ECE 238 Exam 1

Problem 1 10/10

Problem 2 15/15

Problem 3 35/35

Problem 4 15/15

Problem 5 25/25

100

Solutions

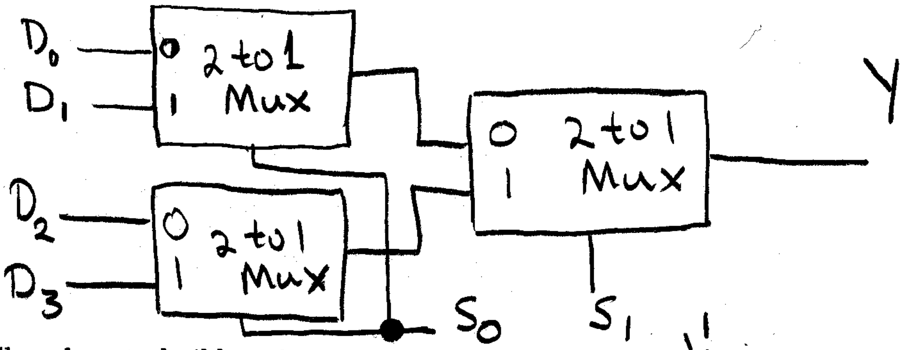
Good Luck!

Problem 1 (10 points)

We would like to use 2-to-1 multiplexers in order to build more complicated multiplexers.

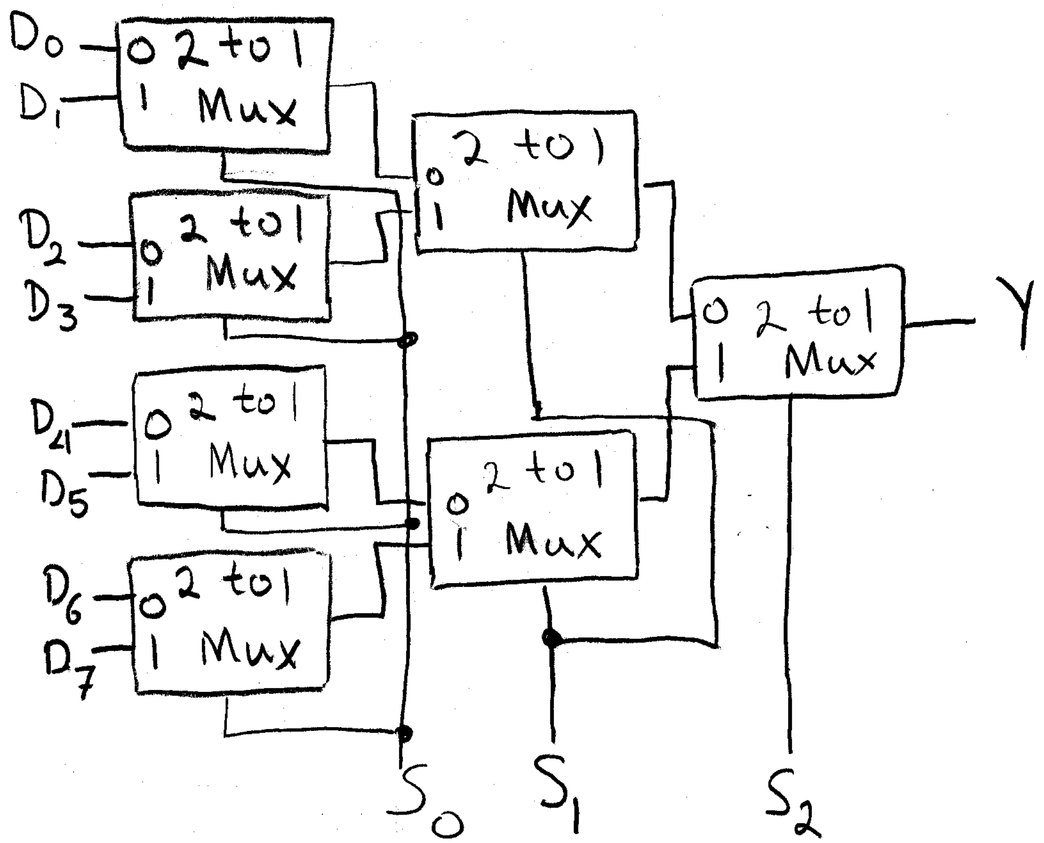
1(a)(5 points) Show how to build a 4-to-1 multiplexer by using 2-to-1 multiplexers only.

