
      L=2*KK-1
      XL=L
      FF=SQRT((2.*XL+1.)/(2.*XL*(XL+1.)))
C     SELECT ETHETA COEFF. FROM ARRAY READ FROM TAPE 2
C     MULT. COEFF. BY FACTOR IN ORDER FOR THEM TO BE COFFF OF NORMALIZED
C     ASSOCIATED LEGENDRE POLYNOMINALS
      B(KK)=F(J+KK-1)/FF
      DO 180 JJ=1,KK
      KZ=KZ+1
      PCEN(KZ)=CEN(KZ)
      XE=EXP(P(KZ)*DT)
      XA=Q(KZ)*DT
C     IS DELTA TAU LARGE OR SMALL
C     IF LARGE GO TO 160   IF SMALL GO TO 170
      IF (DI*RR(KZ)-SM) 170,170,160
  160 CONTINUE
C     DELTA TAU IS LARGE
C     USE SECOND SCHEME TO EVALUATE ICLN AND ISLN
      CEN(KZ)=XE*(CEN(KZ)*COS(XA)-SEN(KZ)*SIN(XA))
      R01=1./(RR(KZ)*RR(KZ))
      G=(B(KK)-BLAST(KK))/DT
      CEN(KZ)=CEN(KZ)-R01*((1.-XE*COS(XA))*(P(KZ)*BLAST(KK)+G*R01*PMQ(KZ
     1))-XE*SIN(XA)*(BLAST(KK)*Q(KZ)+2.*G*PQ(KZ)*R01)+G*DT*P(KZ))
      SEN(KZ)=XE*(PCEN(KZ)*SIN(XA)+SEN(KZ)*COS(XA))+R01*((1.-XE*COS(XA))
     1*(Q(KZ)*BLAST(KK)+2.*G*PQ(KZ)*R01)+XE*SIN(XA)*(P(KZ)*BLAST(KK)+G*R
     201*PMQ(KZ))+G*DT*Q(KZ))
      GO TO 180
  170 CONTINUE
C     DELTA TAU IS SMALL
C     USE TRAPIZODIAL METHOD TO EVALUATE ISLN AND ICLN
      CEN(KZ)=XE*((CEN(KZ)+DEL*BLAST(KK))*COS(XA)-SEN(KZ)*SIN(XA))+DEL*B
     1(KK)
      SEN(KZ)=XE*((PCEN(KZ)+DEL*BLAST(KK))*SIN(XA)+SEN(KZ)*COS(XA))
  180 CONTINUE
      LMT=L+2
C     CALCULATE  THE C QUANTITIES
      DO 200 K=1,LMT
      SUM=0.
      DO 190 N=1,KK
      KN=KN+1
      SUM=SUM+RK(KN)*((BETA(KP+N)*XS(KN)+GAMMA(KP+N)*XC(KN))*CEN(KP+N)+(
     1BETA(KP+N)*XC(KN)-GAMMA(KP+N)*XS(KN))*SEN(KP+N))
  190 CONTINUE
C     THIS IS POINT 4
      IP=IP+1
```

```
      CK(IP)=SUM
200   CONTINUE
210   CONTINUE
      DO 220 IJ=1,20
      BDUM(IJ)=B(IJ)
220   BLAST(IJ)=B(IJ)
      DO 230 IJ=1,450
230   CKDM(IJ)=CK(IJ)
C     WRITE  THE C QUANTITIES ON TAPE 1
      WRITE (1) CKDM
C     WRITE THE NORMALIZED COEFF. OF ETHETA ON TAPE 4
      WRITE (4) BDUM
      IRCC=IRCC+1
      TPRM=TAU
C     THIS IS POINT 2A
C     HAVE 8 OBSERVER NOS. BEEN SCANNED
240   CONTINUE
C     IF YES THEN GO BACK TO 1A TO READ IN MORE COEFF
      GO TO 60
C     THIS IS POINT 4A
250   LIMIT=IN-1
      WRITE (1) CKDM
      WRITE (4) BDUM
C     READ RUN NO. RADIUS,ANGLE OF EXTRAPOLATION
260   READ 400, NOBS,RL,ANG
      NNJ=(NOBS/100)*100
      THETA=(ANG/180.)*PI
C     IS CARD BLANK
      IF (RL) 390,390,270
270   CONTINUE
      NJK=LLL*2+4
      RO=R/RL
      CALL LEG (NJK,PN,P1,THETA)
C     CALCULATE THE RATIO OF RADII TO THE J+1 POWER
      DO 280 I=1,NJK
280   ROR(I)=RO**(I)
      DO 281 I=1,6000
281   RK(I)=0.
      REWIND 1
      REWIND 2
      REWIND 4
      READ (4) BDUM
      READ (1) CKDM
C     CALCULATE EXTRAPOLATED ETHETA,ERADIAL, AND BPHI FOR EACH TIME PT.
      DO 340 IIJ=1,LIMIT
      IP=0
      KZ=0
      DO 290 IJL=1,450
290   CK(IJL)=CKDM(IJL)
      DO 300 IJL=1,20
300   B(IJL)=BDUM(IJL)
      READ (1) CKDM
      READ (4) BDUM
      SUM4=0.
      SUM1=0.
      SUM2=0.
      SUM3=0.
      DO 330 M=1,LLL
      L=2*M-1
      L1=L+2
      IP=IP+L1
      SUME=0.
```

