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The Electromagnetic Pulse from an Underground Nuclear Explosion

by

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ABSTRACT

It is argued that because of earth's high electrical conductivity, the electric current density produced by an underground nuclear explosion becomes approximately solenoidal, i.e., free of electric charges, in a negligibly short time. Therefore, the signal radiated from a simple example of a solenoidal current distribution, a point magnetic dipole with delta function time dependence, located underground, is calculated. The underground signal consists of a magnetic field associated with a much weaker electric field, both of which propagate through the conducting earth in a way that resembles diffusion. The signal is reflected or transmitted at the surface of the ground in a way that depends upon the orientation of the source dipole. It is shown that for a horizontal dipole, a measurable signal could be transmitted to remote points above ground.

I. INTRODUCTION

There are two processes that one should expect to drastically reduce the amplitude of the electromagnetic pulse (EMP) produced above the surface of the ground by an underground nuclear explosion. One of these processes is absorption due to the earth's electrical conductivity; the other is reflection at the surface of the ground due to the abrupt change in conductivity and dielectric constant there. This report deals with the propagation of a signal from an underground source through the underground region and through the ground surface to remote points in the atmosphere and is thus concerned mainly with these two processes. Mechanisms whereby an underground nuclear explosion could produce an EMP source are not discussed.

It is shown that the nature of the radiated signal is almost completely determined by the electrical properties of the earth, rather than by the source, if the source duration is a few microseconds or less. The reason for this is that the signal's high-frequency components are much more strongly absorbed than its low-frequency components. The signal propagates through the earth by a process more similar to diffusion than to radiation. Therefore, the signal's source is assumed to have a delta function time dependence. The results obtained would be the same if the source were a pulse in the microsecond range.

Furthermore, the source is assumed to be completely immersed in a conducting medium; there are no insulators separating a current distribution from the surrounding earth. The medium's conductivity is assumed to be so high that any charge distributions decay in a time that is very short compared to any time of interest in this problem. Therefore, the underground current density is taken to have zero divergence, consistent with the assumption of zero charge density. Thus, it is assumed that the underground nuclear explosion produces locally a current distribution that lasts much longer than an underground charge distribution does, but a much shorter time than that characteristic of an electromagnetic pulse that has propagated through a few hundred meters of earth.

Because the ground's conductivity tends to eliminate the high frequencies, the wavelengths in the radiated signal are much longer than the source dimensions. Therefore, the source is taken to be a delta function of position, as well as of time. The earth is treated as a uniform, isotropic, conducting medium; at the earth's surface the conductivity is assumed to go discontinuously from its uniform underground value to zero above ground. This approximation is justified because the wavelengths in the radiated signal are extremely long compared to the distance over which the conductivity varies significantly. Note that with this approximation the signal calculated at the ground surface just above the source is probably only qualitatively correct. The reason is that, as is shown below, part of the signal is reflected at the surface where the conductivity changes from its underground value to its above ground value of zero. In actual earth this surface is effectively somewhat below ground; the earth's electrical properties vary rapidly but not discontinuously near the ground surface. Therefore, to calculate the signal accurately within this transition region, one must probably consider the variation of conductivity with depth. Since this is not known very well, the calculation of the signal in the region near the source was done without considering surface effects, i.e., the earth was taken to be of infinite extent. Consequently, the results of this part of the calculation can only be regarded as a qualitative representation of possible experimental results.

However, in calculating the signal that propagates through the surface to remote points in the atmosphere, both underground absorption and reflection effects can be treated meaningfully by assuming uniform underground conductivity. This calculation shows that if the energy in the electromagnetic field at a time when it has propagated underground 30 m from its source is equivalent to 10^{-6} kt of TNT, the EMP should be observable above ground a few kilometers from ground zero for a shot at a depth of 300 m. The signal from a source with delta function time dependence would consist of a pulse in the 0.1 msec range followed by a slow tail of much smaller amplitude. Far from ground zero, the EMP would be a vertically polarized pulse propagating in the earth-ionosphere waveguide.

II. THE UNDERGROUND FIELD

Maxwell's equations in a medium of conductivity σ and dielectric constant ε are

$$\nabla \times \vec{E} + \frac{1}{c}\dot{\vec{B}} = 0, \quad \nabla \cdot \vec{B} = 0,$$

$$\nabla \times \vec{B} - \frac{\varepsilon}{c}\dot{\vec{E}} = \frac{4\pi}{c}(\sigma\vec{E} + \vec{J}_0), \quad \nabla \cdot \vec{E} = 4\pi\rho.$$

The source current density is $\vec{J}_0$; since it is assumed to be solenoidal, i.e., $\nabla \cdot J_0 = 0$, it can be expressed as the curl of a vector. Thus,

$$\vec{J}_0 = c\nabla \times \vec{M}(\vec{r},t),$$

and if the magnetic field $\vec{B}$ is written as

$$\vec{B} = \vec{H} + 4\pi\vec{M}(\vec{r},t),$$

Maxwell's equations become

$$\nabla \times \vec{E} + \frac{1}{c}\dot{\vec{H}} = -\frac{4\pi}{c}\dot{\vec{M}}, \tag{1}$$

$$\nabla \times \vec{H} - \frac{\varepsilon}{c}\dot{\vec{E}} - \frac{4\pi\sigma}{c}\vec{E} = 0. \tag{2}$$

The introduction of the vector $\vec{M}$ does not imply that the medium is magnetic. It is merely a mathematically convenient way of expressing the source current. $\vec{M}$ is taken to be

$$\vec{M}(\vec{r},t) = \vec{m}\delta(\vec{r})\ \delta(t);$$

i.e., the source is assumed to be a point magnetic dipole of moment $\vec{m}\delta(t)$ at the origin of the coordinate system. The field $\vec{H}$ is identical to $\vec{B}$ outside the source region.

Equations (1) and (2) are most conveniently solved by introducing a Hertz vector $\vec{\Pi}$, in terms of which

$$\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\nabla \times \vec{\Pi}, \tag{3}$$

$$\vec{H} = \nabla\nabla \cdot \vec{\Pi} - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\varepsilon}{c}\frac{\partial}{\partial t} + \frac{4\pi\sigma}{c}\right)\vec{\Pi}, \tag{4}$$

where the Cartesian components of $\vec{\Pi}$ satisfy

$$-\nabla^2\vec{\Pi} + \frac{\varepsilon}{c^2}\ddot{\vec{\Pi}} + \frac{4\pi\sigma}{c^2}\dot{\vec{\Pi}} = 4\pi\vec{M}(\vec{r},t). \tag{5}$$

At this point it is useful to introduce a quantity

$$q = \frac{c}{4\pi\sigma} \quad ,$$

which has the units of length. The ground conductivity is taken to be $\sigma \simeq 10^{-2}$ mho/m or, in Gaussian units, $\sigma \simeq 9 \times 10^{7}\ \mathrm{sec}^{-1}$, so $q \simeq 30$ cm. With this substitution for σ and the above expression substituted for $\vec{M}(\vec{r},t)$, the equation for $\vec{\Pi}$ is

$$-\nabla^2\vec{\Pi} + \frac{\varepsilon}{c^2}\ddot{\vec{\Pi}} + \frac{1}{qc}\dot{\vec{\Pi}} = 4\pi\vec{m}\ \delta(\vec{r})\ \delta(t) \quad . \qquad (6)$$

From the spherically symmetrical nature of the source, it is clear that for each component of the vector $\vec{\Pi}$ there is a particular solution that is spherically symmetrical. Thus, a particular solution for $\vec{\Pi}$ is

$$\vec{\Pi}_0 = \frac{\vec{m}}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\,\frac{e^{i\Gamma r}}{r} \quad , \qquad (7)$$

where

$$\Gamma^2 = \frac{i\omega}{qc} + \varepsilon\frac{\omega^2}{c^2} \quad . \qquad (8)$$

It is easily verified that this is a solution by substituting it in Eq. (6) and noting that

$$(-\nabla^2 - \Gamma^2)\frac{e^{i\Gamma r}}{r} = 4\pi\delta(\vec{r}) \quad , \qquad (9)$$

and

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t} = \delta(t) \quad . \qquad (10)$$

For Eq. (7) to represent a solution that propagates outward from the source and is attenuated as it goes, the real part of Γ must have the same sign as ω and the imaginary part must be positive for either sign of ω. From its definition it is clear that there is a square root of Γ^2 that satisfies these requirements for either sign of ω; hereafter Γ will be taken to mean that square root. With Γ so defined, it is clear that if Γ is replaced by $-\Gamma$ in Eq. (7) the result is again a solution for $\vec{\Pi}$, but it does not represent an outward-propagating signal and therefore must be rejected. If $\sigma = 0$ or $q = \infty$, the solution given by Eq. (7) becomes $(m/r)\ \delta(t - \sqrt{\varepsilon}\ r/c)$, as can be seen by comparing Eq. (10). This represents a thin shell propagating outward at velocity $c/\sqrt{\varepsilon}$ and decreasing in amplitude as $1/r$.

