Theoretical Notes
Note 88

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COMPUTATIONS ON THE PARAMETER VALUES INSIDE OF THE E. M. SOURCE

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1. The Radial Electric Field

The radial electric field inside the conducting sphere is most easily calculated if we assume complete spherical symmetry so that all quantities, such as gamma-ray intensity, electron and charge densities, etc., depend only on radius and time. The electric field and the current density have only r-components which we write as E and J respectively, without subscripts. The gamma-ray flux is a function of r and of t, which we write

$$\gamma\text{-ray flux} = \Phi(r,t). \tag{1.1}$$

This flux leads, in turn, to a current of Compton betas directed radially, given by

$$J_\beta = J_0\Phi(r,t). \tag{1.2}$$

The units of J_β are to be amperes per square meter, and the direction of current flow is radially inward. The passage of the gamma radiation leaves behind ions and electrons which render the air conducting. This conductivity can readily be calculated if we assume the relevant attachment times and mobilities known. If τ is the electron attachment time, then the electron density is given by

$$\frac{dn_e}{dt} + \frac{n}{\tau} = a\,\Phi(r,t),$$

which yields

$$n_e = ae^{-(t/\tau)} \int^t \Phi(r,x)e^{x/\tau}\,dx,$$

and the electronic conductivity is

$$\sigma_e = e\,w_e\,ae^{-(t/\tau)} \int^t \Phi(r,x)e^{x/\tau}\,dx, \qquad (1.3)$$

where w_e is the electron mobility in $(\text{meters})^2$ per volt second. The number of positive ions present is simply

$$n_+ = a \int^t \Phi(r,x)dx,$$

and the number of negative ions is $n_+ - n_e$. Assuming all ions have the same mobility, we obtain for the ionic conductivity

$$\sigma_i = ew_i a \left[2 \int^t \Phi(r,x)dx - e^{-(t/\tau)} \int^t \Phi(r,x)e^{t/\tau}dx \right], \qquad (1.4)$$

and for the total conductivity

$$\sigma(r,t) = \sigma_e + \sigma_i. \qquad (1.5)$$

As the ionic mobility is much less than the electronic mobility, σ_i is unimportant except at such late times that the electron density is very low compared with the ion density--that is to say, well into the period of delayed gamma radiation.

The passage of the gamma-rays gives rise to charge separation and therefore to a charge density $\rho(r,t)$. This in turn gives rise to a radial electric field $E(\rho,t)$, and to a conduction current σE. The total current density is therefore

$$J_{tot} = \sigma E + J_\beta . \tag{1.6}$$

The relevant Maxwell equations are Poisson's equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E) = \frac{\rho}{\epsilon} , \tag{1.7}$$

and the equation of charge conservation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \sigma E) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 J_0 \Phi) = 0. \tag{1.8}$$

Substituting (1.7) into (1.8) yields

$$\frac{\partial E}{\partial t} + \frac{\sigma}{\epsilon} E = - \frac{J_0}{\epsilon} \Phi \tag{1.9}$$

whence

$$E = - \frac{J_0}{\epsilon} e^{-\int^t \frac{\sigma}{\epsilon} dt} \int^t \Phi(r,t) \, e^{\int^t \frac{1}{\epsilon} \sigma(r,x) dx} dt. \tag{1.10}$$

If one wishes, the charge density can be calculated from (1.7).

Equation (1.10) is a full solution to the problem of electrical relaxation of the conducting sphere. All that one needs is the gamma-ray flux vs. time and distance function $\Phi(r,t)$ and the constants τ, w_e, and w_i. The only approximation is that spherical symmetry was assumed. To work out the complete time history, one would need to perform the quadratures numerically, or else approximate the function $\Phi(r,t)$ by an analytic expression. If we confine our attention to short times, everything works out quite simply. We then have

$$\Phi \sim \frac{1}{r^2} \, e^{\alpha t - \frac{r}{\Lambda}}$$
$$\frac{1}{\Lambda} \overset{\text{def}}{=} \frac{1}{\lambda} + \frac{\alpha}{c} \tag{1.11}$$

where λ is the gamma-ray mean-free-path. Normally, $\frac{1}{\alpha}$ is much shorter than τ. Therefore, only electronic conduction matters, and

$$\sigma = \frac{\Sigma}{r^2} \, e^{\alpha t - \frac{r}{\Lambda}} . \tag{1.12}$$

Also,

$$J_\beta = \frac{J_0}{r^2} \, e^{\alpha t - \frac{r}{\Lambda}} = \frac{J_0}{\Sigma} \, \sigma , \tag{1.13}$$

which yields

$$E = - \frac{J_0}{\Sigma} \left\{ 1 - \exp\left[\frac{\Sigma}{\epsilon r^2} \, e^{-\alpha t + \frac{r}{\Lambda}} \right] \right\} . \tag{1.14}$$

