Theoretical Notes
Note 304

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This report considers several possible effects of dust raised by a nuclear explosion on the EMP due to a second explosion. The effects include change of the dielectric constant, capture of electrons and ions by dust particles, and heating of the dust cloud by thermal radiation. The last effect appears most significant.

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SECTION 1
INTRODUCTION AND CONCLUSIONS

In this report we estimate several effects of dust raised by surface or near-surface nuclear explosions on the EMP produced by later explosions. The effects that occurred to us included:

- changes in the dielectric constant of air;
- changes in the gamma flux and Compton current;
- capture of electrons and ions by dust particles, thus changing the conductivity;
- heating of the dust-bearing air by thermal radiation to temperature at which attachment of electrons to O_2 will not occur.

In Section 2 we describe the dust model used, and estimate the various effects in later sections. The only effect which appears to be likely to modify the EMP seriously is the heating effect. Full evaluation of that effect remains to be done.

We are of the opinion that experiments on the electron and ion capture and on the heating effect would be valuable and justifiable.

SECTION 2
PARTICLE SIZES AND DENSITIES

In order to guide our considerations as to regimes of practical interest, we need some information on the likely distribution of particle sizes and the densities that may occur. We contacted Major Gary Ganong of AFWL, who summarized briefly what is known or guessed about dust raised by nuclear explosions, and lent us copies of some old Weapon Test (WT) reports. The following simplified description is based on information thus provided by Major Ganong; he is not, of course, responsible for any misinterpretations that we may make.

A burst above but near the surface raises, by various processes, a layer of dust above the ground surface. For a 20 KT burst at a height of 400 feet (WT-1102, Operation Teapot, Shot 12), the radius of the dust layer is about 700 meters, and its thickness is about 100 meters. For a scaled 1 MT burst, we assume that the radius and thickness would be about 2500 meters and 300 meters respectively. This layer would remain in place for a period of the order of 10 seconds, after which much of it tends to get sucked up in the stem of the rising fireball. One of the problems of interest to us concerns the effect of the dust on the EMP from a second burst occurring nearby before the dust layer gets sucked into the stem. Such effects would occur within a radius of about 2500 meters from the first burst, which is a significant distance for MX planning.

A burst on the surface apparently raises less dust in the layer described above but, because of cratering, carries more dust along with the cloud to high altitudes. The other problem of interest concerns the effect of this dust on the EMP from a high-altitude burst.

We shall make rough estimates of the size distribution and densities for these two cases, and use these as central values in our estimates of the effects they produce. However, we shall also vary these parameters about the central values.

DUST LAYER

We consider first the dust layer near the ground. WT-519 (Operation Tumbler) presents results of measurements of size distribution in dust raised ahead of the shock wave. These measurements were made in Frenchman's Flat, a dry lake bed, and showed very fine particles. Over the range of particle diameters measured, from 0.02 to about 30 microns, the size distribution found was approximately log-normal, i.e., the number of particles N(a) per cm^3 having radius a in interval da was approximately given by

$$N(a)da = A \exp\left[-\frac{1}{2\sigma^2}\left(\ell n \frac{a}{a_m}\right)^2\right]\frac{da}{a}\,, \tag{1}$$

where A is a constant, σ is the root-mean-square value of $\ell n(a/a_m)$ (standard deviation), and a_m is the median particle radius; thus half of the particles have radii smaller than a_m. The median radius and standard deviation in these measurements were about

$$\left.\begin{aligned} a_m &\approx 0.1 \text{ micron}\,, \\ \sigma &\approx 4\,. \end{aligned}\right\} \tag{2}$$

Since $e^\sigma \approx 50$, the distribution in particle size is several decades wide.

If the distribution of particles is given by Equation 1, the distributions of particle surface areas and particle masses (volumes) are proportional to:

Surface Area:

$$a^2N(a)da = a_m^2 A e^{2\sigma^2} \exp\left[-\frac{1}{2\sigma^2}\left(\ell n \frac{a}{a_m} - 2\sigma^2\right)^2\right]\frac{da}{a}\,, \tag{3}$$

Mass:

$$a^3N(a)da = a_m^3 A e^{9\sigma^2/2} \exp[-\frac{1}{2\sigma^2}(\ln\frac{a}{a_m} - 3\sigma^2)^2]\frac{da}{a}. \qquad (4)$$

It can be seen that these are also log-normal distributions, in which the radii a_{ms} of median surface area and a_{mm} of median mass are:

$$a_{ms} = a_m e^{2\sigma^2},$$

$$a_{mm} = a_m e^{3\sigma^2}.$$

Note, however, that these are very large radii; for the values of Equation 2 we would have:

$$a_{ms} = 0.1 \times e^{32} \text{ microns} \approx 10^9 \text{ cm},$$

$$a_{mm} = 0.1 \times e^{48} \text{ microns} \approx 10^{16} \text{ cm}!$$

Since these values are far beyond the range of particles collected and counted, it is clear that Equations 3 and 4 cannot represent totally the surface and mass distributions. A plausible assumption for the data presented in WT-519 is that Equation 1 gives the particle distribution up to the largest radius a_M collected and counted. For radii less than a_M, Equations 3 and 4 are correct, and nothing can be inferred about the surface area and mass in larger particles.