For the present problem, the term $i\omega/qc$ in Γ^2 is dominant. This can be seen most clearly if Γ is written as $1/q$ times a function of $\omega q/c$. Thus,

$$e^{i\Gamma r} = \exp\left[i(r/q)\sqrt{i\omega q/c + \varepsilon(\omega q/c)^2}\right] \quad . \qquad (11)$$

Clearly, if $r >> q$ then $e^{i\Gamma r}$ is very small unless $\frac{\omega}{c}q << 1$, and this implies that the term in Γ that is proportional to ε is negligible. Thus, the frequencies that make the main contribution to Π_0 in Eq. (7) for $r >> q$ are those for which displacement currents, which are proportional to ε, are negligible compared to conduction currents, which are proportional to $1/q$. Therefore, in the underground region for r many times greater than q,

$$\vec{\Pi}_0 \simeq \frac{\vec{m}}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\,\frac{e^{ir\sqrt{i\omega/qc}}}{r} \quad . \qquad (12)$$

To evaluate this integral, it is desirable to distort the integration path into the complex ω plane. In so doing, care must be exercised to see that the signs of the real and imaginary parts of $\sqrt{i\omega/qc}$ remain the proper ones discussed above. This can be done by setting

$$\sqrt{\frac{i\omega}{qc}} = ue^{i\frac{\psi}{2}} \quad ,$$

or

$$i\omega = qcu^2 e^{i\psi} \quad ,$$

where ψ is an angle between 0 and 2π, and u is real and positive. For real, positive ω the value of ψ is $\psi_+ = \pi/2$, and for real, negative ω the value of ψ is $\psi_- = 3\pi/2$. For an arbitrary point in the complex ω plane, ψ is the angle measured counterclockwise between the negative imaginary axis and a straight line from the origin to the point in question. Because of the branch point at $\omega = 0$ in the integrand of Eq. (12), a branch cut is taken from the origin along the negative imaginary axis to $-i\infty$. The integration path can now be distorted by choosing different values of ψ_+ and ψ_- between 0 and 2π, and the signs of the real and imaginary parts of $\sqrt{i\omega/qc}$ will remain those that correspond to a solution that propagates outward from the source.

For positive values of t, the integration path in Eq. (12) can be closed in the negative imaginary half-plane of ω, except for a circuit around the branch cut. Only the branch-cut part contributes; consequently the integration path can be distorted to that shown in Fig. 1. The path runs from $-i\infty$ to zero along the left-hand side of the branch cut and then back to $-i\infty$ along the right-hand side. On the left side $\psi = 2\pi$, so $e^{ir\sqrt{i\omega/qc}} = e^{iur}$, while on the right side $\psi = 0$, so $e^{ir\sqrt{i\omega/qc}} = e^{+iur}$. Thus, if the integration variable in Eq. (12) is changed to u, i.e., $d\omega = -2iqcudu$, then that equation becomes

$$\vec{\Pi}_0 = \frac{\vec{m}}{2\pi r}(-2iqc)\left(\int_{\infty}^{0} udu\, e^{-qctu^2}\, e^{-iur} + \int_{0}^{\infty} udu\, e^{-qctu^2}\, e^{+iur}\right)$$

$$= \frac{-i\vec{m}qc}{\pi r}\int_{-\infty}^{\infty} udu\, e^{-qctu^2 + iur} \; .$$

If the square of the exponent is completed, the result is

$$\vec{\Pi}_0 = \frac{-i\vec{m}qc}{\pi r}\, e^{-\frac{r^2}{4qct}} \int_{-\infty}^{\infty} udu\, e^{-qct(u-ir/2qct)^2}$$

$$= \frac{-i\vec{m}qc}{\pi r}\, e^{-\frac{r^2}{4qct}} \int_{-\infty}^{\infty} dx\left(x + \frac{ir}{2qct}\right) e^{-qctx^2}$$

$$= \frac{\vec{m}}{2\pi t}\, e^{-\frac{r^2}{4qct}} \sqrt{\frac{\pi}{qct}}$$

$$= 4\pi qc\vec{m}(4\pi qct)^{-3/2}\, e^{-r^2/4qct} \; , \qquad (13)$$

where x is substituted for $u - ir/2qct$, and it is noted that in the resulting integral only the part of the integrand that is even in x contributes.

Note that $\vec{\Pi}_0$ as a function of r is a Gaussian function whose width increases with $\sqrt{t}$. This is characteristic of the solutions of problems of heat conduction or diffusion where the source is a delta function of position and time as discussed by Sommerfeld.[1] Equation (6) with ε set equal to zero is a diffusion equation. It is well known that the magnetic field in a conducting medium at rest behaves in a way that can be described as diffusion. (For example, see Cowling.[2]) Thus, it is not surprising that the radiative solution that is obtained in the case of zero conductivity is so drastically altered if the medium is conducting.

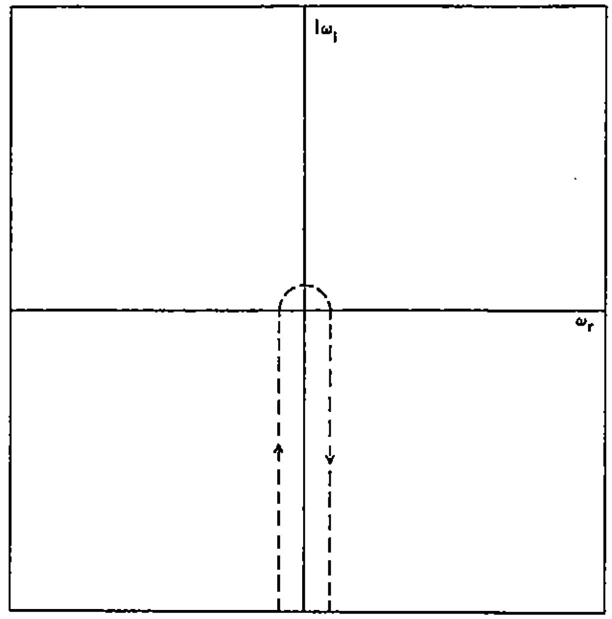

Fig. 1. Integration path used in evaluating the integral of Eq. (12) for positive values of t.

For negative values of the time t, the integral in Eq. (12) must be evaluated by closing the contour in the positive imaginary half-plane of ω; because the integrand contains no poles or branch cuts in this region, the result of the integration is zero. Thus, at r = 0 the vector $\vec{\Pi}_0$ jumps discontinuously from zero for negative times to an infinite value at t = 0; for r > 0 $\vec{\Pi}_0$ is always finite and continuous as a function of time and position. The integral of $\vec{\Pi}_0$ over all space is $4\pi qc\vec{m}$ for any t greater than zero. From this it is seen that $\vec{\Pi}_0$ starts as a delta function of position at t = 0, then decreases in amplitude and spreads out over all space as t increases. It always retains the character of a delta function in the sense that its integral over space is constant.

