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TIME DEPENDENT, ANALYTIC, ELECTROMAGNETIC SOLUTION FOR A HIGHLY CONDUCTING SPHERE

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Topical Report

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The time-dependent, analytic solution for the electromagnetic, axisymmetric, highly conducting sphere problem is derived in terms of arbitrary, spatial current sources. In particular, the skin current at the surface is shown to be determined by the modes of oscillation of the sphere together with the field due only to the spatial current sources, at the surface of the sphere.

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SECTION 1

INTRODUCTION

This report is one of a continuing series of reports dealing with the analytic investigation of the SGEMP problem. In particular, this report is the first to deal with the electromagnetic response of a conducting system.

Photoelectrons, emitted by an object which has been struck by X rays, act as sources for electromagnetic fields. These electromagnetic fields interact with the object producing skin currents and surface charge densities. When the photoelectrons are produced inside the system the problem is usually referred to as an IEMP (Internal Electromagnetic Pulse) problem. When the electrons are produced outside the system the problem is referred to as an external SGEMP problem. The subject of this report will be the external SGEMP problem for a conducting sphere.

It is intended that a two-fold purpose be served by this report. The first is the presentation of the complete, time-dependent, analytic solution for the electromagnetic, axisymmetric sphere problem. This solution is predicated upon a knowledge of the time dependence of photoelectron currents (the sources) in the space external to the sphere. Other reports, in this series[1,2,3], deal with the calculation of the photoelectron currents. The analytic solution, for the sphere, is valuable, in itself, because it provides insight into the external SGEMP problem as a whole. It is also valuable because it allows one to predict the entire electromagnetic response, under arbitrary prescribed source conditions, for at least one system. In contrast, computer codes[4] dealing with the same problem are limited by stability conditions and grid size.

The second purpose of this report is to emphasize that the SGEMP response of a system is really a two-dimensional surface problem. This point of view has been recognized by the EMP community for some time[5,8]. In this report we explicitly demonstrate, for the case of a sphere, that the surface response of a system is determined by the modes of that system together with the fields at the surface, due to the photoelectron sources alone. One of the difficulties, in predicting the SGEMP response of complex objects, is the three-dimensional nature of the problem. The possibility of transforming part of the treatment of that problem, from the realm of three dimensions, to a two-dimensional realm is attractive indeed.

In Sections 2 through 4 of this report we derive the time-dependent solution for the conducting sphere. A future report will provide examples of the use of this solution. The future report will also discuss several specific aspects of the SGEMP problem, based upon the mathematical solution derived in this report.

SECTION 2

DISCUSSION AND FIELD EQUATIONS

Given a highly conducting sphere in space and a piecewise continuous (spatially as well as timewise) current, emanating from the sphere or elsewhere in space, the electromagnetic fields anywhere in space and time can be found. The problem discussed in this report is assumed to be axisymmetric for simplicity. However, the generalization to the non-axisymmetric case is clear and can be done, if desired.

The method of solution used here is to first expand the fields and current in terms of the associated Legendre polynomials $P_\ell^0(\cos\theta)$, $P_\ell^1(\cos\theta)$. The spatial coordinates are spherical coordinates. The problem is independent of the azimuthal angle ψ. The fields are solved in the frequency domain and later transformed back to the time domain by means of a Fourier integral. The modes of oscillations of the sphere are manifest as poles in the complex plane with the transform method.

The θ dependence is solved for by an expansion in Legendre polynomials. A transformation to the frequency domain, ω, removes the time dependence. The remaining frequency domain equation is expressed in terms of ℓ,ω and the spherical coordinate r. These equations are solved by means of a Green's function in the r coordinate. The particular Green's function chosen is one in which: (1) spherical waves of radiation exist at large distances from the source, (2) the low-frequency limit gives the quasi-static Green's function and (3) damped oscillations as opposed to non-damped oscillations are part of the solution.