The total surface area and total mass of the particles per cm^3 with radii in the observed range are then given approximately by

$$S \equiv \text{surface area}/cm^3 = 4\pi\int_0^{a_M} a^2N(a)da$$

$$\approx 2\pi a_M^2 A \exp[-\frac{1}{2\sigma^2}(\ln\frac{a_M}{a_m})^2], \qquad (5)$$

$$\rho_d = \text{mass/cm}^3 = \frac{4\pi}{3}\,\rho \int_0^{a_M} a^3 N(a)\,da$$

$$\approx \frac{4\pi}{9}\,\rho a_M^3 A \exp[-\frac{1}{2\sigma^2}(\ln \frac{a_M}{a_m})^2] \,. \qquad (6)$$

Here ρ is the mass density of each dust particle (all assumed to be the same) and ρ_d is the total observed dust mass per cm^3. In deriving the approximate results here we have made use of the fact that the factors a^2 and a^3 in the integrals vary more rapidly with a than does $N(a)$ near the upper limit $a = a_M$, and that most of the contributions to the integrals come near this limit. (A correction for the variation of $N(a)$ can be made easily: multiply the right hand sides of Equations 5 and 6 by the factors,

$$\text{for (5)}\ ,\quad \frac{2}{2 - \frac{1}{\sigma^2}(\ln \frac{a_M}{a_m})}\ , \qquad (7)$$

$$\text{for (6)}\ ,\quad \frac{3}{3 - \frac{1}{\sigma^2}(\ln \frac{a_M}{a_m})}\ . \qquad (8)$$

Use of these corrections is probably not justified by the accuracy of the data available.

Equation 6 is useful for determining the constant A if the total observed mass density ρ_d is known. This quantity was not given in WT-519. However, in WT-1113 (Operation Teapot, Shot 12), the following parameters are reported for the dust behind the shock wave, at 3 and 10 feet above the ground, in or near Frenchman's Flat:

$$\left.\begin{aligned} &a_m \approx 0.6 \text{ microns}\ ,\\ &a_{mm} \approx 8 \text{ microns},\\ &\sigma \approx 2.9\ ,\\ &\rho_d \approx 10^{-3}\ \frac{gm}{cm^3} \text{ (about equal to } \rho_{air}) \ . \end{aligned}\right\} \qquad (9)$$

The maximum particle radius collected and counted was not stated, but, from the value of a_{mm} and the arguments given above, we assume it was about

$$a_M \approx 10 \text{ microns} . \tag{10}$$

(This is consistent with the range collected and counted in WT-519.) Using Equation 6 with $\rho = 2$ gm/cm^3, we then find

$$\frac{1}{2\sigma^2} (\ell n \frac{a_M}{a_m})^2 \approx 0.47 \quad , \quad e^{-0.47} \approx 0.62 \; , \tag{11}$$

$$A = 0.5 \times 10^6/\text{cm}^3 \; . \tag{12}$$

In the measurements, particles with $a < 0.1$ micron were not counted. Over the range $0.1 < a < 10$ microns, the exponential in Equation 1 does not vary much, as shown by Equation 11. Thus we may take as a rough estimate of the particle distribution from this experiment,

$$N(a)da \approx 10^6 \, \frac{da}{a}/\text{cm}^3 \quad , \quad 0.1 < a < 10 \text{ microns} . \tag{13}$$

We shall use this result as a rough guide to the regimes of interest in the dust layer in the size range indicated.

It is stated in WT-1113 that the particle size distribution observed in the dust was very near that in the soil before the shot. This result is to be expected for the radius range observed, since all particles in this range are easily carried by the winds involved in the shock wave. Probably the native soil would well represent the dust over a considerably wider size range. Thus measurements at the MX site would be useful if the dust distribution turns out to be important.

For larger particle sizes it is conventional to use a distribution, for dust carried to high altitudes, of the form

$$N(a)da = A' \, \frac{da}{a^{3.5}} \; , \tag{14}$$

where A' is again a constant. This form diverges at small a and must be terminated there in some applications. In addition, the particle mass diverges at large a, so that a largest radius must also be assumed. Following the suggestion of Major Ganong, we assume a largest radius of

$$a_L = 0.1 \text{ cm} = 10^3 \text{ microns} . \tag{15}$$

Such a particle has a terminal fall velocity in the atmosphere of the order

$$v_t \approx 10^3 \text{ cm/sec} . \tag{16}$$

Particles of this size will certainly be lofted in the dust layer, where wind speeds are of the order of the sound speed, 3×10^4 cm/sec. However, they will fall out of the dust layer (10^4 cm thick) in times of the order of 10 seconds. We also choose A' to join the distribution (14) continuously on to Equation 13 at a = 10 microns, and find

$$N(a)da = \frac{1}{30} \frac{da}{a^{3.5}}/\text{cm}^3 \quad , \quad 10^{-3} < a < 10^{-1} \text{ cm} . \tag{17}$$

Thus the total mass of dust per cm^3 is

$$\rho_d \approx \frac{8}{3} \frac{\sqrt{a_L}}{30} = 0.2 \text{ gm/cm}^3 . \tag{18}$$

This seems like a large number, but it appears that we have little choice but to use it.