The electric and magnetic fields can now be calculated using Eqs. (3) and (4). For this purpose, it will be assumed that $\vec{m}$ is directed in the +z direction in a Cartesian-coordinate system. For the time being, the actual direction in space will not be specified, i.e., the +z direction is not necessarily upward. The Cartesian components of the electric and magnetic fields are easily calculated from the above equations. If these components are combined to express the fields in spherical coordinates with m along the polar axis, the nonvanishing components are:

$$H_r = \frac{16mqc}{\sqrt{\pi}} \frac{e^{-r^2/4qct}}{(4qct)^{5/2}} \cos\theta \quad , \tag{14}$$

$$H_\theta = \frac{-16mqc}{\sqrt{\pi}} \left(1 - \frac{r^2}{4qct}\right) \frac{e^{-r^2/4qct}}{(4qct)^{5/2}} \sin\theta \quad , \tag{15}$$

$$E_\phi = \frac{80mq^2c}{\sqrt{\pi}} \left(1 - \frac{2}{5}\frac{r^2}{(4qct)}\right) \frac{re^{-r^2/4qct}}{(4qct)^{7/2}} \sin\theta \quad . \tag{16}$$

From these equations, it is seen that the characteristic time of the field variation is

$$t_r = r^2/4qc \quad . \tag{17}$$

For $r \simeq 300$ m and $q \simeq 30$ cm, $t_r \simeq 0.25 \times 10^{-3}$ sec. Thus, a nuclear explosion at 300 m underground should produce a VLF signal at the surface of the ground. The signal's time dependence is determined by the ground's electrical properties if the source's time dependence is much faster than VLF.

The time dependences of H_r, H_θ, and E_ϕ are shown in Figs. 2, 3, and 4, respectively.

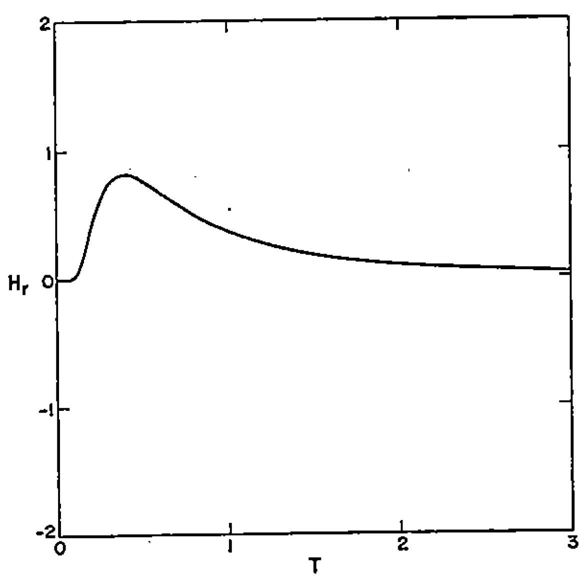

Fig. 2. Plot of $T^{-5/2}e^{-1/T}$ where $T = t/t_r$. This gives the time dependence of H_r underground. For units, see text.

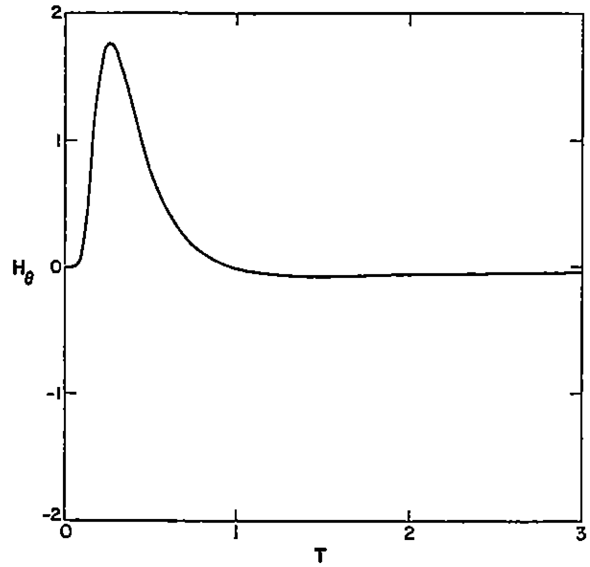

Fig. 3. Plot of $-T^{-5/2}(1-T^{-1})e^{-1/T}$ where $T = t/t_r$. This gives the time dependence of H_θ underground. For units, see the text.

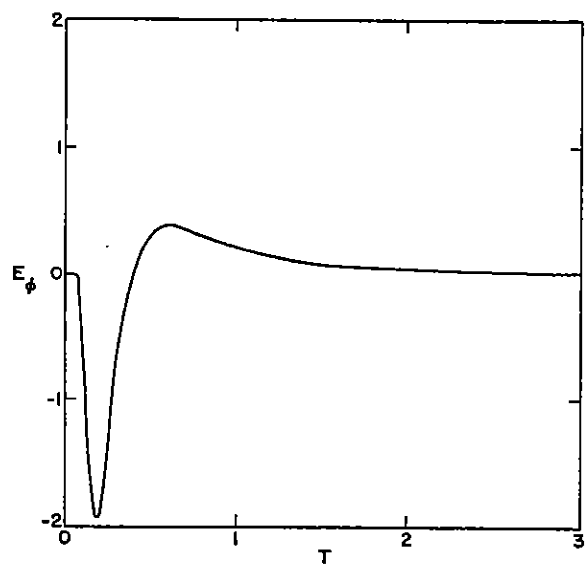

Fig. 4. Plot of $T^{-7/2}(1-0.4T^{-1})e^{-1/T}$ where $T = t/t_r$. This gives the time dependence of E_ϕ underground. For units, see the text.

In these figures, time is expressed in units of t_r. The units of H_r and H_θ are $16mqc/(r^5\sqrt{\pi})$ multiplied by $\cos\theta$ for H_r and $\sin\theta$ for H_θ. For E_ϕ, the units are $80\ mq^2c\ \sin\theta/(r^6\sqrt{\pi})$. Thus, the electric field is smaller than the magnetic field by a factor of about $5q/r$ for $r >> q$. These figures give the time dependence of the underground signal for any r that is large compared to q in terms of the value of t_r appropriate to that r. Note, however, that t_r is proportional to the ground conductivity, which may differ by as much as an order of magnitude from the value assumed here.

Tatro[3] has observed the EMP produced at the ground surface near ground zero by an underground nuclear explosion, using magnetometers and 300-ft-radius, horizontal, loop antennas. His instrumentation was intended for frequencies in the ELF to dc range; however, the largest amplitude part of the signal he observed was an initial pulse with a width that appears to be in the millisecond range. Since this experiment was intended to measure signals of longer time duration, the details of the recorded pulse cannot be very well determined because of the slow sweep speed. Moreover, because of the sensors used it probably did not give a good representation of the actual signal, particularly for the magnetometers. However, this pulse does seem to have the time dependence of the signals calculated here. Its amplitude was recorded as ∿ 50 gammas (γ). This initial pulse was followed by a more slowly varying signal of smaller amplitude. Possibly this slowly varying signal is partly due to the slowly varying tails of the calculated signals in Figs. 2 and 3.

Since the time dependence of the recorded pulses was outside the intended frequency response of the instrumentation, it is probably reasonable to regard 50 γ as a lower limit to the amplitude of the observed signal. With this assumption, (Eqs. (14) - (16) can be used to extrapolate the signal back to the region near the burst point. Both H_r and H_θ are functions of the form $(1/r^3)F(t/t_r)$. The signal was observed at a distance $r = h \simeq 1000q$ from the burst point. If the signal was 50 γ with millisecond time dependence at $r = h$, then at $r = h/10 = 100q$ it would be 50 G with time dependence in the 10 μsec range. The peak energy density in the magnetic field would be about 100 ergs/cm^3 at this distance. If this energy density occupied a volume equivalent to that of a sphere of 30 m radius, the total energy would be ∿ 10^{13} ergs. For comparison, 1 kt of TNT is equivalent to 4.2×10^{19} ergs. Since the peak electric field is ∿ $5q/r$ times the peak magnetic field, this implies that at $r = h/10 = 100q$ the electric field $E_\phi \simeq H_\theta/20 \simeq 2.5$ statvolt/cm = 7.5×10^4 V/m, and if the conductivity is still 10^{-2} mho/m this close to the burst point the peak current density would be 750 A/m^2. As noted above, these values probably represent lower limits to the field strengths and current densities consistent with the signal observed at the ground surface. This gives an indication of the nature of the source that would be required to explain the observed signal.