Equation (1.13) shows that the E-field rises exponentially at very early times. However, owing to the double exponential time behavior, a saturation field strength of $-J_0/\Sigma$ is very quickly reached and then maintained. Ultimately, of course, the signal strength decays again to zero. To follow this decay requires a numerical integration of equation (1.10) together with (1.3), (1.4), and (1.5) for the conductivity.

2. The Azimuthal Magnetic Field

When the explosion takes place at ground level, the conducting sphere is really a hemisphere. The charge densities, and therefore the resulting currents and electric field, must be pretty much the same as discussed in the last section, except that the radial current now flows only in the upper hemisphere and therefore gives rise to an azimuthal magnetic field (H_φ).

The total radial current density is given by

$$J_{Total} = \sigma E + J_0 \Phi \tag{2.1}$$

or, according to equation (1.9),

$$J_{Total} = \epsilon \frac{\partial E}{\partial t} . \tag{2.2}$$

If we use equation (1.14) for the early signal, we see that this is an exceedingly short pulse. As the frequencies involved will be of the order of 100 megacycles or more, it is not likely that this signal can give any trouble.

The smaller but more bothersome signal will be that associated with the decay of the E-field after the gamma-ray flux has passed its peak. With the form for $\Phi(r,t)$ which I shall give, the integrals of (1.3) and (1.4) can be done to obtain an analytic expression for σ, but I see little hope of carrying out the integration of (1.10) analytically. I suggest, therefore, that (1.10) be carried out numerically and that J_{tot} be calculated by (2.1) or by (2.2). The numerically-obtained values of J_{tot} should then be fit by an analytic expression, preferably as a function of r,t and of yield. The functional form of J_{tot} should be so chosen that the integral for the retarded vector potential* can be evaluated. The alternative of evaluating this integral numerically is to be avoided, if possible, as the integration is over three dimensions and must be carried out for a variety of radii, of times, and of yield.

For the evaluation of the integrals, I suggest the following functional forms:

$$\Phi \approx \frac{7 \times 10^{20}\, Y\, e^{-r/\lambda}}{4\pi r^2 (t - r/c)} \quad \text{for } t \geq t_0 = 2 \times 10^{-7} \text{ sec.}$$

(Prior to t_0 the rise may be taken exponential or, more conveniently, as a delta function.)

r = distance from explosion

λ = gamma-ray mean-free-path, about 250 meters

Y = yield in kilotons

(2.3)

$J_0 = ec\,\frac{\delta}{\lambda} \cdot \Phi$ where δ = electron range in air. Numerically, we have

$$J_0 = \frac{2 \times 10^{11}\, Y}{4\pi r^2}\;\frac{e^{-r/\lambda}}{(t - r/c)} \quad \text{for } t \geq t_0 = 2 \times 10^{-7} \text{ sec.}$$

(2.4)

For the conductivity calculation, the constant ea in equations (1.3) and (1.4) may be taken to be 6×10^{-18} coulombs per meter per gamma; w_e, the electron mobility, is about 0.3 (meters/sec) per (volt/meter); and w_i should be w_e reduced by the square root of the mass ratio, e. g. $w_i \approx$ .0013 (meters)^2/volt sec.

*See, e. g. J. A. Stratton, "Electromagnetic Theory," McGraw-Hill (1941) p. 428.

As only the lower frequency components are significant under ground, I suggest that the calculation ignore the effects of retarded time, both in the expressions (t - r/c) in the denominators of (2.3) and (2.4), which may be replaced simply by t, and in the calculation of the vector potential which may simply be taken as

$$A_z(R,t) = \frac{\mu}{4\pi} \int \frac{J_{tot}(r,t)}{|\vec{R} - \vec{r}|} \; r^2 \sin\theta \, dr \, d\theta \, d\varphi.$$

The azimuthal magnetic field is, of course,

$$B_\varphi = -\frac{\partial A_z(R,t)}{\partial R} .$$