The integrals of a^n over the total distribution, for $n = 0, 1, 2, 3$, are given in Table 1, with the contributions from small particles (Equation 13) and large particles (Equation 17) listed separately. These quantities will be useful in calculations of the effects of the dust. For example, the mean free path of an optical photon is approximately the reciprocal of the total value for $n = 2$ multiplied by π, or about 0.13 cm.

Table 1. Integrals of a^n for dust layer.

n	$\int a^n N(a)da$ Small Particles	Large Particles	Total
0	$4.6 \times 10^6/cm^3$	$0.4 \times 10^6/cm^3$	$5 \times 10^6/cm^3$
1	10^3 cm/cm^3	0.7×10^3 cm/cm^3	1.7×10^3 cm/cm^3
2	0.5 cm^2/cm^3	2 cm^2/cm^3	2.5 cm^2/cm^3
3	3×10^{-4} cm^3/cm^3	2×10^{-2} cm^3/cm^3	2×10^{-2} cm^3/cm^3

DUST CLOUD

Following the suggestion of Major Ganong, we assume that the dust cloud, just after reaching the stabilization altitude, is roughly a sphere 10 km in diameter, extending from 10 to 20 km in altitude, and containing 3×10^{11} grams of dust. (We assume here a 1-megaton surface burst.) Thus the density of dust is about

$$\rho_d \approx 6 \times 10^{-6} \text{ gm/cm}^3 . \tag{19}$$

We shall assume that the shape of the size distribution here is the same as in the dust layer. Since most of the mass is contained in the larger-radius distribution (14), we can evaluate A' by matching Equation 19 with

$$\rho_d = \frac{8\pi}{3} A' \rho \sqrt{a_L} , \tag{20}$$

which gives $A' \approx 10^{-7}$. Then matching the distribution (14) to the form of (13) for smaller particles at a = 10 microns, we obtain the results

$$N(a)da = 10^{-7} \frac{da}{a^{3.5}}/\text{cm}^3 , \quad 10^{-3} < a < 10^{-1} \text{ cm} , \tag{21}$$

$$N(a)da = 3\,\frac{da}{a}/cm^3 \quad , \quad 10^{-5} < a < 10^{-3}\ \text{cm} . \qquad (22)$$

We shall use these estimates as the central values for considerations of the effects in high-altitude EMP.

The integrals of a^n for the distribution given by Equations 21 and 22 are obtained by multiplying the values in Table 1 by a factor of 3×10^{-6}. Thus

$$(\int a^n N(a)da)_{cloud} = 3 \times 10^{-6} (\int a^n N(a)da)_{layer} \qquad (23)$$

Since the air density at 20 km altitude is down only by a factor of 20 below the sea level value, the relative density of dust to air in the cloud is down by a factor of about 10^4 from that in the layer.

SECTION 3
EFFECTS OF DUST UPON EMP

Electromagnetic effects are governed by Maxwell's equations, hence the effect of dust upon the electromagnetic fields must be manifested in either of two categories: (1) the specification of the electromagnetic environment, i.e, the dielectric constant, ε, and the magnetization, μ, or (2) the electromagnetic sources, i.e., the currents, J, or the electron and/or ion densities which give rise to the electrical conductivity, σ. Both of these categories will now be discussed individually identifying areas of concern.

In specifying the electromagnetic environment, there is no reason to suspect that dust has any significant magnetic moment nor magnetic susceptibility. Hence the change in magnetization of dust filled air is probably not an effect. If the dust, however, in a particular area has a large amount of iron present, there may be an effect. For silicon dioxide (which will be taken to be the usual major component of dust) no effect would be observed. The dielectric constant will, however, show more of an effect. The dielectric constant of an air-dust mixture, ε, is given by a volume reciprocal average of the dielectric constants of the components,

$$\varepsilon^{-1} = (\%V_{AIR} \times 100)\ \varepsilon_{AIR}^{-1} + (\%V_{DUST} \times 100)\ \varepsilon_{DUST}^{-1}\ , \qquad (24)$$

where %V is the percent volume of the individual components. Roughly, $\varepsilon_{DUST} \cong 2\varepsilon_{AIR} = 2$, hence

$$\varepsilon \cong \frac{\varepsilon_{AIR}}{1 - (\%V_{DUST} \times 100)/2}\ . \qquad (25)$$