```
      DO 130 IJL=1,IRCC
      READ (1) CK
      READ (4) B
130   CONTINUE
      CALL SECOND(CPTM)
      XT=CPTM+XDOT
C     PAUSE IN ORDER FOR SS 5 TO BE TURNED OFF
      PAUSE 5
140   CONTINUE
C     THIS IS POINT 3A
150   CONTINUE
      J=J+22
      IP=0
      KN=0
      KZ=0
      DO 210 KK=1,LLL
      KP=KZ
      L=2*KK-1
      XL=L
      FF=SQRT((2.*XL+1.)/(2.*XL*(XL+1.)))
C     SELECT ETHETA COEFF. FROM ARRAY READ FROM TAPE 2
C     MULT. COEFF. BY FACTOR IN ORDER FOR THEM TO BE COEFF OF NORMALIZED
C     ASSOCIATED LEGENDRE POLYNOMINALS
      B(KK)=F(J+KK-1)/FF
      DO 180 JJ=1,KK
      KZ=KZ+1
      PCEN(KZ)=CEN(KZ)
      XE=EXP(P(KZ)*DT)
      XA=Q(KZ)*DT
C     IS DELTA TAU LARGE OR SMALL
C     IF LARGE GO TO 160   IF SMALL GO TO 170
      IF (DT*RR(KZ)-SM) 170,170,160
160   CONTINUE
C     DELTA TAU IS LARGE
C     USE SECOND SCHEME TO EVALUATE ICLN AND ISLN
      CEN(KZ)=XE*(CEN(KZ)*COS(XA)-SEN(KZ)*SIN(XA))
      R01=1./(RR(KZ)*RR(KZ))
      G=(B(KK)-BLAST(KK))/DT
      CEN(KZ)=CEN(KZ)-R01*((1.-XE*COS(XA))*(P(KZ)*BLAST(KK)+G*R01*PMQ(KZ
     1))-XE*SIN(XA)*(BLAST(KK)*Q(KZ)+2.*G*PQ(KZ)*R01)+G*DT*P(KZ))
      SEN(KZ)=XE*(PCEN(KZ)*SIN(XA)+SEN(KZ)*COS(XA))+R01*((1.-XE*COS(XA))
     1*(Q(KZ)*BLAST(KK)+2.*G*PQ(KZ)*R01)+XE*SIN(XA)*(P(KZ)*BLAST(KK)+G*R
     201*PMQ(KZ))+G*DT*Q(KZ))
      GO TO 180
170   CONTINUE
C     DELTA TAU IS SMALL
C     USE TRAPIZODIAL METHOD TO EVALUATE ISLN AND ICLN
      CEN(KZ)=XE*((CEN(KZ)+DEL*BLAST(KK))*COS(XA)-SEN(KZ)*SIN(XA))+DEL*B
     1(KK)
      SEN(KZ)=XE*((PCEN(KZ)+DEL*BLAST(KK))*SIN(XA)+SEN(KZ)*COS(XA))
180   CONTINUE
      LMT=L+2
C     CALCULATE  THE C QUANTITIES
      DO 200 K=1,LMT
      SJM=0.
      DO 190 N=1,KK
      KN=KN+1
      SJM=SJM+RK(KN)*((BETA(KP+N)*XS(KN)+GAMMA(KP+N)*XC(KN))*CEN(KP+N)+(
     1BETA(KP+N)*XC(KN)-GAMMA(KP+N)*XS(KN))*SEN(KP+N))
190   CONTINUE
C     THIS IS POINT 4
      IP=IP+1
```

```
      CK(IP)=SUM
200   CONTINUE
210   CONTINUE
      DO 220 IJ=1,20
      BDUM(IJ)=B(IJ)
220   BLAST(IJ)=B(IJ)
      DO 230 IJ=1,450
230   CKDM(IJ)=CK(IJ)
C     WRITE  THE C QUANTITIES ON TAPE 1
      WRITE (1) CKDM
C     WRITE THE NORMALIZED COEFF. OF ETHETA ON TAPE 4
      WRITE (4) BDUM
      IRCC=IRCC+1
      TPRO=TAU
C     THIS IS POINT 2A
C     HAVE 8 OBSERVER NOS. BEEN SCANNED
240   CONTINUE
C     IF YES THEN GO BACK TO 1A TO READ IN MORE COEFF
      GO TO 60
C     THIS IS POINT 4A
250   LIMIT=IN-1
      WRITE (1) CKDM
      WRITE (4) BDUM
C     READ RUN NO. RADIUS,ANGLE OF EXTRAPOLATION
260   READ 400, NOBS,RL,ANG
      NNJ=(NOBS/100)*100
      THETA=(ANG/180.)*PI
C     IS CARD BLANK
      IF (RL) 390,390,270
270   CONTINUE
      NJK=LLL*2+4
      RO=R/RL
      CALL LEG (NJK,PN,P1,THETA)
C     CALCULATE THE RATIO OF RADII TO THE J+1 POWER
      DO 280 I=1,NJK
280   ROR(I)=RO**(I)
      DO 281 I=1,6000
281   RK(I)=0.
      REWIND 1
      REWIND 2
      REWIND 4
      READ (4) BDUM
      READ (1) CKDM
C     CALCULATE EXTRAPOLATED ETHETA,ERADIAL, AND BPHI FOR EACH TIME PT.
      DO 340 IIJ=1,LIMIT
      IP=0
      KZ=0
      DO 290 IJL=1,450
290   CK(IJL)=CKDM(IJL)
      DO 300 IJL=1,20
300   B(IJL)=BDUM(IJL)
      READ (1) CKDM
      READ (4) BDUM
      SUM4=0.
      SUM1=0.
      SUM2=0.
      SUM3=0.
      DO 330 M=1,LLL
      L=2*M-1
      L1=L+2
      IP=IP+L1
      SUME=0.
```