Maxwell's equations are

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} , \tag{2-1}$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} . \tag{2-2}$$

Taking the curl of Equation 2-1 and substituting Equation 2-2 into the result we have

$$\nabla \times \nabla \times \vec{B} = - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{B} + \frac{4\pi}{c} \nabla \times \vec{J} . \tag{2-3}$$

For an axisymmetric system, in the spherical coordinates r, θ, and ψ, $\vec{B}$ exists only in the ψ direction. Under this circumstance, Equation 2-3 becomes

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} rB + \frac{1}{r^2} \frac{\partial}{\partial \theta} \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \sin\theta B - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} B$$
$$= \frac{4\pi}{c} \frac{1}{r} \left(\frac{\partial}{\partial \theta} J^r - \frac{\partial}{\partial r} rJ^\theta\right) , \tag{2-4}$$

where J^r and J^θ are the r and θ component of the spatial current. E^r and E^θ are the only non-zero $\vec{E}$ field components. They are obtained by solving Equation 2-1. In terms of components, Equation 2-2 is

$$\frac{1}{c} \frac{\partial E^r}{\partial t} = \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta} \sin\theta B - \frac{4\pi}{c} J^r , \tag{2-5}$$

and

$$\frac{1}{c} \frac{\partial E^\theta}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} (rB) - \frac{4\pi}{c} J^\theta . \tag{2-6}$$

Equations 2-4 through 2-6 can be solved by an expansion in the Legendre polynomials $P_\ell^0(\cos\theta)$ and $P_\ell^1(\cos\theta)$. The expansion is made as follows:

$$B = \sum_{c=1}^{\infty} B_\ell(r,t)\ P_\ell^1(\cos\theta) \ , \tag{2-7}$$

$$E^r = \sum_{\ell=0}^{\infty} E_\ell^r(r,t)\ P_\ell^0 \ , \tag{2-8}$$

$$E^\theta = \sum_{\ell=1}^{\infty} E_\ell^\theta(r,t)\ P_\ell^1 \ , \tag{2-9}$$

$$J^r = \sum_{\ell=0}^{\infty} J_\ell^r(r,t)\ P_\ell^0 \ , \tag{2-10}$$

and

$$J^\theta = \sum_{\ell=1}^{\infty} J_\ell^\theta(r,t)\ P_\ell^1 \ . \tag{2-11}$$

After the substitution of Equations 2-7 through 2-11, Equations 2-4 through 2-6, for each ℓ become

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} rB_\ell - \frac{1}{r^2}\ell(\ell+1)B_\ell - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}B_\ell = -\frac{4\pi}{c}\frac{1}{r}\left(J_\ell^r + \frac{\partial}{\partial r} rJ_\ell^\theta\right) \ , \quad \ell \geq 1 \tag{2-12}$$

$$\frac{1}{c}\frac{\partial}{\partial t}E_\ell^r = \ell(\ell+1)\frac{B_\ell}{r} \ , \quad \ell \geq 1 \ , \tag{2-13}$$

$$\frac{1}{c}\frac{\partial}{\partial t}E_0^r + \frac{4\pi}{c}J_0^r = 0 \ , \tag{2-14}$$

and

$$\frac{1}{c}\frac{\partial}{\partial t}E_\ell^\theta = -\frac{1}{r}\frac{\partial}{\partial r}(rB_\ell) - \frac{4\pi}{c}J_\ell^\theta \ , \quad \ell \geq 1 \ . \tag{2-15}$$

The method of solution is to first solve Equation 2-12, using the boundary condition that at the sphere E_ℓ^θ is zero. Equation 2-15 will be used to state the boundary condition at the spherical surface. After B_ℓ is solved for, the other fields can be obtained from Equations 2-13 through 2-15. To this end we express B_ℓ, J_ℓ^r and J_ℓ^θ in terms of a Fourier integral, removing the time dependence. The integral transformation for B_ℓ is

$$B_{\ell\omega} = \int_{-\infty}^{\infty} e^{+i\omega t} B_{\ell}(r,t)dt \ , \tag{2-16}$$

and

$$B_{\ell}(r,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} B_{\ell\omega} d\omega \ , \tag{2-17}$$

for example. With this integral transform, Equation 2-12 becomes

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} rB_{\ell\omega} - \frac{1}{r^2} \ell(\ell + 1)B_{\ell\omega} + k^2 B_{\ell\omega} = - \frac{4\pi}{c} \frac{1}{r} (J^r_{\ell\omega} + \frac{\partial}{\partial r} (rJ^{\theta}_{\ell\omega})) \ , \tag{2-18}$$

where

$$k^2 \equiv \omega^2/c^2 \ . \tag{2-19}$$

Equation 2-18 is a basic equation for the solution of this problem. $J^r_{\ell\omega}$ and $J^{\theta}_{\ell\omega}$ are the Fourier transforms of J^r_{ℓ} and J^{θ}_{ℓ} respectively.

SECTION 3

QUASI-STATIC SOLUTION

One of the conditions imposed on the general time-dependent solution is that it approach the quasi-static solution, for the low-frequency limit ($k \to 0$). The quasi-static solution will now be derived as it gives insight into the solution for the general problem.