The percent volume of dust may be determined from the density of the dust in air and the density of the dust particles themselves which will be taken to be that of sand, $\rho = 2.2$ grams/cm^3. For dust of about the same density as that of air,

$$\varepsilon \cong 1.0002\ \varepsilon_{AIR}, \tag{26}$$

and for the very dense dust as predicted in the distribution derived in Section 2, Equation 18,

$$\varepsilon \cong 1.04\ \varepsilon_{AIR}\ . \tag{27}$$

The resulting 2 percent effect in the speed of electromagnetic wave propagating in the dust-air mixture could be significant when the electrical conductivity is low and electromagnetic signals propagate over large distances. Note, however, that gammas do not penetrate many meters into dust of this high density. Also, ε is unimportant in the diffusion and quasi-static phases, when the dominant fields occur.

For the category of electromagnetic sources, there is expected to be little change in Compton current due to a gamma radiation source since the increase in Compton current caused by the larger Compton scattering cross section in the denser dust particles is approximately canceled by the decrease in Compton current due to the reduction of the electron's mean-forward-range caused by energy less in dust particles. The gamma source that normally comes from neutron capture in the ground would come from a few meters near the top of the very dense dust layer, and it would be about 10 times weaker but last 10 times longer than normal.

Effects which manifest themselves in the electrical conductivity are confined to changes in the free-electron and ion densities since no noticeable change in the mobilities is anticipated. Changes in electron and ion density will be manifest in two ways: (1) the dust particles will constitute a site for electron and ion capture and eventual neutralization, and (2) dust will absorb optical and infrared radiation which will raise the temperature of air to the point where electron detachment significant,

$$O_2^- \rightarrow O_2 + e^- \; .$$

Both of these effects will be discussed in greater detail in later sections.

SECTION 4
CAPTURE OF FREE ELECTRONS BY DUST PARTICLES

At early times in the EMP problem the electrical conductivity is dominated by free electrons. Since electrons that strike a dust particle are likely to stick either to the surface or to trapping centers inside the SiO_2 crystal, the presence of dust will reduce the density of free electrons. However, in order to appreciably change the free electron density, the rate of capture of electrons by dust must be comparable with the attachment rate to O_2 (forming O_2^-). The lifetime for attachment is about 10^{-8} second, since the attachment rate is

$$\alpha \approx 10^8/\text{sec} \quad \text{in sea level air} . \tag{28}$$

The question, then, is whether in this short time an electron is likely or unlikely to strike a dust particle.

We can get a first estimate of the answer to this question by assuming that electrons and dust particles are randomly distributed. Then for dust particles of radius a (cm) and density N_d/cm^3 the capture rate β per electron would be

$$\beta = N_d \pi a^2 v , \tag{29}$$

where v is the thermal speed of the free electrons,

$$v(\text{cm/sec}) \approx 7 \times 10^7 \sqrt{T_e} \text{ (eV)} . \tag{30}$$

Here T_e is the free electron temperature in eV. Typically we have

$$T_e \approx 0.1 \text{ eV} \quad , \quad v \approx 2 \times 10^7 \text{ cm/sec} . \tag{31}$$

For a distribution of particle sizes, Equation 29 has to be integrated over the distribution, so that we need the integral of πa^2. From Table 1 for $n = 2$, we obtain

$$\beta \approx \pi \times 2.5\ v \approx 1.6 \times 10^8/\text{sec} \ . \tag{32}$$

According to this estimate, capture by dust particles does indeed compete with attachment to O_2.

However, there are two effects which reduce the capture rate. First, the dust particles become negatively charged and repel further electrons. Second, a dust particle may capture all of the electrons in its immediate neighborhood, but then further electrons have to diffuse to its location. The solution of the total problem including both effects is fairly complicated, although it could be done. Fortunately, the second effect by itself is sufficient to reduce electron capture to a negligible rate compared with attachment.

We shall therefore solve the diffusion problem for the case without electric field, i.e., we ignore the effect of charging of the dust particle on the diffusion. The flux of electrons is

$$\vec{F} = -\frac{\lambda v}{3} \nabla N_e \ , \tag{33}$$

where N_e is the electron density and λ is the scattering mean free path of electrons

$$\lambda \approx 4 \times 10^{-5} \text{ cm in sea level air} \ . \tag{34}$$

If ion pairs are produced at rate $\dot{\gamma}/\text{cm}^3$ sec, the continuity equation for electrons is

$$\frac{\partial N_e}{\partial t} = -\nabla \cdot \vec{F} - \alpha N_e + \dot{\gamma} \ . \tag{35}$$