```
      SUMR=0.
      SUMR=0.
      DO 320 JJ=1,L1
      L3=IP-JJ+1
      L4=IP-JJ
      KZ=KZ+1
      SUMF=SUME+V(KZ)*ROR(JJ)*CK(L3)
      IF (JJ-L1) 310,320,320
310   SUMR=J(KZ)*ROR(JJ+1)*CK(L4)+SUMR
      SUMR=J(KZ)*ROR(JJ)*CK(L3)+SUMR
      IF(RDJM)320,320,311
311   IF(JJ-1)312,312,320
312   SUMRD=SUMF
320   CONTINUE
      SUM1=SUM1-(SUMR*PN(L+1))*ZZ(M)
      SUM2=SUM2+RO*R(M)*P1(L+1)+SUME*P1(L+1)
      SUM3=SUM3+RO*R(M)*P1(L+1)+SUMR*P1(L+1)
      SUM4=SUM4+RO*R(M)*P1(L+1)+SUMRD*P1(L+1)
330   CONTINUE
      IT=IIJ+1
      E(IT)=SUM2/R
      EB(IT)=SUM3/(C*R)
      ER(IT)=SUM1
      RK(IT)=SUM4/R
340   CONTINUE
      IF(RDJM)345,345,341
341   DO 342  ILM=1,IN,50
      PRINT430 ,NNJ,PG
      PRINT440,RL,ANG
      PRINT450
      MM=ILM+49
      MJ=ILM
      PRINT460,(T(IIK),RK(IIK),IIK=MJ,MM)
450   FORMAT(1H0,36X*TIME*31X*RADIATION TERM*)
460   FORMAT(1H ,31X2(E14.7,20X))
342   CONTINUE
      DO 371 KK=1,4
      CALL GRAPH (KK,IN,4,4,10HLOCAL-TIME,10HRADIATION ,NNJ,T,RK,8.5,6.,
     13.,0.)
      CALL SYMBOL (6.6,.14,.14,8H(        .,0.,8)
      CALL SYMBOL (7.56,.14,.14,6H,    .),0.,6)
      CALL NUMBER (6.72,.14,.14,RL,0.,4HF6.0)
      CALL NUMBER (7.68,.14,.14,ANG,0.,4HF3.0)
371   CONTINUE
C     PRINT AND PLOT THE FIELDS
345   CONTINUE
      DO 350 ILM=1,IN,50
      CALL STPG (PG,NNJ,RL,ANG)
      MM=ILM+49
      MJ=ILM
      PRINT 420, (T(IIK),ER(IIK),E(IIK),EB(IIK),IIK=MJ,MM)
350   CONTINUE
      IF (IN-MM) 370,370,360
360   MM=MM+1
      CALL STPG (PG,NNJ,RL,ANG)
      PRINT 420, (T(IIK),ER(IIK),E(IIK),EB(IIK),IIK=MM,IN)
370   CONTINUE
      DO 380 KK=1,4
      CALL GRAPH (KK,IN,4,4,10HLOCAL-TIME,10HE-THETA    ,NNJ,T,E,8.5,6.,3
     1.,0.)
      CALL SYMBOL (6.6,.14,.14,8H(        .,0.,8)
      CALL SYMBOL (7.56,.14,.14,6H,    .),0.,6)
```

```
      CALL NUMBER (6.72,.14,.14,RL,0.,4HF6.0)                          J    278
      CALL NUMBER (7.68,.14,.14,ANG,0.,4HF3.0)                         J    279
      CALL GRAPH (KK,IN,4,4,10HLOCAL-TIME,10HE-R         ,NNJ,T,ER,8.5,6.,J    280
     13.,0.)                                                           J    281
      CALL SYMBOL (6.6,.14,.14,8H(        .,0.,8)                      J    282
      CALL SYMBOL (7.56,.14,.14,6H,    .),0.,6)                        J    283
      CALL NUMBER (6.72,.14,.14,RL,0.,4HF6.0)                          J    284
      CALL NUMBER (7.68,.14,.14,ANG,0.,4HF3.0)                         J    285
      CALL GRAPH (KK,IN,4,4,10HLOCAL-TIME,10HB-PHI       ,NNJ,T,ER,8.5,6.,J    286
     13.,0.)                                                           J    287
      CALL SYMBOL (6.6,.14,.14,8H(        .,0.,8)                      J    288
      CALL SYMBOL (7.56,.14,.14,6H,    .),0.,6)                        J    289
      CALL NUMBER (6.72,.14,.14,RL,0.,4HF6.0)                          J    290
      CALL NUMBER (7.68,.14,.14,ANG,0.,4HF3.0)                         J    291
                                                                       J    292
  380 CONTINUE                                                         J    293
      CALL SECOND (CPTM)                                               J    295
      GO TO 260                                                        J    296
  390 CONTINUE                                                         J    297
C     CARD WAS BLANK    TERMINATE THE PROGRAM                          J    298
C                                                                      J    299
  400 FORMAT (I10,2E10.3)
  410 FORMAT (1H ,*RESTART GROUP NO *I5,* IS COMPLETE   LAST TIME POINT
     1WAS*E10.3)                                                       J    301
  420 FORMAT (1H ,31X4(E14.7,6X))
  430 FORMAT (1H1,48X*RUN NUMBER*I5,9X*. . . PAGE *F5.0,*.*)
  440 FORMAT (48X*(*F6.0,*.,*F3.0,*.)*)                                J    302
      END
```