S_1	S_0	Y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3



1(b)(5 points) Show how to build an 8-to-1 multiplexer by using 2-to-1 multiplexers only.

S_2	S_1	S_0	Y
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7



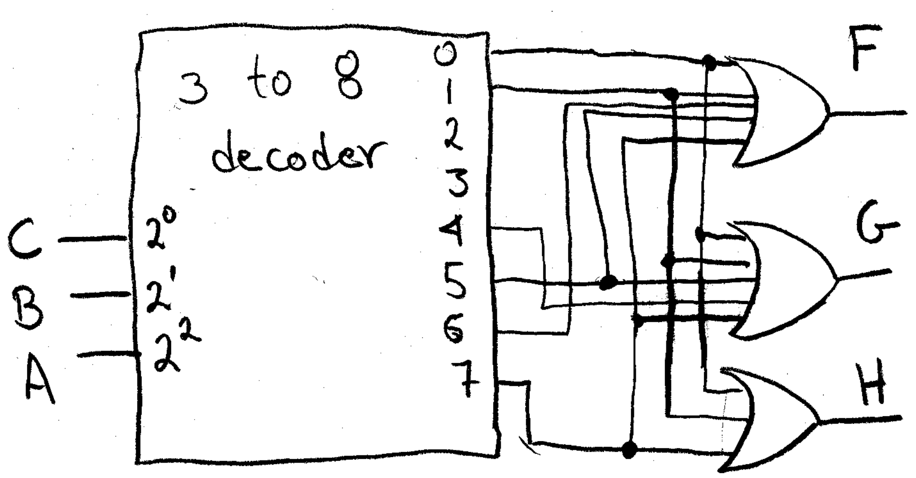
Problem 2 (15 points)
Use a 3-to-8 decoder and three OR-gates to realize the following functions:

$$F(A,\ B,\ C) = A(B + C) + A'B'$$
$$G(A,\ B,\ C) = \sum m(0,\ 1,\ 4,\ 5,\ 7)$$
$$H(A,\ B,\ C) = ABC + A'B'$$

$$F(A,B,C) = AB + AC + A'B'$$
$$= AB(C+C') + A(B+B')C + A'B'(C+C')$$
$$= ABC + ABC' + \cancel{ABC} + AB'C + A'B'C + A'B'C'$$

$$H(A,B,C) = ABC + A'B'(C+C')$$
$$= ABC + A'B'C + A'B'C'$$

A	B	C	F	G	H
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	1	0	0
1	1	1	1	1	1



Problem 3 (35 points).
In what follows, we want to implement the function $f(x) = x^2 - 1$ for three-bit integer inputs. Please read the instructions carefully and answer each part separately.

3(a)(5 points) For the inputs, we want to use three bit **signed numbers** $(a_2a_1a_0)_2$. Give all signed integer numbers that can be represented using three bits, assuming that 2's complement is used for representing negative numbers.

$a_2a_1a_0$	number
0 0 0	0
0 0 1	1
0 1 0	2
0 1 1	3

increasing

$a_2a_1a_0$	number
1 0 0	-4
1 0 1	-3
1 1 0	-2
1 1 1	-1

decreasing

3(b)(3 points) Plug in all the numbers represented in 3(a) into $f(x) = x^2 - 1$. What is the smallest number? What is the largest number?

x	x^2-1
-4	15
-3	8
-2	3
-1	0
0	-1
1	0
2	3
3	8

largest

smallest

3(c)(2 points) Based on your result in 3(b), how many bits do we need in order to represent the result? For your answer, assume that we are using two's-complement for representing negative numbers.

Five bits: $\begin{cases} 01111 \text{ is } 15 \\ 11111 \text{ is } -1 \end{cases}$

Four bits are not enough!
In four-bits: 15 is 1111 = -1 !

3(d)(10 points) Provide a truth table where the input is $x = (a_2a_1a_0)_2$ and the output is $f(x) = x^2 - 1 = (b_nb_{n-1}\dots b_0)_2$, where n is determined from your answer in 3(c).

a_2	a_1	a_0	x	$f(x)$	b_4	b_3	b_2	b_1	b_0
0	0	0	0	-1	1	1	1	1	1
0	0	1	1	0	0	0	0	0	0
0	1	0	2	3	0	0	0	1	1
0	1	1	3	8	0	1	0	0	0

3(e)(10 points) Use three-variable Karnaugh maps to determine minimum sums of products for each one of $b_n, b_{n-1}, \dots, b_0$.