If $\dot{\gamma}$ changes only little in the attachment time, we may be satisfied with the steady state solution of Equation 35, which becomes

$$\frac{\lambda v}{3} \nabla^2 N_e - \alpha N_e + \dot{\gamma} = 0 \ . \tag{36}$$

Without dust, the solution of this equation would be

$$N_e = \dot{\gamma}/\alpha \ . \tag{37}$$

With a dust particle present, we must enforce the boundary condition that $N_e = 0$ at the particle surface. It is easy to see that the solution then is

$$N_e = \frac{\dot{\gamma}}{\alpha} \left\{ 1 - \frac{a}{r} \exp\left[- \frac{(r-a)}{d} \right] \right\} \ , \tag{38}$$

where

$$d = \sqrt{\frac{\lambda v}{3\alpha}} \approx 1.6 \times 10^{-3} \text{ cm} \ . \tag{39}$$

(Note that d is the mean distance diffused in a time $1/\alpha$.) The total flux of electrons into the dust particle can be calculated from this solution, and is

$$\mathscr{F} = -4\pi a^2 F = \frac{4\pi}{3} \lambda v \ a^2 \left. \frac{\partial N_e}{\partial r} \right)_{r=a}$$

$$= \frac{4\pi}{e} \lambda v \left[a + \frac{a^2}{d} \right] \frac{\dot{\gamma}}{\alpha} \text{ sec}^{-1} \text{ per dust particle} \ . \tag{40}$$

This result must now be integrated over the distribution of dust particles per unit volume to give the total electron capture rate per unit volume, and then divided by $N_e \approx \frac{\dot{\gamma}}{\alpha}$ to obtain the capture rate per electron,

$$\beta \approx \frac{4\pi}{3} \lambda v \left[\int a N(a) da + \frac{1}{d} \int a^2 N(a) da \right] \ . \tag{41}$$

Using results from Table 1 and Equations 31, 34 and 39, we estimate

$$\beta = 10^7/\text{sec} \ , \tag{42}$$

which is about 10% of the attachment rate.

Charging of the dust particles will reduce the capture rate to much lower values. The approximate result of charging is to reduce the electron diffusion rate to the ion diffusion rate, which is roughly a thousand times slower, because of the lower thermal speed of ions.

Before concluding that capture of electrons by dust particles is totally negligible, there is one other effect to be considered. In the presence of an EMP electric field E, the electrons drift at a speed

$$v_d = \mu_e E \tag{43}$$

where μ_e is the electron mobility,

$$\mu_e \approx 10^6 \text{ cm/sec esu} . \tag{44}$$

Since $E \leq 10$ esu typically,

$$v_d \leq 10^7 \text{ cm/sec} . \tag{45}$$

The rate at which electrons drift into uncharged dust particles is

$$\begin{aligned} \beta &= \pi v_d \int a^2 N(a) da \\ &\leq \pi \times 10^7 \times 2.5 \approx 8 \times 10^7/\text{sec} \end{aligned} \tag{46}$$

This rate is not negligible. However, again the dust particles become negatively charged, with the result that the total electric field lines skirt around the particle, and no further electrons drift into the particle. This condition is achieved when the field due to electrons on the particle cancels E at the particle surface, or when

$$\frac{\mathcal{N} e}{a^2} \approx E , \tag{47}$$

where $\mathcal{N}$ is the number of electrons stuck to the particle. Thus the total number N_s of stuck electrons per unit volume is

$$\begin{aligned} N_s &= \frac{E}{e} \int a^2 N(a) da \approx 2.5 \frac{E}{e} \\ &\leq 5 \times 10^{10}/\text{cm}^3 . \end{aligned} \tag{48}$$

Normally in air the free electron density is

$$N_e \approx \frac{\dot{Y}}{\alpha} \approx \frac{2\times 10^9}{\alpha} \dot{D} \approx 20\dot{D}/cm^3 , \quad (49)$$

where $\dot{D}$ is the dose rate in rads/second. N_s and N_e will be equal for a dose rate of 2.5×10^9 rad/sec. At this relatively low dose rate the maximum field E is about 1 esu instead of 10 esu, and the capture rate reduces from Equation 46 to

$$\beta \leq 8 \times 10^6/sec \quad (50)$$

This rate is small compared with the attachment rate.

Since we have used a rather high total dust density, we conclude that dust probably does not appreciably affect the free electron density. The chief uncertainty in this statement arises from the uncertainty in the dust density. However, it might be useful to refine further the calculation of the capture rate. Note that 20 percent of the integral of a^2 in Table 1 comes from the small particles, which have a mass density only comparable to the air density, and is probably not unreasonably high.

In the dust cloud (i.e., at stabilization altitudes) capture rates are some 10^4 times slower, and are too slow to affect the main part of the high altitude EMP, which occurs in times less than 10^{-6} second.

SECTION 5
CAPTURE OF IONS BY DUST PARTICLES

At late times, the air conductivity is determined by positive and negative ions, rather than free electrons. Thus we also need to estimate the effect of the dust on the ion densities.