```
      SUBROUTINE LEG (NN,P,P1,THETA)
      DIMENSION P(45), P1(45)
      DOUBLE PRECISION P,P1
      P(1)=1.
      P(2)=COS(THETA)
      DO 10 I=3,NN
      XN=I-2
      P(I)=((2.*XN+1.)*COS(THETA)*P(I-1)-XN*P(I-2))/(XN+1.)
10    CONTINUE
      N=NN-1
      DO 20 I=1,N
      XN=I
      P1(I)=XN*(COS(THETA)*P(I)-P(I+1))/SIN(THETA)
20    CONTINUE
      P(1)=P(1)*(0.5)**.5
      DO 30 I=2,N
      XN=I-1
      FACTN=SQRT((2.*XN+1.)/2.)
      FACT=SQRT((2.*XN+1.)/(2.*XN*(XN+1.)))
      P(I)=P(I)*FACTN
      P1(I)=P1(I)*FACT
30    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GREN (P,Q,BETA,GAMMA,PMQ,RR,THE,PQ,U,V)
      DIMENSION U(500), V(500), P(220), Q(220), BETA(220), GAMMA(220), P
     1MQ(220), PQ(220), RR(220), THE(220)
      DOUBLE PRECISION ZM,FY,UX,VX,FJ
      PI=3.141592653589793
      KZ=0
      DO 30 I=1,20
      L=I*2-1
      M=(L+1)/2
      KX=KZ+1
      KY=KZ+M
      READ 120, (P(KP),Q(KP),KP=KX,KY)
      DO 10 KP=KX,KY
      READ 130, BETA(KP),GAMMA(KP)
10    CONTINUE
      DO 20 J=1,M
      KZ=KZ+1
      RR(KZ)=(P(KZ)*P(KZ)+Q(KZ)*Q(KZ))**.5
      THE(KZ)=PI-ASIN(Q(KZ)/RR(KZ))
      PQ(KZ)=P(KZ)*Q(KZ)
      PMQ(KZ)=P(KZ)*P(KZ)-Q(KZ)*Q(KZ)
20    CONTINUE
30    CONTINUE
      INIT=1
      KZ=0
      DO 110 I=1,20
      L=2*I-1
      L1=L+1
      L2=L+2
      X=L
      DO 100 J=1,L2
      Y=J
      ZM=1.
      DO 40 K=1,J
      SS=K
      ZM=ZM*(X+SS-1.)*(X-SS+2.)
40    CONTINUE
      IF (J-1) 50,50,60
50    FJ=1.
      GO TO 70
60    FJ=FJ*(Y-1.)
70    CONTINUE
      KZ=KZ+1
      UX=ZM/(X*(X+1.)*FJ*2.**(J-1))
      U(KZ)=UX
      IF (L-(J-1)) 80,90,90
80    V(KZ)=V(KZ-1)
      GO TO 100
90    CONTINUE
      VX=U(KZ)*(X*(X+1.)+(Y-1.)*(Y-2.))/(X*(X+1.)-(Y-1.)*(Y-2.))
      V(KZ)=VX
100   CONTINUE
110   CONTINUE
      RETURN
C
120   FORMAT (9XF13.0,7XF13.0)
130   FORMAT (2E22.15)
      END
```

```
      SUBROUTINE STPG (PG,ING,RL,ANG)
      PRINT 10, ING,PG
      PRINT 20, RL,ANG
      PRINT 30
      PG=PG+1.
      RETURN
C
10    FORMAT (1H1,48X*RUN NUMBER*I5,9X*. . . PAGE *F5.0,*.*)
20    FORMAT (48X*(*F6.0,*.,*F3.0,*.)*)
30    FORMAT (1H0,36X*TIME*15X*ERADIAL*13X*ETHETA*15X*BPHI*)
      END
```