continuing 3(d)

a_2	a_1	a_0	x	$f(x)$	b_4	b_3	b_2	b_1	b_0
1	0	0	-4	15	0	1	1	1	1
1	0	1	-3	8	0	1	0	0	0
1	1	0	-2	3	0	0	0	1	1
1	1	1	-1	0	0	0	0	0	0

By inspection: $b_0 = b_1 = \bar{a}_0$

$a_2 \backslash a_1a_0$	00	01	11	10
0	1	0	0	0
1	0	0	0	0

$a_2 \backslash a_1a_0$	00	01	11	10
0	1	0	1	0
1	1	1	0	0

$a_2 \backslash a_1a_0$	00	01	11	10
0	1	0	0	0
1	1	0	0	0

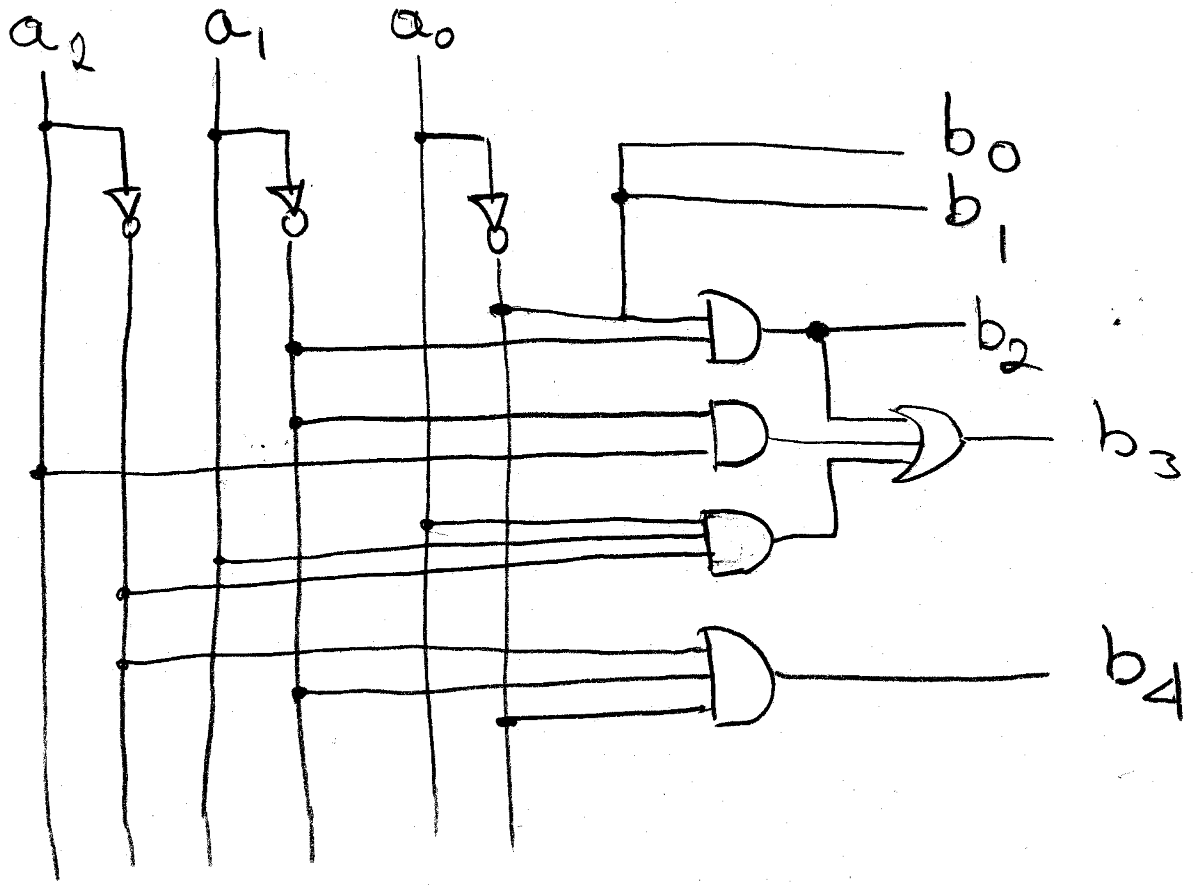
$b_4 = \bar{a}_2\bar{a}_1\bar{a}_0$

also by inspection!