The ions have very nearly the same temperature as the air, since they can transfer energy effectively in collisions with air molecules, which have comparable mass. Thus the ion temperature is

$$T_i \approx \frac{1}{40} \text{ eV} = 4 \times 10^{-14} \text{ erg} . \tag{51}$$

It is likely that positive ions striking a dust particle will either stick to the surface or rebound as a neutral molecule, for the binding energy available for the missing electron in the positive ion (≈ 12 eV) is larger than the few eV required to pull an electron out of the SiO_2 crystal. It is also likely that negative ions, like O_2^-, will stick or neutralize, since the binding energy of the electron in O_2^- (≈ 0.43 eV) is less than the few eV available to bind it to the crystal. If one type of ion sticks, then the resulting electric field will make the other type stick. If free electrons have charged the dust particles, then positive ions will be drawn in. In any case, the dust particles are likely to be effective recombination centers for whatever ions strike their surfaces.

The mass M of the ions is not well known, because of their tendency to form clusters with neutral molecules. The commonly accepted value for the mobility of the ions is

$$\mu_i \approx 750 \text{ cm/sec esu} . \qquad (52)$$

From this we can find the diffusion coefficient without knowledge of the ion mass. In terms of the ion mean free path λ and the thermal speed v,

$$\mu_i = \frac{e\lambda}{Mv} \quad \text{or} \quad \lambda = \frac{Mv\mu_i}{e} \qquad (53)$$

Thus we find the diffusion coefficient

$$\frac{v\lambda}{3} = \frac{1}{3} Mv^2 \frac{\mu_i}{e}$$

$$= T_i \mu_i / e \qquad (54)$$

$$= 0.06 \text{ cm}^2/\text{sec} \qquad (55)$$

Since T_i and T_e do not differ greatly, Equation 54 shows that the diffusion coefficient of electrons and ions have about the same ratio as their mobilities. It follows that when ion conductivity dominates that of electrons, ion diffusion also dominates that of electrons. Thus when ion conductivity is dominant, we can treat the capture of ions by neglecting the presence of electrons, and consider only positive and negative ions.

Since positive and negative ions have about the same mobility, and therefore about the same diffusion constant, the diffusive capture of ions will proceed with the dust particles remaining neutral, and we can use the field-free diffusion theory developed in Section 4 for electrons.

In that theory the attachment rate α played a role. For ions, attachment is replaced by mutual neutralization, e.g.,

$$N_2^+ + O_2^- \rightarrow N_2 + O_2 , \qquad (56)$$

usually with the help of a third body. The rate of neutralization per ion is

$$\alpha_i = kN_i \ , \tag{57}$$

where N_i is the density of ions of either species. The rate constant k is subject to the same uncertainties as the ion masses. The value used in EMP calculations is

$$k = 2.3 \times 10^{-6} \ cm^3/sec \quad \text{at sea level.} \tag{58}$$

The dependence on the ion density of the ion neutralization rate α_i makes the ion diffusion problem nonlinear. However, we shall treat α_i as if it were a constant in the diffusion solution with N_i set equal to its asymptotic value far from the dust particle. This value is the steady state solution of the equation

$$\frac{\partial N_i}{\partial t} = \dot{\gamma} - kN_i^2 \ , \tag{59}$$

or

$$N_i = \sqrt{\dot{\gamma}/k} \ . \tag{60}$$

The proportionality of N_i to $\sqrt{\dot{\gamma}}$, whereas N_e is proportional to $\dot{\gamma}$ (Equation 37), is the reason ion conductivity dominates at late times as $\dot{\gamma}$ falls. Taking into account the fact that electron mobility is about 10^3 times that of ions and the two ion species, ion conductivity will dominate when

$$2\sqrt{\frac{\dot{\gamma}}{k}} > 10^3 \ \frac{\dot{\gamma}}{\alpha} \ ,$$

or when

$$\dot{\gamma} < 4 \times 10^{-6} \ \frac{\alpha^2}{k}$$

$$< 1.7 \times 10^{16}/cm^3 \ sec \rightarrow \dot{D} \approx 10^7 \ \text{rads/sec} \ . \tag{61}$$