In Fig. 5 the field lines of H, which lie in planes of constant ϕ, are plotted. Also, in this figure the values in the equatorial plane of E_ϕ in units of $80mq^2c/(\sqrt{\pi}\,(4qct)^3)$ and H_θ in units of $16mqc/(\sqrt{\pi}\,(4qct)^{5/2})$ are plotted vs distance from the burst point. The distances x and z are expressed in units of

$$a = \sqrt{4qct} \quad . \tag{18}$$

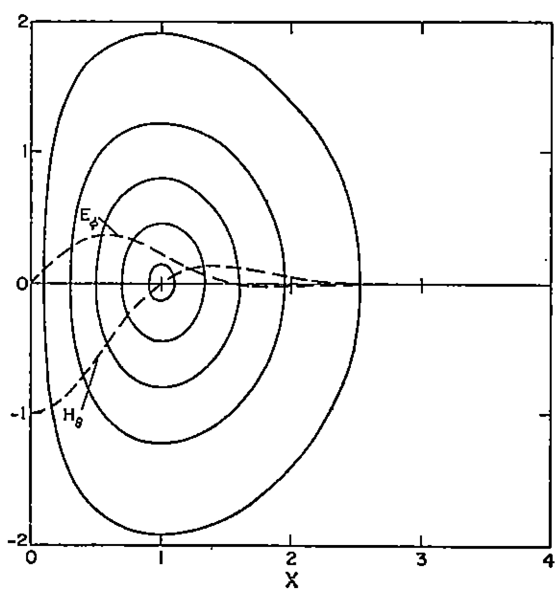

Fig. 5. The magnetic lines of force in the underground region. These lines of force (shown as solid lines) lie in a plane of constant ϕ which in this figure is taken to be the x-z plane. The field has axial symmetry about the polar, or z, axis. Also shown are the values of E_ϕ and H_θ in the equatorial or x-y plane. The scale on the vertical axis gives the value of z as well as of E_ϕ and H_θ. For units, see the text.

For $t = 10^{-6}$ sec, $a \simeq 20$ m and for $t = 10^{-4}$ sec $a \simeq 200$ m. Thus, the electromagnetic field requires $\sim 10^{-4}$ sec to diffuse from the burst point to the ground surface. This time delay can also be seen in Figs. 2 - 4.

Figure 5 shows that the electric current density in the ground, σE_ϕ, flows in a ring about the z axis. This ring expands like a smoke ring and carries its associated magnetic field with it. Because of the high electrical conductivity of the medium, the electric field necessary to drive the current is small compared to the magnetic field. It was shown above that E_ϕ is $\sim 5q/r$ times H_θ, so the disturbance becomes largely a magnetic phenomenon at a distance of a few hundred meters from the source. Both $\vec{E}$ and $\vec{J}$ are solenoidal vectors, so there is no electric charge distribution associated with the disturbance, as must surely be the case for times later than a few microseconds within such a highly conducting medium. A more general underground EMP could consist of a sum of solutions of this type, each with its source occurring within a time span of a few microseconds and within a volume of dimensions several meters or less. However, far from the sources, such a sum of solutions would look approximately like a single solution of the above type with $\vec{m}$ representing the vector sum of all of the source dipoles if this sum did not vanish. Thus, it is reasonable to believe that the above solution represents a qualitatively correct picture of the underground EMP. The important assumptions on which it is based are that the medium's conductivity is so high that the current must be mainly solenoidal and that the magnetic-dipole part of the source is more important than higher order multipoles.

Since the magnetic dipole in the source is not necessarily vertical, the ring current associated with the emitted field will intersect the ground surface, which it cannot flow through, and therefore be perturbed in some complicated way. This will in turn perturb the signal produced at the ground surface and consequently further complicate the interpretation of measured field strengths in terms of the source strength or orientation. Clearly, the only way to obtain quantitative results is to include the effect of the ground surface in the calculation of the electric and magnetic fields.

III. SURFACE EFFECTS AND THE RADIATED SIGNAL

To satisfy the boundary conditions at the ground surface, one must add to the Hertz vector $\vec{\Pi}_0$ calculated above a vector that is a solution to the homogeneous form of Eq. (6). Each component of this vector is of the form

$$\Pi_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} F_\omega(\vec{r}) \quad , \tag{19}$$

where $F_\omega(\vec{r})$ satisfies

$$(\nabla^2 + \Gamma^2) F_\omega(\vec{r}) = 0 \quad . \tag{20}$$

Below the ground surface, Γ^2 is as defined in Eq. (8) except that ε is taken to be zero, as already discussed, while above ground $\Gamma = \omega/c$. Thus, with the ground surface at $z = h$

$$\Gamma^2 = \begin{cases} \omega^2/c^2 & \text{for} \quad z > h \\ i\omega/qc & \text{for} \quad z < h \end{cases} \quad .$$

The function $F_\omega(\vec{r})$ can be expressed as a sum of functions of the form (for example, see Sommerfeld[1])

$$u(n,\alpha,\omega) = J_n(\alpha\rho) e^{\pm in\phi} f_\omega(z) \quad , \tag{21}$$

where ρ, ϕ, and z are cylindrical coordinates with the z axis directed vertically upward, and α and n are real constants. The parameter n must be an integer for this function to be single-valued. $J_n(\alpha\rho)$ is a Bessel function of the first kind and is therefore finite at $\rho = 0$. The function $f_\omega(z)$ satisfies

$$\left(\frac{d^2}{dz^2} + \Gamma^2 - \alpha^2\right) f_\omega(z) = 0 \quad . \tag{22}$$

In the underground region $f_\omega(z)$ is thus a linear combination of functions of the form $e^{\pm i\gamma z}$ where

$$\gamma = \sqrt{i\omega/qc - \alpha^2} \quad . \tag{23}$$

The sign of the square root is to be chosen such that the real part of γ has the same sign as ω and the imaginary part is positive for either sign of ω when both ω and α are real.

Above ground, f(z) is of the form

$$e^{i\beta z} \quad ,$$

where

$$\beta = \sqrt{\omega^2/c^2 - \alpha^2} \quad . \tag{24}$$

Above ground, only the upward-propagating wave is present because the source of radiation is underground. The particular linear combination of upward and downward waves to be used underground is to be determined by the boundary condition that $\vec{\Pi}$ must satisfy at the surface.

It is also necessary to express the solution of the inhomogeneous equation $\vec{\Pi}_0$ as a sum of products of Bessel functions and upward- or downward-propagating waves. This can be done by using the relation

$$\frac{e^{i\Gamma r}}{r} = i\int_0^\infty \alpha d\alpha\; J_0(\alpha\rho)\; \frac{e^{i\gamma|z|}}{\gamma} \quad , \tag{25}$$

which may be derived, for example, by the method used by Sommerfeld (Ref. 1, pg. 242). The right-hand side of this equation is equivalent to a sum of functions of the form given in Eq. (21) with n = 0. As expected, this function contains only upward-propagating waves above the source and downward-propagating waves below it. With this result, $\vec{\Pi}_0$, given by Eq. (12), can be written in the region above the source, which is still taken to be at the origin of the coordinate system, and below the ground surface, taken to be the plane z = h, as

$$\vec{\Pi}_0 = \frac{i\vec{m}}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^\infty \alpha d\alpha\; J_0(\alpha\rho)\; \frac{e^{i\gamma z}}{\gamma} \quad . \tag{26}$$

The boundary conditions on $\vec{\Pi}$ are determined from those that $\vec{E}$ and $\vec{H}$ must satisfy; i.e., the tangential components of $\vec{E}$ and $\vec{H}$ must be continuous at z = h.