If the time derivative of B is neglected in Equation 2-12 this equation becomes:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} rB_\ell - \frac{1}{r^2}\ell(\ell + 1)B_\ell = -\frac{4\pi}{c}\frac{1}{r}\left(J_\ell^r + \frac{\partial}{\partial r} rJ_\ell^\theta\right) . \qquad (3\text{-}1)$$

The method of solution will be to find the Green's function for Equation 3-1. The solutions to the homogeneous equation are

$$B_\ell = r^\ell \ , \ r^{-(\ell+1)} \ , \qquad (3\text{-}2)$$

The Green's function $G_\ell(r,r')$ must satisfy

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} rG_\ell(r,r') - \frac{1}{r^2}\ell(\ell + 1)G_\ell = \frac{\delta(r - r')}{r} . \qquad (3\text{-}3)$$

Since G is continuous at $r = r'$, from Equation 3-3 we must have

$$\left.\frac{\partial}{\partial r} rG_\ell(r,r')\right|_{r\to r'+} - \left.\frac{\partial}{\partial r} rG_\ell(r,r')\right|_{r\to r'-} = 1 . \qquad (3\text{-}4)$$

We construct a Green's function (continuous at r') from the solutions expressed by Equation 3-2 as follows:

$$G_\ell(r,r')_> \equiv r^{-(\ell+1)}(dr'^{-(\ell+1)} + br'^{\ell}), r > r' \;, \qquad (3\text{-}5)$$

$$G_\ell(r,r')_< \equiv r'^{-(\ell+1)}(dr^{-\ell+1} + br^{\ell}), r' > r \;, \qquad (3\text{-}6)$$

where b and d are arbitrary constants to be determined by Equation 3-4 and the condition at the spherical boundary. Equation 3-4 determines the value of b. $E^\theta = 0$ at the spherical boundary gives the value for d. If d = 0 the fields would exist as if the sphere were not present. The problem is thus separated (through the Green's function) into fields due to the source, $\vec{J}$, alone (d = 0, b ≠ 0) and those due to the sources interacting with the sphere. Substituting Equations 3-5 and 3-6 into 3-4 we find that

$$b = -\frac{r'}{2\ell + 1} \;. \qquad (3\text{-}7)$$

For r' > r, B_ℓ is given by

$$B_\ell = -\frac{4\pi}{c}\int_R^\infty G_{\ell<}\left(J_\ell^r + \frac{\partial}{\partial r'}(r'J_\ell^\theta)\right)dr' \;, \qquad (3\text{-}8)$$

where R is the radius of the sphere. Substituting Equation 3-8 into Equation 2-15, setting $E^\theta = 0$ at r = R, and using Equation 3-6 and 3-7 we find that

$$d = -\frac{(\ell + 1)}{2\ell^2 + \ell} R^{2\ell+1} r' \;. \qquad (3\text{-}9)$$

Equations 3-5, 3-6, 3-7 and 3-9 are all the relations necessary for the solution of the problem. They are, in a sense, the solution to the problem.

One important quantity that must be considered in SGEMP problems is the skin current. The ℓ^{th} component of the skin current, $\mathcal{K}_\ell$, is $-\frac{c}{4\pi}$ $B_\ell(R,t)$. Using Equations 3-5 through 3-9 we find that

$$\mathcal{K}_\ell = -R^\ell\left(\frac{\ell + 1}{2\ell^2 + \ell} + \frac{1}{2\ell + 1}\right)\int_R^\infty r'^{-1}\left(J_\ell^r + \frac{\partial}{\partial r'}(r'J_\ell^\theta)\right)dr' \;. \qquad (3\text{-}10)$$

In Equation 3-10, the term $\frac{1}{2\ell+1}$ arises from the sources alone, the term $\frac{\ell+1}{2\ell^2+1}$ arises from the source interaction with the sphere. For large values of ℓ, the contribution to the skin current, from the sources alone (coming directly from the B field of the sources), contributes half the total skin current. For $\ell = 1$ the sources alone contribute 1/3 of the total skin current. Integrating by parts and combining terms, Equation 3-10 becomes

$$\mathcal{K}_\ell = -\frac{R^\ell}{\ell}\int_\omega^\infty r'^{-\ell}(J_\ell^r + \ell J_\ell^\theta)dr' \ . \tag{3-11}$$

Equation 3-11 corresponds to Equation 24 of Reference 4 which was derived by another method.