In this regime,

$$N_i < 10^{11}/cm^3, \tag{62}$$

$$\alpha_i = \sqrt{k\dot{\gamma}} \quad (63)$$

$$< 2 \times 10^5/\text{sec} . \quad (64)$$

The diffusion length, d, for the ion problem can now be taken over from Equation 39,

$$d = \sqrt{\frac{\lambda v}{3\alpha_i}} = \sqrt{\frac{\lambda v}{3}} \Big/ (k\dot{\gamma})^{1/4} \quad (65)$$

$$> 5.5 \times 10^{-3} \text{ cm} . \quad (66)$$

The capture rate for ions can be taken over from Equation 41 (for the dust parameters of Table 1),

$$\beta = 4\pi(0.06)\left[1.7 \times 10^3 + \frac{2.5}{d}\right] . \quad (67)$$

It can be seen that for d satisfying Equation 66, the second term in the bracket here is not dominant, and we have

$$\beta \approx 1.5 \times 10^3/\text{sec} , \quad (68)$$

roughly independent of the ion density. Let us compare this capture rate with the neutralization rate (63). We see that the capture rate exceeds the neutralization rate for sufficiently small $\dot{\gamma}$, i.e., for

$$\dot{\gamma} < \frac{\beta^2}{k} \quad (69)$$

$$< 10^{12}/\text{cm}^3 \text{ sec} \rightarrow \dot{D} = 500 \text{ rads/sec} . \quad (70)$$

At this critical ionization rate the Compton current density, air conductivity and saturated field would be

$$J = 10^{-10} \text{ abamps/cm}^2 = 10^{-5} \text{ amps/m}^2 \quad (71)$$

$$\sigma = 3 \times 10^{-9} \text{ cm}^{-1} \approx 10^{-9} \text{ mho/m} \quad (72)$$

$$E_s = \frac{J}{\sigma} = 0.03 \text{ esu} \approx 10^3 \text{ V/m} \quad (73)$$

At lower ionization rates, β exceeds α_i, and the saturated field remains near the result (73), instead of falling as $\sqrt{\dot{\gamma}}$.

We thus see that diffusive capture of ions raises the (vertical) electric field at late times, but the increased field is still relatively small, for the dust layer assumed in Table 1.

We now consider the capture of ions due to their drift in the electric EMP field. At late times, when ion conductivity dominates, the electric field is believed to be not larger than about 1 esu, so that the drift speeds are

$$v_d < 10^3 \text{ cm/sec} . \tag{74}$$

The capture rate of ion is therefore, from Equation 46,

$$\beta < \pi \times 10^3 \times 2.5 = 8 \times 10^3 \text{/sec} . \tag{75}$$

Since positive and negative ions drift in opposite directions, a dust particle will acquire positive charges on one side and negative charges on the other. If the particle were spherical, the collected surface charge would have a $\cos\theta$ distribution, and the electric field produced would be that of a dipole. The effect of this dipole field is to make the total electric field lines tend to skirt around the particle, and there is a critical surface charge at which the total field lines just do not touch the surface, i.e., the radial component of the total E vanishes just outside the surface. A simple calculation in potential theory shows that the critical surface charge density is

$$q = \frac{3}{8\pi} E_o \cos\theta \quad \text{(cgs esu)}, \tag{76}$$

where E_o is the applied EMP field. Thus the total positive charge on one hemisphere is

$$Q_+ = \frac{3}{8} a^2 E_o , \tag{77}$$

and the number of stuck positive ions per dust particle is

$$\mathcal{N}_+ = \frac{3}{8} a^2 \frac{E_o}{e} \ . \tag{78}$$

This result may be compared with the result (47) for electrons (which was not derived so carefully).

Summing over the dust particles per unit volume and using Table 1 again, we obtain the number of stuck positive ions per unit volume.

$$\begin{aligned} N_{+s} &= \frac{3}{8} \times 2.5 \frac{E_o}{e} = \frac{E_o}{e} \\ &\leq 2 \times 10^9/\text{cm}^3 \end{aligned} \tag{79}$$

To produce this much charge per cm^3 requires a total dose of one rad, which takes 2×10^{-3} second at the dose rate in Equation 70. This is a fairly short time for the ion conductivity regime, and the particles will charge up quickly, or be already charged up from the earlier electron conductivity regime. However, at sufficiently great distances from a nuclear explosion, the dose rate will be small compared with 500 rads/second, and the capture rate β (Equation 75) due to drift will exceed the mutual neutralization rate α_i. The saturated field going with the capture rate (75) is

$$E_s \approx 0.07 \text{ esu} \approx 2000 \text{ V/m} \ . \tag{80}$$

Again, this is not a very serious field.

The Compton currents, ion conductivities, and saturated fields given above have been calculated from the well-known formulae,

$$J \approx 2 \times 10^{-13} \dot{D} \text{ abamps/cm}^2 \tag{81}$$

$$\begin{aligned} \sigma_i &= N_i \mu_i \frac{e}{c} = \frac{2\times10^9 \dot{D}}{\beta} \mu_i \frac{e}{c} \\ &\approx 2.4 \times 10^{-8} \frac{\dot{D}}{\beta} \text{ cm}^{-1} \end{aligned} \tag{82}$$

$$E_s = J/\sigma \ . \tag{83}$$

Note that to produce a given static electric field requires a certain amount of charge separation or time-integrated Compton current, or total dose from Equation 81. To produce a field of 0.07 esu requires a total dose of about one rad, which was the same as that required to produce the charge N_{+s}, Equation 79. Thus if there is not enough dose to saturate the dust particles with ions, there is also not enough to reach the saturated field. The actual field, in regions where ion drift capture is significant, will be less than the result (80).