$b_3 = \bar{a}_1\bar{a}_0 + a_2\bar{a}_1 + \bar{a}_2a_1a_0$

$b_2 = \bar{a}_1\bar{a}_0$

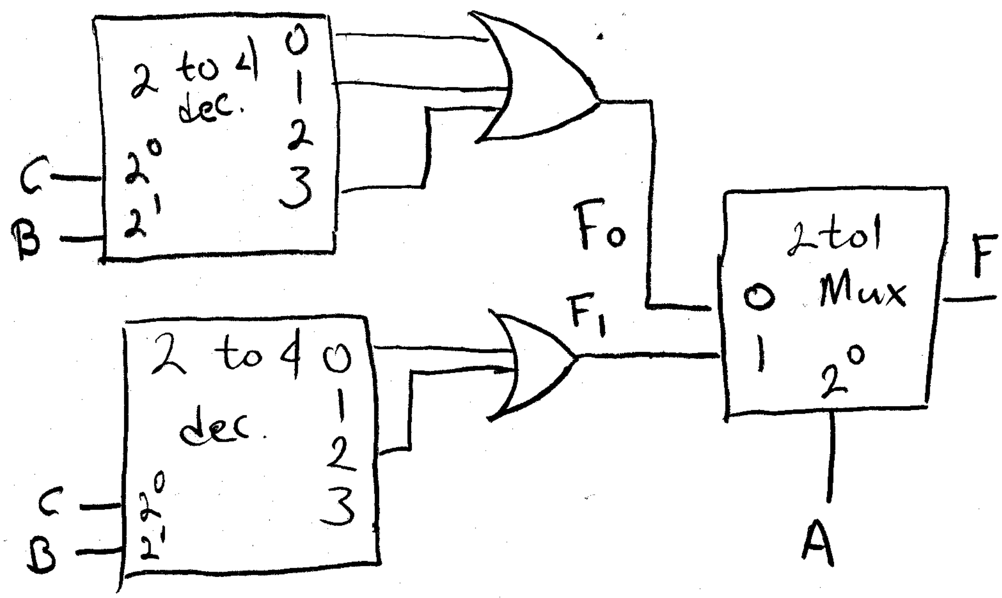
3(f)(5 points) Show the circuits for realizing b_n, b_{n-1}, $\ldots$, b_0 based on your answer from 3(e).



Problem 4(15 points)
Use two **2-to-4** decoders (**do not use a 3-to-8 decoder!**), two OR-gates, and a 2-to-1 multiplexer in order to realize the following three-variable function:

$$F(A,\ B,\ C) = \sum m(0,\ 1,\ 3,\ 4,\ 6)$$

A	B	C	F	
0	0	0	1	
0	0	1	1	F_0
0	1	0	0	
0	1	1	1	
1	0	0	1	
1	0	1	0	F_1
1	1	0	1	
1	1	1	0	



You could also combine the two decoders into one! (See problem 2).

The basic idea is that:

$$\underbrace{F(A,B,C)}_{\text{3-Vars}} = \bar{A}\, F_0 \underbrace{(B,C)}_{\text{2-Vars}} + A\, F_1 \underbrace{(B,C)}_{\text{2-Vars}}$$

Combine using 2to1 Mux.

Problem 5(25 points)

5(a)(10 points) Give the two's complement *binary, hexadecimal, and octal* representations of −1, −5, −8.

$-1: 0001$, 1110, $+1$; b: $(1111)_2$; h: $(F)_{16}$; o: $(17)_8$

$-5: 0101$, 1010, $+1$; b: $(1011)_2$; h: $(B)_{16}$; o: $(13)_8$

$-8: 1000$, 0111, $+1$; b: $(1000)_2$; h: $(8)_{16}$; o: $(10)_8$

4 bits are enough for representing −8 to +7.

5(b)(5 points) Show the binary results for the following operations: 4 − 5, 0 − 1.

$4: 0100$
$-5: 1011$
$-1: 1111$

$0: 0000$
$-1: 1111$
$-1: 1111$

5(c)(10 points) Give the binary representation of 1/10 assuming that only 8 bits can be used for representing the fractional part.

$\frac{1}{10} \times 2 = \frac{2}{10} \rightarrow b_{-1} = 0$

$\frac{2}{10} \times 2 = \frac{4}{10} \rightarrow b_{-2} = 0$

$\frac{4}{10} \times 2 = \frac{8}{10} \rightarrow b_{-3} = 0$

$\frac{8}{10} \times 2 = 1\frac{6}{10} \rightarrow b_{-4} = 1$

$\frac{6}{10} \times 2 = 1\frac{2}{10} \rightarrow b_{-5} = 1$

repeats (see b_{-2})

$\frac{1}{10} = (0.0\overline{0011})_2$

In eight-bits:

$(0.00011001)_2$