SECTION 4

TIME-DEPENDENT SOLUTION

We begin with Equation 2-18 and construct a Green's function in a manner analogous to our solution for the quasi-static case in Section 3. The Green's function $G_{\ell\omega}(r,r')$ satisfies:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rG_{\ell\omega}(r,r')) - \frac{\ell}{r^2}(\ell + 1)\, G_{\ell\omega}(r,r') + k^2 G_{\ell\omega}(r,r') = \frac{1}{r}\,\delta(\vec{r} - \vec{r}\,') \; . \tag{4-1}$$

Solutions to the homogeneous version of Equation 4-1 are spherical Bessel functions (Reference 6, pages 539-540). The spherical Hankel function h^1_ℓ represents outgoing waves at infinity; the spherical Bessel function j_ℓ is finite at the origin. We construct a Green's function continuous at $r = r'$ as follows:

$$G_{\ell\omega}(r,r')_> = h^1_\ell(kr)(dh^1_\ell(kr') + bj_\ell(kr')) \; , \; r > r' \; , \tag{4-2}$$

$$G_{\ell\omega}(r,r')_< = h^1_\ell(kr')(dh^1_\ell(kr) + bj_\ell(kr)) \; , \; r < r' \; . \tag{4-3}$$

Equations 4-2 and 4-3 have the property that for $r > r'$, they represent outgoing waves. h^1_ℓ is chosen as the multiplier of the arbitrary constant* d (b is a constant to be determined, also) because it allows for damped oscillations. When d is determined, this latter statement will be obvious. The choice of functions, in Equations 4-2 and 4-3, also allows the Green's function to approach the quasi-static solution in the limit that $k \to 0$. Equation 4-1 also implies relation 3-4. Substituting Equations 4-2 and 4-3 into Equation 3-4 we find b:

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* d and b are constants only in the sense that they are independent of the spatial coordinates. They are functions of the frequency.

$$b = \frac{r'^{-1}}{j_\ell(kr') \frac{\partial}{\partial r} h^1_\ell \Big|_{r=r'} - h^1_\ell \frac{\partial}{\partial r} j_\ell \Big|_{r=r'}} \tag{4-4}$$

By noting that the homogeneous form of Equation 4-1 can be put into a Sturm-Liouville form, and by using the asymptotic forms[6]

$$j_\ell(kr) \underset{k \to 0}{\to} \frac{(kr)^\ell}{(2\ell + 1)!!} \ , \tag{4-5}$$

and

$$h^1_\ell(kr) \underset{k \to 0}{\to} - i \frac{(2\ell - 1)!!}{(kr)^{\ell+1}} \ , \tag{4-6}$$

it is easy to show that Equation 4-4 becomes

$$b = - i r' k \ . \tag{4-7}$$

We note that if d is set equal to zero, in Equation 4-2 and 4-3, the result would be the Green's function for the situation without the sphere present.

To find the value of d we impose the condition that $E^\theta = 0$ at $r = R$. Utilizing the Fourier transformed version of Equation 2-6 we must satisfy

$$\frac{\partial}{\partial r} (rG_{\ell\omega<}) \Big|_{r=R} = 0 \ . \tag{4-8}$$

Substituting Equation 4-3 and Equation 4-7 into 4-8 we find

$$d = i r' k \frac{\frac{\partial}{\partial r} (rj_\ell(kr))}{\frac{\partial}{\partial r} (rh^1_\ell(kr)} \Big|_{r=R} \ . \tag{4-9}$$

The values of $\omega(k^2 = \omega^2/c^2)$ at which the denominator of Equation 4-9 is equal to zero, will be poles in the complex ω space, when we finally integrate over ω to obtain the time dependence of the fields. These poles are the resonant frequencies of oscillation. The frequencies, then, are defined by the equation

$$\frac{\partial}{\partial r} \left(r h_\ell^1(kr)\right)\Big|_{r=R} = 0 \ . \qquad (4\text{-}10)$$

This is the same equation that appears in Reference 7, page 558, Equation 20, which defines the frequencies of oscillation of a sphere, for electric modes.

Equations 4-2, 4-3, 4-7 and 4-9 constitute the solution for the Green's function in the coordinate r. By using the asymptotic forms (Equations 4-5 and 4-6) it is easy to show that the Green's function we have just found approaches the quasi-static Green's function derived in Section 3 (Equations 3-5, 3-6, 3-7 and 3-9).