First consider the case in which $\vec{m}$ is directed vertically upward, i.e., has only a z component. Then $\vec{\Pi}$ has only a z component, and from Eqs. (3) - (5) the components of $\vec{E}$ are

$$E_x = -\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial \Pi_z}{\partial y} \quad , \tag{27a}$$

$$E_y = +\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial \Pi_z}{\partial x} \quad , \tag{27b}$$

$$E_z = 0 \quad , \tag{27c}$$

and those of $\vec{H}$ are

$$H_x = \frac{\partial}{\partial x}\frac{\partial \Pi_z}{\partial z} \quad , \tag{28a}$$

$$H_y = \frac{\partial}{\partial y}\frac{\partial \Pi_z}{\partial z} \quad , \tag{28b}$$

$$H_z = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_z \quad . \tag{28c}$$

The continuity of the tangential components of E and H implies that both Π_z and $\partial\Pi_z/\partial z$ are continuous at z = h for this case. Since Π_z is continuous, it follows that H_z is also continuous. For a vertical magnetic dipole, the electric field has no vertical component.

Now consider the case of a horizontal magnetic dipole in the x direction. Such a source will produce a $\vec{\Pi}$ vector with an x component; however, the boundary conditions at z = h cannot be satisfied unless $\vec{\Pi}$ also has a vertical component Π_z. This is the same situation that Sommerfeld[1] found for the case of a horizontal electric or magnetic dipole above an arbitrary earth. If $\vec{\Pi}$ has x and z components, then from Eqs. (3) - (5) the components of $\vec{E}$ are

$$E_x = -\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial \Pi_z}{\partial y} \quad , \tag{29a}$$

$$E_y = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial \Pi_x}{\partial z} - \frac{\partial \Pi_z}{\partial x}\right) \quad , \tag{29b}$$

$$E_z = +\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial \Pi_x}{\partial y} \quad , \tag{29c}$$

and those of $\vec{H}$ are

$$H_x = \frac{\partial}{\partial x}\nabla\cdot\vec{\Pi} - \left(\frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)\Pi_x$$

$$= \frac{\partial}{\partial x}\frac{\partial \Pi_z}{\partial z} - \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Pi_x \quad , \tag{30a}$$

$$H_y = \frac{\partial}{\partial y}\nabla\cdot\vec{\Pi} \quad , \tag{30b}$$

$$H_z = \frac{\partial}{\partial z}\nabla\cdot\vec{\Pi} - \left(\frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)\Pi_z$$

$$= \frac{\partial}{\partial z}\frac{\partial \Pi_x}{\partial x} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Pi_z \quad . \tag{30c}$$

The tangential components of $\vec{E}$ and $\vec{H}$ are continuous at $z = h$ if each of the following quantities is continuous there

$$\frac{\partial \Pi_x}{\partial z} , \quad \left(\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)\Pi_x , \quad \Pi_z , \quad \nabla\cdot\vec{\Pi} \quad .$$

Note that this implies that E_z is not continuous at $z = h$. The quantity that is continuous there is

$$\left(\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right)E_z = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\left(\frac{\varepsilon}{4\pi}\frac{\partial E_z}{\partial t} + \sigma E_z\right) \quad .$$

The quantity in parenthesis on the right-hand side is just the sum of the displacement current plus the conduction current. From Eq. (2) it follows that the divergence of this quantity is zero. Therefore, its normal component must be continuous across any surface. This is the reason for the complicated boundary conditions that $\vec{\Pi}$ must satisfy when $\vec{m}$ is not vertical and there is therefore a vertical electric-field component. Since Π_x is not continuous at $z = h$, the divergence of $\vec{\Pi}$ cannot be continuous unless $\vec{\Pi}$ has a z component such that the discontinuity in $\partial\Pi_z/\partial_z$ cancels that in $\partial\Pi_x/\partial_x$. Below ground the displacement current is negligible compared to the conduction current, but above ground the conduction current is zero. Therefore, any vertical conduction current below ground must connect nearly continuously to a vertical displacement current above ground. Thus, if the frequency ω is small compared to $4\pi\sigma/\varepsilon$, then the amplitude of the vertical electric field just above the surface is larger than that just below the surface by the factor $4\pi\sigma/\varepsilon\omega$.

Actually, this mathematical discontinuity is spread through a layer of the earth extending from the surface to a depth of at least several meters because of conductivity variations with depth near the surface. In this region, the vertical electric field (and possibly also the horizontal field) may be expected to vary rapidly with depth and with local variations in the earth conductivity.

Probably the actual source $\vec{m}$ of the observed underground EMP has both horizontal and vertical components. Because the boundary conditions on differ for these two components, it is necessary to consider the cases of horizontal $\vec{m}$ and vertical $\vec{m}$ separately. Most of the discussion will be devoted to the case of a horizontal $\vec{m}$ because it is the most complicated. Also, it turns out that far from the source the signal due to a horizontal $\vec{m}$ is much larger than that due to a vertical $\vec{m}$ of the same magnitude.

For a source $\vec{m}$ in the x direction, the x component of $\vec{\Pi}$ above ground is of the form

$$\Pi_x = \frac{im}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_0(\alpha\rho) \times \frac{be^{i\beta(z-h)}}{\gamma}\, e^{i\gamma h} \qquad z>h , \tag{31}$$

whereas above the source but below the ground surface,

$$\Pi_x = \frac{im}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_0(\alpha\rho) \times \frac{e^{i\gamma(z-h)} + ae^{-i\gamma(z-h)}}{\gamma}\, e^{i\gamma h} \qquad 0<z<h , \tag{32}$$

where a and b are constants that depend on α and ω. From the above discussion it is clear that this Π_x is a solution of the homogeneous form of Eq. (6) both above and below ground for $z > 0$. If a is zero, then Π_x for $0 < z < h$ reduces to Π_0 as expressed in Eq. (26). Underground, the part of Π_x proportional to a represents a wave reflected downward from the surface, whereas above ground Π_x contains only an upward wave that is transmitted through the surface. The factors $e^{i\gamma h}$ (which, like a and b, depend on α and ω) were inserted only to simplify the expressions for a and b.

From the boundary conditions found above for Π_x, it is easily shown that a and b satisfy

$$\beta b = \gamma(1-a) \quad ,$$

and

$$\omega b = 4\pi i\sigma(1+a) \quad , \tag{33}$$

where again in the underground region $\varepsilon\frac{\omega^2}{c^2}$ is neglected compared to $\frac{4\pi i\sigma\omega}{c^2}$. From these equations it follows that

$$a = -\frac{\beta + i\frac{\omega}{c}q\gamma}{\beta - i\frac{\omega}{c}q\gamma} \quad ,$$
$$b = \frac{2\gamma}{\beta - i\frac{\omega}{c}q\gamma} \quad , \tag{34}$$

which completes the determination of Π_x.

The determination of Π_z depends on the continuity of $\nabla\cdot\vec{\Pi}$ which involves $\partial\Pi_x/\partial x$. Since Π_x is independent of ϕ, it follows that

$$\frac{\partial\Pi_x}{\partial x} = \frac{\partial\rho}{\partial x}\frac{\partial\Pi_x}{\partial\rho} = \cos\phi\,\frac{\partial\Pi_x}{\partial\rho} \quad , \qquad (35)$$

and, since $\partial J_0(\alpha\rho)/\partial\rho = -\alpha J_1(\alpha\rho)$, the above expressions for Π_x can be differentiated with respect to x to obtain

$$\frac{\partial\Pi_x}{\partial x} = -\frac{im}{2\pi}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_1(\alpha\rho)\cos\phi \times \alpha b\frac{e^{i\beta(z-h)}}{\gamma}e^{i\gamma h} \qquad z>h \quad , \qquad (36)$$

$$\frac{\partial\Pi_x}{\partial x} = -\frac{im}{2\pi}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_1(\alpha\rho)\cos\phi \times \alpha\frac{e^{i\gamma(z-h)} + ae^{-i\gamma(z-h)}}{\gamma}e^{i\gamma h} \qquad 0<z<h \quad . \qquad (37)$$