We conclude that ion capture, in the dust layer of Table 1, will produce noticable but not serious increases in the late-time vertical electricl field at distances from the burst where this field is fairly small anyway.

Ion conductivity is not believed to be significant for the main part of the high altitude EMP. It appears that dust density in the stabilized cloud is too low by a factor of 10^4 to affect the late-time high altitude EMP. However, this part of the EMP is not very well understood, so firm conclusions cannot be made.

SECTION 6
VAPORIZATION OF DUST BY THERMAL RADIATION

Let us now consider the effect of heating of the dust layer due to the thermal radiation from a surface burst. Since a nuclear burst yields about 35 percent of its energy in the form of thermal radiation from the fireball, the energy delivered to a point at a distance R from a device of yield, Y, is given by

$$W\left[\frac{\text{calories}}{\text{cm}^2}\right] = \frac{0.35 \times 10^{15} Y[\text{Mton}]}{2\pi \times 10^{10} R[\text{km}]^2}$$

$$= 6000\ Y[\text{Mtons}]/R[\text{km}]^2, \tag{84}$$

where we have used the fact that 1 megaton of TNT is equivalent to 1×10^{15} calories.

This thermal energy when it enters a dust cloud causes both the dust and air to achieve a thermal equilibrium since the thermal diffusion time in the dust-air mixture is of the order of tens of microseconds and the duration of the thermal radiation pulse is of the order of several seconds. The air then heats up to the point where the dust sublimates and the air then ceases to absorb the thermal radiation. SiO_2 boils at approximately 2300°C which will be taken as the temperature resulting from this process. In this way, the dust is burned off creating a layer of heated air at a temperature of 2300°C. This is especially significant for EMP effects since temperatures above 600°C cause significant deattachment of electrons from the O_2^- ion, and at 2300°C there will be virtually no attachment.

This will therefore create a layer of high electrical conductivity on the top of the dust cloud. We shall now attempt to estimate the size of this strongly heated layer.

In this analysis, the density of dust present in the air and its opacity to visible and infrared light at high temperatures are crucial effects. We shall assume that the density of dust is about the same as the density of air,

$$\rho_{DUST} \cong 1 \times 10^{-3} \text{ gram/cm}^3 \quad . \tag{85}$$

The optical path for the small particles in Table 1 (which contribute about this density), is about 1 cm. We assume that the dust is a strong absorber of light up to the dust's sublimation point. The heat required for this process is dominated by the latent heat of vaporization, L_v, of the dust which will be taken to be that of SiO_2,

$$L_v = 2300 \text{ cal/gram} \; . \tag{86}$$

The thickness of this heated layer therefore is, assuming all radiation is absorbed,

$$\ell[\text{cm}] \equiv \frac{W[\frac{\text{cal}}{\text{cm}^2}]}{L_v[\frac{\text{cal}}{\text{gm}}]\rho[\frac{\text{gm}}{\text{cm}^3}]} \approx 2500 \frac{Y[\text{Mtons}]}{R[\text{km}]^2} \quad ,$$

or

$$\ell[\text{m}] = 25 \frac{[\text{Mtons}]}{R[\text{km}]^2} \quad . \tag{87}$$

Hence, at a distance of 2 kilometers from a 1 megaton device, this highly heated thermal layer will be 6 meters thick. Reflection of light from the surface of the dust cloud probably reduces this length further possibly as much as 50 percent. Scattering of light may also lead to a much more complex spatial distribution of thermal energy.

The electrical conductivity in the vaporized layer should be much higher than that in normal air. Electrons are removed presumably only by dissociative recombination, e.g.,

$$e + O_2^+ \rightarrow O + O \;, \tag{88}$$

which has a rate constant

$$k \approx 2 \times 10^{-7} \text{ cm}^3/\text{sec} \;. \tag{89}$$

The electron conductivity would be

$$\sigma_e = N_e \mu_e \frac{e}{c} \approx 1.6 \times 10^{-14} \sqrt{\dot{\gamma}/k}$$

$$\approx 1.6 \times 10^{-6} \cdot \dot{D} \quad \text{cm}^{-1} \tag{90}$$

To reach conductivity of typical soils, 10^{-2} mho/m $\approx 3 \times 10^{-3}$ cm^{-1}, requires a dose rate of only

$$\dot{D} \approx 4 \times 10^6 \text{ rads/sec} \;. \tag{91}$$

Such a highly conducting layer would effectively separate the EMP generation above and below the layer, but lead to additional transverse EMP generation at the upper and lower edges of the heated layer. This layer is not likely to be smooth, because of turbulence. We shall not, in this report, attempt to estimate the features of the additional EMP. However, such work should be undertaken, along with firming up the basic physics of the heated layer.