```
      SUBROUTINE DOT (NUM1)                                             J
      DIMENSION L(1), NFILE(65), NFAD(65), NSSUBS(10)                   J
      DATA KRX,NSB,NSSUBS/1,9,3LDOT,6LIFENDF,5LGETBA,6LOUTPTB,6LINPUTR,4J
     1LLOCF,4LXRCL,6LENDFIL,5LUMERR/                                    J
      NUM=NUM1                                                          J
      GO TO (20,120,10), KRX                                            J
10    I=I*J*K*L*M*N*II*IJ*IK*IL*IM*IN*JI*JJ*JK*JL*JM*JN*KI*KJ*KK*KL*KM*KJ
      I=A*B*C*D*E*F*G*H*O*P*Q*S*R*T*A*B*C*D*EF*A*S*T*R*W*Q*X*Z*R*W*Y*L*GJ
      I=I*J*K*L*M*N*II*IJ*IK*IL*IM*IN*JI*JJ*JK*JL*JM*JN*KI*KJ*KK*KL*KM*KJ
      I=A*B*C*D*E*F*G*H*O*P*Q*S*R*T*A*B*C*D*EF*A*S*T*R*W*Q*X*Z*R*W*Y*L*GJ
      I=I*J*K*L*M*N*II*IJ*IK*IL*IM*IN*JI*JJ*JK*JL*JM*JN*KI*KJ*KK*KL*KM*KJ
      I=A*B*C*D*E*F*G*H*O*P*Q*S*R*T*A*B*C*D*EF*A*S*T*R*W*Q*X*Z*R*W*Y*L*GJ
      I=I*J*K*L*M*N*II*IJ*IK*IL*IM*IN*JI*JJ*JK*JL*JM*JN*KI*KJ*KK*KL*KM*KJ
      I=A*B*C*D*E*F*G*H*O*P*Q*S*R*T*A*B*C*D*EF*A*S*T*R*W*Q*X*Z*R*W*Y*L*GJ
      I=I*J*K*L*M*N*II*IJ*IK*IL*IM*IN*JI*JJ*JK*JL*JM*JN*KI*KJ*KK*KL*KM*KJ
20    KRX=2                                                             J
      LM=LOCF(L(1))-1                                                   J
      KK=0                                                              J
      DO 50 J=1,65                                                      J
      KK=KK+1                                                           J
      LOCAL=L(1+J-LM)                                                   J
      NAME=LOCAL.AND.77777777777777000000B                              J
      IF (NAME.EQ.0) GO TO 60                                           J
      NAME=LOCAL.AND.777777B                                            J
      NFILE(KK)=L(NAME-LM).AND.77777777777777000000B                    J
      NFAD(J)=NAME                                                      J
      JJ=J-1                                                            J
      IF (JJ.EQ.0) GO TO 50                                             J
      DO 30 LMN=1,JJ                                                    J
      IF (NAME.EQ.NFAD(LMN)) GO TO 40                                   J
30    CONTINUE                                                          J
      NFAD(KK)=NAME                                                     J
      GO TO 50                                                          J
40    KK=KK-1                                                           J
50    CONTINUE                                                          J
60    NF=KK-1                                                           J
      N=NF+1                                                            J
      ASSIGN 10 TO JK                                                   J
      JM=0                                                              J
70    JK=JK-1                                                           J
      LOCAL=L(JK-LM).AND.77777777777777000000B                          J
      IF (LOCAL.NE.NSSUBS(1)) GO TO 70                                  J
80    JQL=L(JK-LM).AND.777777B                                          J
      JK=JK+JQL                                                         J
      IF (JM.EQ.NSB-1) GO TO 110                                        J
      LOCAL=L(JK-LM).AND.77777777777777000000B                          J
      DO 90 JJ=2,NSB                                                    J
      IF (LOCAL.EQ.NSSUBS(JJ)) GO TO 100                                J
90    CONTINUE                                                          J
      GO TO 80                                                          J
100   JM=JM+1                                                           J
      GO TO 80                                                          J
110   LOCAL=JK                                                          J
      LAST=NFAD(NF)-1                                                   J
      LINEL=L(LAST+8-LM)                                                J
      ASSIGN 130 TO KENT2                                               J
      KFIELD=NFAD(1)                                                    J
      KFIELD=L(KFIELD-LM+4)                                             J
120   WRITE (NUM) LOCAL,KENT2,(L(I-LM),I=401,LOCAL),N,(L(I-LM),I=2,N)   J
      WRITE (NUM) (L(I-LM),I=2,400),(L(I-LM),I=LOCAL,LAST)              J
      END FILE NUM                                                      J
```

```
      RETURN
130   DO 200 J=1,NF
      LFL=NFAD(J)
      L(LFL-LM)=NFILE(J)
      DO 140 K=1,3
140   L(LFL-LM+K)=LFL+8
      IF (J-1) 150,150,160
150   L(LFL-LM+4)=KFIELD
      GO TO 170
160   L(LFL-LM+4)=NFAD(J-1)
170   IF (NFILE(J).NE.17252420252400000000B) GO TO 180
      L(LFL-LM+8)=34000000000000000000B
      L(LFL-LM+2)=L(LFL-LM+2)+1
180   DO 190 K=5,6
190   L(LFL-LM+K)=0
200   L(LFL-LM+7)=LINEL
      READ (NUM) (L(I-LM),I=2,400),(L(I-LM),I=LOCAL,LAST)
210   READ (NUM)
      IF (EOF,NUM) 220,210
220   RETURN
      END
```