From these equations it follows that Π_z must contain $J_1(\alpha\rho)\cos\phi$ rather than $J_0(\alpha\rho)$. Also, because the source of Π_z is the electric current induced in the surface of the ground, Π_z must contain only an upward-propagating wave for $z > h$ and a downward propagating wave for $z < h$. Thus, Π_z can be written as

$$\Pi_z = \frac{im}{2\pi}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_1(\alpha\rho)\cos\phi \times b'\frac{e^{i\beta(z-h)}}{\gamma}e^{i\gamma h} \qquad z>h \quad , \qquad (38)$$

$$\Pi_z = \frac{im}{2\pi}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\int_0^{\infty}\alpha d\alpha\, J_1(\alpha\rho)\cos\phi \times b'\frac{e^{-i\gamma(z-h)}}{\gamma}e^{i\gamma h} \qquad z<h \quad . \qquad (39)$$

Here the same constant b' has been used both above and below the surface because of the boundary condition that Π_z be continuous at $z = h$. The requirement that $\partial\Pi x/\partial x + \partial\Pi z/\partial z$ be continuous at $z = h$ implies that

$$-\alpha b + i\beta b' = -\alpha(1+a) - i\gamma b' \quad ,$$

or

$$b' = \frac{-i\alpha}{\beta+\gamma}\left[b - (1+a)\right] \quad . \qquad (40)$$

In all of the integrals that give expressions for components of $\vec{\Pi}$ near or above the ground surface, the factor $e^{i\gamma h}$ occurs in the integrand. The main contribution to the integrals comes from those values of α and ω for which this factor is ~ 1. Equation (23) shows that if α^2 is small enough to be neglected, the imaginary part of γ is $\sqrt{\omega/(2qc)}$; for larger values of α^2 the imaginary part of γ becomes larger. Thus, the frequencies that contribute significantly are those for which

$$h\sqrt{\omega/(2qc)} = \frac{h}{q}\frac{1}{\sqrt{2}}\sqrt{\omega q/c} \le 1 \quad . \qquad (41)$$

Since $h/q \simeq 10^3$, the important frequencies are those for which

$$|\omega q/c| \le 10^{-6} \quad . \qquad (42)$$

The relative importance of the terms β and $i(\omega/c)q\gamma$ in the expressions for a and b can now be examined. If $\alpha^2 << |\omega/(qc)|$, then $|q\gamma| \simeq |\sqrt{i\omega q/c}| \le 10^{-3}$ for frequencies that satisfy inequality (42), and except for a very small part of the range of α about the point where $\beta = 0$, i.e., where $\alpha = \omega/c$, the term $i(\omega/c)q\gamma$ is negligible compared to β. Suppose, on the other hand, that $\alpha^2 \ge |\omega/(qc)|$. Then

$$\alpha^2 \ge \frac{\omega^2/c^2}{|\omega q/c|} \ge 10^6\ \omega^2/c^2$$

for frequencies that satisfy inequality (42). In this case, from Eqs. (23) and (24) it follows that $|\gamma| \simeq \alpha$ and $|\beta| \simeq \alpha$, so again $|\beta| >> |i(\omega/c)q\gamma|$. The only part of the range of α where the term $i(\omega/c)q\gamma$ is not negligibly small compared to β is near $\alpha = \omega/c$, and then

$$|\beta| \simeq \left|\sqrt{2\frac{\omega}{c}}\sqrt{\frac{\omega}{c} - \alpha}\right| \quad ,$$

$$\left|\frac{\omega}{c}q\gamma\right| \simeq \left|\frac{\omega}{c}\sqrt{i\omega q/c}\right| \le 10^{-3}\left|\frac{\omega}{c}\right| \quad ,$$

where again inequality (42) is assumed to be satisfied, and use is made of the fact just established that if $\alpha \simeq \omega/c$, then $\alpha^2 << \omega/qc$, so $q\gamma \simeq \sqrt{i\omega q/c}$. The range of α over which $|\beta|$ is not much larger than $|\frac{\omega}{c}q\gamma|$ is thus

$$|\omega/c - \alpha| \le 10^{-6}|\ \omega/c|$$

if inequality (42) is satisfied. This range is too small to be significant, and in the expressions for a and b/γ it is good approximation to set $q = 0$. This gives

$$a \simeq -1 \quad ,$$
$$b/\gamma \simeq 2/\beta \quad , \tag{43}$$

which gives for Π_x above and below the surface

$$\Pi_x = \frac{im}{\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^{\infty} \alpha d\alpha \; J_0(\alpha\rho) \times \frac{e^{i\beta(z-h)}}{\beta} e^{i\gamma h} \qquad z>h \; , \tag{44}$$

$$\Pi_x = \frac{im}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^{\infty} \alpha d\alpha \; J_0(\alpha\rho) \times \frac{e^{i\gamma z} - e^{-i\gamma(z-2h)}}{\gamma} \qquad 0<z<h \; . \tag{45}$$

The fact that the integrand of Eq. (44) is infinite at $\alpha = \frac{\omega}{c}$ causes no difficulty because the integral itself is well behaved there.

If the above approximate values of a and b/γ are substituted into Eq. (40) for b', the result is

$$b' = \frac{-2i\alpha\gamma}{\beta(\beta+\gamma)} \quad ,$$

and if also the relation $\partial J_0(\alpha\rho)/\partial x = - \alpha J_1(\alpha\rho) \cos\phi$ is used, it is easy to show from Eqs. (38) and (39) that in this approximation

$$\frac{\partial \Pi_z}{\partial z} = \frac{\partial}{\partial x} \frac{im}{\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^{\infty} \alpha d\alpha \; J_0(\alpha\rho) \times \frac{e^{i\beta(z-h)}}{\beta} \frac{(-\beta)}{\beta+\gamma} e^{i\gamma h} \qquad z>h \; , \tag{46}$$

$$\frac{\partial \Pi_z}{\partial z} = \frac{\partial}{\partial x} \frac{im}{\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^{\infty} \alpha d\alpha \; J_0(\alpha\rho) \times \frac{e^{-i\gamma(z-h)}}{\beta} \frac{\gamma}{\beta+\gamma} e^{i\gamma h} \qquad z<h \; . \tag{47}$$

From Eqs. (44) - (47) it is easy to see that $\partial \Pi_x/\partial z$, Π_z, and $\nabla \cdot \vec{\Pi}$ are all continuous at $z = h$ in this approximation and therefore satisfy their respective boundary conditions. (Π_z is obtained from Eqs. (46) and (47) by simply dividing the integrand by $i\beta$ and $-i\gamma$, respectively). The discontinuity in Π_x at $z = h$ is such that Π_x jumps from zero just beneath the surface to a finite value just above it; thus, these expressions for Π_x and Π_z satisfy boundary conditions corresponding to infinite earth conductivity. The actual earth conductivity is so high that these boundary conditions are a good approximation, as can be seen from the above discussion of the relative importance of the terms β and $i(\omega/c)q\gamma$ in a and b.

Below the surface, Π_x given by Eq. (45) is the sum of the field of a magnetic dipole in the x direction located at the origin plus the field of its image in the minus-x direction located at a height 2h above the origin. For this source configuration, both the vertical electric field and the horizontal magnetic field approach zero as z approaches h from below. The other field components are doubled as z approaches h from below. This can be seen, for example, by calculating the contributions to the field components due to Π_x alone using Eqs. (29) and (30). However, since Π_z is not zero, this image picture does not correctly represent the underground electric and magnetic fields. In particular, it is easy to see that the contributions of Π_z to H_x and H_y do not in general vanish just below the surface.

Because both $\vec{E}$ and $\vec{H}$ have horizontal components at $z = h$, the energy flux through the ground surface, given by $\frac{c}{4\pi} \vec{E} \times \vec{H}$, can differ from zero. If the electric field has a vertical component above ground, there must be a vertical displacement current there, which, as was pointed out above, must connect almost continuously to a vertical conduction current underground. If the underground conductivity is large, a significant vertical current can exist there even if the underground vertical electric field is small. This vertical current is the source of a horizontal magnetic field, which must be present if there is an upward energy flux through the surface. The underground vertical current leads to a charge distribution on the ground surface, which is the source of the vertical electric field above ground.