Using the Green's function we have just derived we find an expression for the P_ℓ^1 coefficient of the skin current $K_\ell(t)$ on the sphere. From Equation 2-17

$$K_\ell(t) = -\left(\frac{c}{4\pi}\right)\left(\frac{1}{2\pi}\right)\int_{-\infty}^{\infty} e^{i\omega t} B_{\ell\omega}(R)\,d\omega \ . \qquad (4\text{-}11)$$

By using Equation 2-18, 4-2, 4-3, 4-7 and 4-9 and the fact that

$$B_{\ell\omega}(R) = -\frac{4\pi}{c}\int_R^{\infty} G_{\ell\omega}(R,r')_<\left(J_{\ell\omega}^r + \frac{\partial}{\partial r'}\left(r' J_{\ell\omega}^\theta\right)dr'\right) , \qquad (4\text{-}12)$$

we find that

$$B_{\ell\omega}(R) = -\frac{4\pi}{c}\, i k \left[\frac{\frac{\partial}{\partial R}\left(R j_\ell(kR)\right)}{\frac{\partial}{\partial R}\left(R h_\ell^1(kR)\right)} h_\ell^1(kR)\int_R^{\infty} r' h^1(kr')\left(J_{\ell\omega}^r + \frac{\partial}{\partial r'}\left(r' J_{\ell\omega}^\theta\right)\right)dr' \right.$$

$$\left. - j_\ell(kR)\int_R^{\infty} r' h_\ell^1(kr')\left(J_{\ell\omega}^r + \frac{\partial}{\partial r'}\left(r' J_{\ell\omega}^\theta\right)\right)dr'\right] \ . \qquad (4\text{-}13)$$

Letting $k = \omega/c$ and substituting Equation 4-13 into Equation 4-11 we find

$$K_{\ell}(t) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega \left[\frac{\omega}{c} \frac{\frac{\partial}{\partial R}(Rj_{\ell}(\frac{\omega}{c}R))}{\frac{\partial}{\partial R}(Rh^1_{\ell}(\frac{\omega}{c}R))} e^{-i\omega t} h^1_{\ell}(\frac{\omega}{c}R) \int_R^{\infty} r' h^1(\frac{\omega}{c}r')(J^r_{\ell\omega} + \frac{\partial}{\partial r'}(r' J^{\theta}_{\ell\omega}))dr' \right]$$

$$- \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega \left[\frac{\omega}{c} j_{\ell}(\frac{\omega}{c}R) e^{-i\omega t} \int_R^{\infty} r' h^1_{\ell}(\frac{\omega}{c}r')(J^r_{\ell\omega} + \frac{\partial}{\partial r'}(r' J^{\theta}_{\ell\omega}))dr' \right] \quad (4\text{-}14)$$

Equation 4-14, which expresses the skin current in terms of the sources, is one of the desirable relations we wish to find. The second term in Equation 4-14 is proportional to the magnetic field at the spherical surface due to the sources alone. Because of the denominator, the first term in Equation 4-14 is expressible in terms of the modes of the sphere.

We could construct the solution for any of the fields, E^r_{ℓ}, E^{θ}_{ℓ}, B_{ℓ} at any point in space, in exactly the same manner that Equation 4-14 was formed, that is, by using the determined Green's function and integrating over ω in the complex plane. The ω integration brings out many of the features of the problem. The fields are retarded in time and those parts of the fields which arise from the sphere are characterized by specific frequencies and damping constants. When the explicit functional form, for the spherical Bessel functions, is inserted into the Green's function, powers of ω^{-1} result. This has the effect of representing the time-dependent solution for the fields in terms of time integrals of the known time dependence of the source. These and other characteristics of the solution will be discussed in a succeeding report.

SECTION 5

CONCLUSION

The solution to the time dependent sphere problem, in terms of the source currents in space, has been presented. Equation 4-14 expresses the ℓ^{th} component of the skin current in terms of the ℓ^{th} components of the source current components. One of the notable features of this equation is that the reaction of the sphere can be expressed in terms of the modes of the sphere and the field of the sources alone, at the surface of the sphere. The modes are an inherent electromagnetic characterization of the surface of a system. This fact emphasizes the two-dimensional nature of the surface response even though the Green's function solution presented here was three dimensional.

If we are only interested in the surface currents and surface charge densities on a system we would, in a sense, be getting a lot of unnecessary information by finding the electromagnetic solution, in all of three-dimensional space. Surface currents and charge densities are the important SGEMP quantities. It therefore seems reasonable that some part of future research, in this area, should be devoted to characterizing systems and their SGEMP response in terms of surface modes.

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