```
      SUBROUTINE GRAPH (IA,JM,IU,JC,IR,IL,IV,BR,AA,AP,BC,BS,AB)
COPYRIGHT 1966 CHRIS ASHLEY
      DIMENSION BR(JM),AA(JM)
      DATA JD,IC,IM,IW,BT,AC,AR,BE,JE,ID,CN,CO,BU,CQ,CF,CG,CH/-3,-2,2,10
     1,90.,.07,.08,.14,2HI9,4HE8,1,.02,1.32,.135,.33,.273704789,.6065102
     263,.5278592375/
      AS=0.
      BF=0.
      JF=1
      IK=JM
      IF (IK.LE.0.O.IK.GT.100000) IK=1
      IF (IE.NE.3) GO TO 50
      IF (IA.EQ.5) GO TO (300,530,460,530),IY
      CALL PLOT (AS,BF,JD)
      GO TO 60
50    CALL PLOT (IN,0,1)
      IF (IN.EQ.0) GO TO 60
      CALL REMARK (28H * FILMPL IS NOT DECLARED. *)
      STOP 9
60    IE=3
      AD=0.
      AT=0.
      IY=IA
      IG=IU
      IO=JC
      IZ=IV
      BV=AP
      AE=BC
      AU=BS
      BG=AB
      JH=BV
      AF=JH
      IF (IY.LE.2) AU=0.
      IF (IY.LT.1.O.IY.GT.4) IY=3
      IF (IY.LE.1.O.IY.EQ.4) BG=0.
      IF (IG.LT.1) IG=1
      IF (IO.LT.1) IO=1
      IF (BV.LE.0..O.BV.GT.8.95) BV=8.
      IF (AE.LE.0..O.AE.GT.8.95) AE=8.
      IF (IZ.GT.999999999) IZ=999999999
      DO 70 JG=1,IK
      AV=ABS(BR(JG))
      BH=ABS(AA(JG))
      IF (AD.LT.AV) AD=AV
70    IF (AT.LT.BH) AT=BH
      IF (AD.LE.0.) AD=1.
      IF (AT.LE.0.) AT=1.
      JI=ALOG10(AD)
      IH=ALOG10(AT)
      IF (10.**JI.GT.AD) JI=JI-1
      IF (10.**IH.GT.AT) IH=IH-1
      GO TO (110,80,80,110),IY
80    IP=BG
      BI=IP
      BI=BG-BI
      BW=BV-BG
      II=BW
      AW=II
      BJ=BW-AW
      JJ=IP+II
      AG=BV-BJ
```

```
      JN=JI-3
90    CI=10.**JN
      DO 93 JO=1,9
      CJ=JO
93    IF(CJ*CI*BW.GE.AU)GO TO 97
      JN=JN+1
      GO TO 90
97    BX=CJ*CI
      BK=BX*BW
      CALL NUMBER (BI+CN,AU-BU,AC,(BI-BG)*BX,AH,ID)
      CALL PLOT (AH,AU,IE)
      CALL PLOT (BI,AU,IM)
      CALL PLOT (BI,AU-.05,IM)
      CALL PLOT (BI,AU,IM)
      IQ=(BV+BG)*.5
      DO 100 JA=1,JJ
      BY=JA
      AS=BY+BI
      CALL PLOT (AS,AU,IM)
      CALL PLOT (AS,AU-.05,IM)
      CALL PLOT (AS,AU,IM)
      IF (JA.LT.JJ) CALL NUMBER (AS-CF,AU-BU,AC,(AS-BG)*BX,AH,ID)
      IF (JA.EQ.JJ) CALL NUMBER (AS-CG,AU-BU,AC,(AS-BG)*BX,AH,ID)
      IF (JA.EQ.IQ) CALL SYMBOL ((BV+BG)*.5-CH,AU-CQ,BE,IB,AH,IW)
100   CALL PLOT (AS,AU,IE)
      CALL PLOT (BV,AU,IM)
      IF (IZ.NE.0) CALL NUMBER (BV-CO,AE*.05,BE,IZ,AH,JE)
      CALL PLOT (BV,AH,IE)
      GO TO (170,170,140,140),IY
110   AI=IG
      JR=JI+1-IG
      AY=JR
      BL=10.**JR
      CALL NUMBER (AH+CN,AU-BU,AC,BL,AH,ID)
      CALL PLOT (AH,AU,IE)
      BZ=BV/AI
      AJ=-BZ
      IQ=AI*.5
      DO 130 IR=1,IG
      BX=IR
      AJ=AJ+BZ
      DO 120 JA=1,10
      BY=JA
      BM=AJ+ALOG10(BY)*BZ
      CALL PLOT (BM,AU,IM)
      CALL PLOT (BM,AU-.05,IM)
120   CALL PLOT (BM,AU,IM)
      IF (IR.LT.IG) CALL NUMBER (BM-CF,AU-BU,AC,10.**(JR+IR),AH,ID)
      IF (IR.EQ.IG) CALL NUMBER (BM-CG,AU-BU,AC,10.**(JR+IG),AH,ID)
      IF (IR.EQ.IQ) CALL SYMBOL (BV*.5-CH,AU-CQ,BE,IB,AH,IW)
130   CALL PLOT (BM,AU,IE)
      IF (IZ.NE.0) CALL NUMBER (BV-CO,AE*.05,BE,IZ,AH,JE)
      CALL PLOT (BV,AH,IE)
      GO TO (170,170,140,140),IY
140   JK=AU
      AK=JK
      AK=AU-AK
      AZ=AE-AU
      JB=AZ
      CB=JB
      AL=AZ-CB
      IS=JK+JB
```