The infinite ground conductivity approximation has been made only in the calculation of a and b. The integrals of Eqs. (44) and (46) still depend on the conductivity through the function γ. In particular, it was shown that the factor $e^{i\gamma h}$ limits the frequencies that contribute significantly to the integrals to low values. This factor would also tend to cut off the α integrals for large α if they were not cut off sooner for other reasons. It was shown above that the dependence of γ on α is negligible unless α is much greater than $|\omega/c|$, and if this is so, $\beta \simeq i\alpha$, and the α integrals will be cut off sooner by the factors $e^{i\beta(z-h)}$ than by $e^{i\gamma h}$ if $z - h$ is much greater than h. Suppose, on the other hand, that $z - h \simeq 0$ but ρ is much larger than h. Then when α is large compared to $|\omega/c|$, so that $\beta \simeq i\alpha$, and $\alpha\rho$ is large compared to 1, so that $J_0(\alpha\rho)$ approaches its asymptotic form,

$$\frac{\alpha J_0(\alpha\rho)}{\beta} \simeq - i\frac{\cos(\alpha\rho-\pi/4)}{\sqrt{\pi\alpha\rho/2}}$$

which oscillates rapidly as a function of α with slowly decreasing amplitude. If ρ is much greater than h, this function always varies more rapidly with α than does $e^{i\gamma h}$. The larger ρ is, the more completely the contributions to the integrals in Eqs. (44) and (46) from alternate half-cycles cancel each other, so again the α integral will cut off before α becomes large enough to contribute significantly to γ.

It is useful to define a distance r' from the point on the ground surface just above the source to the point where the field is evaluated;

$$r' = \sqrt{(z-h)^2 + \rho^2} \quad , \tag{48}$$

where it is to be understood that $z - h$ is never less than zero. Then if $r' \gg h$, the α dependence of γ can be ignored in Eqs. (44) and (46). In these equations,

$$\gamma \simeq \sqrt{i\omega/qc} = \frac{\omega}{c}\sqrt{ic/\omega q} \quad , \tag{49}$$

and from Eq. (42) it follows that for the important part of the range of α and ω, $|\gamma| \geq 10^3|\omega/c|$. Also, since $|\gamma| \gg |\alpha|$ for the important part of the range of α and ω, it follows that

$$|\gamma| \gg |\beta| \ .$$

Equations (44) and (46) show that the equations for Π_x and Π_z above ground differ only in that the Π_z equation contains a derivative with respect to x and the factor $i/(\beta+\gamma)$ in the integrand. Because the x derivative is proportional to α, the Π_z integrand is smaller in absolute value by roughly the factor $|\alpha/(\beta+\gamma)|$, which is much less than 1. Thus, above ground for $r' \gg h$, Π_z is negligible compared to Π_x.

In this connection, it is interesting to compare the case in which $\vec{m}$ is in the z direction, rather than in the x direction. Then $\vec{\Pi}$ has only a z component, which is given by expressions of the same form as the right-hand sides of Eqs. (31) and (32). However, in this case the boundary conditions on Π_z imply that

$$b = 1 + a \quad ,$$

$$\beta b = \gamma(1-a) \quad ,$$

or

$$a = \frac{\gamma - \beta}{\gamma + \beta} \quad ,$$

$$\frac{b}{\gamma} = \frac{2}{\gamma + \beta} \quad .$$

Comparing these equations with Eq. (43) shows that because $|\gamma| \gg |\beta|$, Π_z due to a vertical source is very small compared to Π_x due to a similar horizontal source for $r' \gg h$.

In summary, the factor $e^{i\gamma h}$ in the integrands of the equations for the Hertz vector in the above-ground region limits the contributing frequencies to low values. In particular, Eq. (41) is equivalent to the condition

$$|\omega/c| \leq q/h^2 \quad .$$

The structure of the integrals giving the components of $\vec{\Pi}$ above ground far from the source is such that most of the contribution comes from values of α and ω such that

$$\alpha^2 \leq \omega^2/c^2 \quad , \quad \beta^2 \leq \omega^2/c^2 \quad ,$$

and

$$|\gamma^2| = |\omega/qc| = \frac{\omega^2}{c^2}\left|\frac{c}{\omega q}\right| \geq \frac{\omega^2}{c^2}\frac{h^2}{q^2} \quad .$$

Because $h/q \approx 10^3$, α and β are negligibly small compared to γ and $(\omega/c)q\gamma$ is negligibly small compared to α and β for the important part of the range of α and ω.

This makes it possible to use convenient approximations for the quantities $be^{i\gamma h}$ and $b'e^{i\gamma h}$ that occur in these integrals when they are evaluated to give the remote above-ground signal. These approximations are equivalent to taking the limit zero q, zero h with q/h^2 held constant. In this limit, for z greater than h the Hertz vector has only a horizontal component whose source is the horizontal component of $\vec{m}$. This is because of the different boundary conditions that Π_x and Π_z satisfy, which, in turn, is due to the fact that only the horizontal component of the source dipole produces a vertical electric field, which must be much larger just above than just below the ground surface. Consequently, Π_x, which gives the vertical electric field, must be much larger just above than just below the surface, whereas Π_z is continuous there. The radiated signal for r' much greater than h can therefore be obtained from Eqs. (29) and (30) with Π_z set equal to zero. However, in the underground region Π_z is not negligible.

To calculate the signal produced at a distance from the source, it remains only to evaluate the integral in Eq. (44). Because the dependence of γ on α is not important, the factor $e^{i\gamma h}$ can be removed from under the integral sign of the α integral. Then this equation becomes

$$\Pi_x = \frac{im}{\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\, e^{ih\sqrt{i\omega/qc}} \times \int_0^{\infty} \alpha d\alpha\, J_0(\alpha\rho)\, \frac{e^{i\beta(z-h)}}{\beta} \qquad z>h\ . \tag{50}$$

The α integral can be evaluated from Eq. (25). If, in Eq. (25), Γ is set equal to ω/c, then γ, which is $\Gamma^2-\alpha^2$, becomes β. This leads to

$$\frac{e^{i(\omega/c)r'}}{r'} = i\int_0^{\infty} \alpha d\alpha\, J_0(\alpha\rho)\, \frac{e^{i\beta(z-h)}}{\beta}\ , \tag{51}$$

where r' is defined in Eq. (48). If a retarded time

$$\tau = t - \frac{r'}{c} \tag{52}$$

is now introduced, Eq. (50) can be written as

$$\Pi_x = \frac{m}{\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega\tau}\, \frac{e^{ih\sqrt{i\omega/(qc)}}}{r'}\ . \tag{53}$$

The ω integral in this equation is the same as that in Eq. (12), which has already been evaluated. Thus, the result for Π_x above the ground surface and a distance at least several times h from ground zero is

$$\Pi_x = \frac{2h}{r'}\,(4\pi qcm)\,(4\pi qc\tau)^{-3/2}\, e^{-h^2/(4qc\tau)}$$

$$= \frac{8mqc}{\sqrt{\pi}\, r'h}\, T^{-3/2}\, e^{-1/T}\ , \tag{54}$$

where

$$T = \frac{4qc\tau}{h^2}\ . \tag{55}$$

Here the characteristic time $t_h = h^2/4qc$ is a constant, rather than increasing with r^2 as it does underground. Also Π_x above ground depends on the retarded time τ; it is a hemispherical wave centered on the surface just above the source. It propagates radially with velocity c while its amplitude decreases as $1/r'$.