```
      BN=AE-AL
      CK=AE-AU
      JP=IH-3
150   CL=10.**JP
      DO 153 JQ=1,9
      CM=JQ
153   IF(CM*CL*CK.GE.AT) GO TO 157
      JP=JP+1
      GO TO 150
157   BB=CM*CL
      AM=BB*BW
      CALL PLOT (BV,AK,IM)
      CALL PLOT (BV+.05,AK,IM)
      CALL PLOT (BV,AK,IM)
      DO 160 JA=1,IS
      BY=JA
      BF=BY+AK
      CALL PLOT (BV,BF,IM)
      CALL PLOT (BV+.05,BF,IM)
160   CALL PLOT (BV,BF,IM)
      CALL PLOT (BV,AE,IM)
      GO TO (220,200,200,220),IY
170   BA=IO
      JS=IH+1
      BO=JS-IO
      CD=10.**BO
      AN=AE/BA
      AJ=-AN
      DO 190 IR=1,IO
      AJ=AJ+AN
      DO 190 JA=1,10
      BY=JA
      BM=AJ+ALOG10(BY)*AN
      CALL PLOT (BV,BM,IM)
      CALL PLOT (BV+.05,BM,IM)
190   CALL PLOT (BV,BM,IM)
      GO TO (220,200,200,220),IY
200   CALL PLOT (AG,AE,IM)
      CALL PLOT (AG,AE+.05,IM)
      CALL PLOT (AG,AE,IM)
      DO 210 JA=1,JJ
      BY=JA
      AS=AG-BY
      CALL PLOT (AS,AE,IM)
      CALL PLOT (AS,AE+.05,IM)
210   CALL PLOT (AS,AE,IM)
      CALL PLOT (AH,AE,IM)
      CALL PLOT (BG,AE,IE)
      GO TO (270,270,250,250),IY
220   CALL PLOT (BV,AE+.05,IM)
      CALL PLOT (BV,AE,IM)
      AJ=BV+BZ
      DO 240 IR=1,IG
      AJ=AJ-BZ
      DO 240 JA=1,10
      BY=11-JA
      BM=AJ-BZ+ALOG10(BY)*BZ
      CALL PLOT (BM,AE,IM)
      CALL PLOT (BM,AE+.05,IM)
240   CALL PLOT (BM,AE,IM)
      GO TO (270,270,250,250),IY
250   IQ=CK*.5
```

```
      CALL PLOT (BG,BN,IM)
      CALL PLOT (BG-.05,BN,IM)
      CALL PLOT (BG,BN,IM)
      CALL NUMBER (BG-BU,BN-CG,AC,CR*BB,BT,ID)
      CALL PLOT (BG,BN,IE)
      DO 260 JA=1,IS
      BY=JA
      BF=BN-BY
      CALL PLOT (BG,BF,IM)
      CALL PLOT (BG-.05,BF,IM)
      CALL PLOT (BG,BF,IM)
      IF (JA.LT.IS) CALL NUMBER (BG-BU,BF-CF,AC,(BF-AU)*BB,BT,ID)
      IF (JA.EQ.IS) CALL NUMBER (BG-BU,BF+CN,AC,(BF-AU)*BB,BT,ID)
      IF (JA.EQ.IQ) CALL SYMBOL (BG-CO,(AE+AU)*.5-CH,BE,IL,BT,IW)
260   CALL PLOT (BG,BF,IE)
      CALL PLOT (BG,AH,IM)
      GO TO (300,530,460,530),IY
270   CALL PLOT (BG-.05,AE,IM)
      CALL PLOT (BG,AE,IM)
      AJ=AE+AN
      IQ=BA*.5
      CALL NUMBER (BG-BU,AE-CG,AC,10.**JS,BT,ID)
      CALL PLOT (BG,AE,IE)
      DO 290 IR=1,IO
      AJ=AJ-AN
      DO 280 JA=1,10
      BY=11-JA
      BM=AJ-AN+ALOG10(BY)*AN
      CALL PLOT (BG,BM,IM)
      CALL PLOT (BG-.05,BM,IM)
280   CALL PLOT (BG,BM,IM)
      IF (IR.LT.IO) CALL NUMBER (BG-BU,BM-CF,AC,10.**(JS-IR),BT,ID)
      IF (IR.EQ.IO) CALL NUMBER (BG-BU,BM+CN,AC,10.**(JS-IO),BT,ID)
      IF (IR.EQ.IQ) CALL SYMBOL (BG-CO,AE*.5-CH,BE,IL,BT,IW)
290   CALL PLOT (BG,BM,IE)
      GO TO (300,530,460,530),IY
300   DO 450 JG=1,IK
      BP=AS
      CE=BF
      IJ=1
      IF (BR(JG).LT.BL) IJ=3
      IF (AA(JG).LT.CD) IJ=IJ+1
      GO TO (430,310,390,410),IJ
310   IF (AA(JG).GT.-CD) GO TO 350
      IT=1
      AS=(ALOG10(BR(JG))-AY)*BZ
320   BF=(ALOG10(-AA(JG))-BO)*AN
330   IF (JF.EQ.3) GO TO 340
      JF=3
      GO TO 380
340   BQ=BQ+SQRT((AS-BP)**2+(BF-CE)**2)
      IF (BQ.LT..5) GO TO 440
      BQ=0.
      IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL SYMBOL (AS,BF,AR,IT,AH,IC)
      GO TO 450
350   AS=(ALOG10(BR(JG))-AY)*BZ
360   BF=0.
370   JF=1
380   IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
```

```
      CALL PLOT (AS,BF,IE)
      GO TO 450
390   IF (BR(JG).GT.-BL) GO TO 400
      IT=9
      AS=(ALOG10(-BR(JG))-AY)*BZ
      BF=(ALOG10(AA(JG))-BO)*AN
      GO TO 330
400   AS=0.
      BF=(ALOG10(AA(JG))-BO)*AN
      GO TO 370
410   IF (BR(JG).GT.-BL.O.AA(JG).GT.-CD) GO TO 420
      IT=2
      AS=(ALOG10(-BR(JG))-AY)*BZ
      GO TO 320
420   AS=0.
      GO TO 360
430   AS=(ALOG10(BR(JG))-AY)*BZ
      BF=(ALOG10(AA(JG))-BO)*AN
      IF (JF.EQ.2) GO TO 440
      JF=2
      GO TO 380
440   IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL PLOT (AS,BF,IM)
450   CONTINUE
      RETURN
460   DO 520 JG=1,IK
      AS=BR(JG)/BX+BG
      BF=AA(JG)/BB+AU
      IJ=1
      IF (AS.LT.0) IJ=3
      IF (BF.LT.0) IJ=IJ+1
      GO TO (470,500,480,490),IJ
470   IF (JG.EQ.1) GO TO 510
      IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL PLOT (AS,BF,IM)
      GO TO 520
480   AS=0.
      GO TO 510
490   AS=0.
500   BF=0.
510   IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL PLOT (AS,BF,IE)
520   CONTINUE
      RETURN
530   IT=1
      IF (IY.EQ.2) GO TO 540
      BX=BB
      BO=AY
      AN=BZ
      CD=BL
      BG=AU
      IT=9
540   JL=IT
      DO 660 JG=1,IK
      BP=AS
      CE=BF
      IF (IY.NE.4) GO TO 550
      AO=BR(JG)
      BR(JG)=AA(JG)
```

```
      AA(JG)=AO
550   AS=BR(JG)/BX+BG
      IF (AS.GE.0.) GO TO 560
      AS=0.
      JF=1
560   IF (AA(JG).GT.CD) GO TO 620
      IF (AA(JG).GT.-CD) GO TO 590
      BF=(ALOG10(-AA(JG))-BO)*AN
      IF (IY.NE.4) GO TO 570
      AO=AS
      AS=BF
      BF=AO
570   IF (JF.EQ.3) GO TO 580
      JF=3
      GO TO 610
580   BQ=BQ+SQRT((AS-BP)**2+(BF-CF)**2)
      IF (BQ.LT..5) GO TO 640
      BQ=0.
      IF (AS.LT.BG.O.BF.LT.AU) IT=2
      IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL SYMBOL (AS,BF,AR,IT,AH,IC)
      IT=JL
      GO TO 650
590   BF=0.
      IF (IY.NE.4) GO TO 600
      AO=AS
      AS=BF
      BF=AO
600   JF=1
610   IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL PLOT (AS,BF,IF)
      GO TO 650
620   BF=(ALOG10(AA(JG))-BO)*AN
      IF (IY.NE.4) GO TO 630
      AO=AS
      AS=BF
      BF=AO
630   IF (JF.EQ.2) GO TO 640
      JF=2
      GO TO 610
640   IF (AS.GT.BV) AS=BV
      IF (BF.GT.AE) BF=AE
      CALL PLOT (AS,BF,IM)
650   IF (IY.NE.4) GO TO 660
      AO=BR(JG)
      BR(JG)=AA(JG)
      AA(JG)=AO
660   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SYMBOL(X,Y,HT,LBL,ROT,NCHAR)
      DIMENSION DELX(11,3),DELY(11,3),NDEL(3),LBL(2)
      DATA DELX/0.,-.01,-.02,-.02,-.01,.01,.02,.02,.01,0.,0.,0.,-.02,.02
     1,0.,0.,0,0,0,0,0,0,.02,0.,0.,0.,-.02,0.,/
      DATA DELY/.02,.02,.01,-.01,-.02,-.02,-.01,.01,.02,.02,0.,.02,-.02,
     1-.02,.02,0.,0,0,0,0,0,0,.02,0.,-.02,0.,.02,0./
      DATA NDEL/11,5,6/
      IP=3
      IF (NCHAR.LT.-1) IP=2
      CALL PLOT(X,Y,IP)
      IF(NCHAR)6,5,8
8     ISIZ=7
      IF (HT.LT.(.220750)) ISIZ=6
      IF (HT.LT.(.155525)) ISIZ=5
      IF (HT.LT.(.110375)) ISIZ=4
      IO=0
      IF(ROT.GT.45)IO=1
      CALL PLOT (LBL(1),IO,ISIZ)
5     RETURN
6     LVL=LBL
      IF(LVL.NE.1.AND.LVL.NE.2.AND.LVL.NE.9)LVL=1
      IF(LVL.EQ.9)LVL=3
      NM=NDEL(LVL)
      SF=HT/.04
      DO 7 I=1,NM
7     CALL PLOT (X+DELX(I,LVL)*SF,Y+DELY(I,LVL)*SF,2)
      RETURN
      END
```

```
SUBROUTINE NUMBER(X,Y,HT,FNUM,ROT,FORMAT)
DIMENSION FORM(3),TEMP(3)
ISIZE=7
IF (HT.LT.(.220450)) ISIZE=6
IF (HT.LT.(.155525)) ISIZE=5
IF (HT.LT.(.110375)) ISIZE=4
CALL PLOT (X,Y,3)
IOR=0
IF(ROT.GT.45)IOR=1
FORM(1)=1H(
FORM(2)=FORMAT
FORM(3)=1H)
ENCODE(30,FORM,TEMP)FNUM
CALL PLOT (TEMP(1),IOR,ISIZE)
RETURN
END
```