The vertical electric field is given by

$$E_z = \frac{1}{c}\frac{\partial}{\partial t}\frac{\partial\Pi_x}{\partial y} = \frac{1}{c}\frac{\partial}{\partial t}\sin\theta\,\sin\phi\,\frac{\partial\Pi_x}{\partial r'}\ ,$$

where θ and ϕ are the spherical-coordinate angles of r' in a coordinate system with its origin on the surface directly above the source; θ is measured from the vertical, and ϕ is measured from the direction of $\vec{m}$. From the above expression for Π_x, the vertical electric field is

$$E_z = -\frac{16mqc}{\sqrt{\pi}\, r'h^4}\left(8\,\frac{q^2}{h^2}\right)\sin\theta\,\sin\phi \times \left(\frac{h^2}{4qr'}\frac{\partial}{\partial T} + \frac{\partial^2}{\partial T^2}\right)T^{-3/2}\, e^{-1/T}\ . \tag{56}$$

If h = 300 m, then $t_h = h^2/4qc = 0.25 \times 10^{-3}$ sec and ct_h = 75 km. When r' is small compared to ct_h, the first term in the above expression is dominant. This is the induction term; the induction zone extends ∿ 75 km from the source. Within the induction zone, the time dependence of the radiated signal depends on distance from the source. This is shown in Figs. 6 and 7 which give the time dependence of the signal at $r' = 0.2\ ct_h$ and $r' = 1.6\ ct_h$, respectively. The quantity plotted there is

$$S(T) \equiv \left(\frac{h^2}{4qr'} \frac{\partial}{\partial T} + \frac{\partial^2}{\partial T^2} \right) T^{-3/2} e^{-1/T} \quad .$$

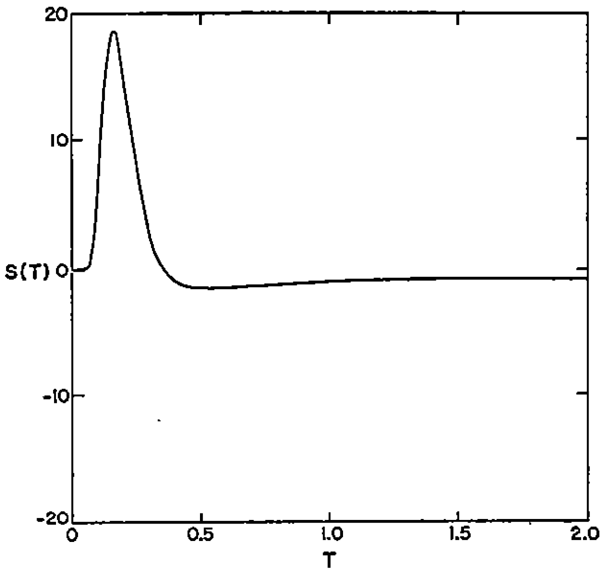

Fig. 6. Time dependence of E_z at a distance $r' = 0.2\ ct_h$. The time variable is $T = \tau/t_h$. For units, see the text.

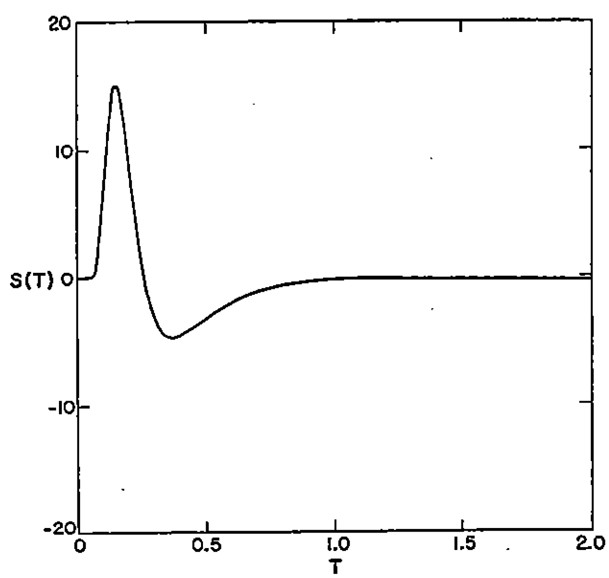

Fig. 7. Time dependence of E_z at a distance $r' = 1.6\ ct_h$. The time variable is $T = \tau/t_h$. For units, see the text.

If Tatro's measurements, in conjunction with Fig. 2 or 3, are interpreted to mean that

$$\frac{16mqc}{\sqrt{\pi}\ h^5} \approx 50\gamma = 15\ \text{V/m} \quad , \qquad (57)$$

and $8q^2/h^2 = 8 \times 10^{-6}$, this gives for E_z, from Eq. (56),

$$E_z \approx -\left(120\ \mu\text{V/m}\right) \frac{h}{r'} \sin\theta \sin\phi\ S(T) \quad .$$

As indicated, this should probably be interpreted as a lower limit to the expected signal. It was shown that a signal of this amplitude extrapolated back to the time when the peak magnetic field was only 30 m from the underground source contained an energy of $\sim 10^{13}$ ergs, which is less than 10^{-6} kt.

Note the dependence of the calculated signal on the assumed value of the conductivity. If σ is 10^{-3}, instead of 10^{-2} mho/m, q is increased by a factor of 10, and the characteristic time and the length of the induction zone are each reduced by a factor of 10. If Eq. (57) is still assumed to hold, the source amplitude $\vec{m}$ is reduced by a factor of 10, and E_z, given by Eq. (56), is increased by a factor of 100 for $r' \gg ct_h$.

CONCLUSION

According to "The Effects of Nuclear Weapons," p. 67,[4] an underground nuclear explosion expands the chamber containing the burst point to a radius of a few tens of meters in a time of a few hundredths of a second. During the explosion and the initial part of the expansion, an EMP may be produced which will propagate through the ground faster than the expansion of the chamber.

An electromagnetic field in the earth causes an electric current, which is itself a source of an electromagnetic field. Thus, shortly after the detonation of a bomb underground, the resulting electromagnetic field will be contained in a 20- to 30-m-radius sphere in the surrounding earth, and the induced currents in that sphere may be regarded as the source of the subsequent EMP. Equation (18) showed that the field is contained within a radius of ~ 20 m after about 10^{-6} sec, whereas for reasonable values of the electrical conductivity and dielectric constant of the earth, the decay time of an electric charge distribution, $\varepsilon/(4\pi\sigma)$, is less than 10^{-7} sec. It therefore seems reasonable to take the source of the underground EMP to be a solenoidal current distribution contained in a sphere of ∿ 20- to 30-m radius with a time duration in the μsec range. The simplest such source is a magnetic dipole. The events that occur between the initiation of the explosion and the time when the electromagnetic field has expanded to occupy a sphere of this size undoubtedly involve production of electric charge distributions and, therefore, nonsolenoidal current densities, but if the charge distributions decay, the current density must become solenoidal. Also, higher order multipoles are probably produced, but they can be represented by sums of magnetic dipoles. Because of the diffusion-like propagation of electromagnetic fields in a conducting medium, an electromagnetic pulse will be spread in time after propagating some distance underground. Consequently, the details of the space and time dependence of its source will be lost. Therefore, it is appropriate to investigate the EMP produced by an underground point dipole with delta function time dependence.

The underground magnetic-dipole problem was solved in the time domain. The nature of the underground signal during the time before the pulse arrives at the ground surface was determined. This signal consists of a magnetic field diffusing through the conducting earth, with a much smaller associated electric field. The electric field causes a current to flow in the earth, which results in Joule heating with a consequent loss of energy from the electromagnetic disturbance. If the source has delta function time dependence, the signal after propagating 300 m, is a pulse ∿ 10^{-4} sec wide.

The effect of the ground surface was also considered. A careful examination of the consequences of the boundary conditions at this surface indicates that the signal due to a horizontal magnetic dipole is much more strongly transmitted through the surface than is that due to a vertical magnetic dipole. The physical reason for this phenomenon is examined in detail. The signal produced by a horizontal magnetic dipole and transmitted through the ground surface to remote points in the atmosphere is calculated. It was shown that if the electromagnetic energy in the disturbance at a time when it is contained in a 30-m-radius sphere is equivalent to ∿ 10^{-6} kt, the EMP should be observable above ground out to a few kilometers from ground zero. The time dependence of the remote, above-ground signal depends on the depth of the burst below the surface. For a 300-m depth the signal is a pulse in the 0.1 msec range.

REFERENCES

1. Arnold Sommerfeld, Partial Differential Equations in Physics (Academic Press, Inc., New York, 1949).

2. T. G. Cowling, Magnetohydrodynamics (Interscience Publishers, Inc., New York, 1957).

3. L. D. Tatro, Los Alamos Scientific Laboratory, unpublished data.

4. The Effects of Nuclear Weapons (United States Atomic Energy Commission, 1962).