

lecture #1

✓ 1. Number systems and conversions
Rethinking Decimals

Chapter 1 + ~~p134-137~~

①

write 543.21 to represent:

$$5 \cdot 10^2 + 4 \cdot 10^1 + 3 \cdot 10^0 + 2 \cdot 10^{-1} + 1 \cdot 10^{-2}$$

The Base is $b = 10$.

The digits: 5, 4, 3, 2, 1 satisfy:

$$0 \leq 5, 4, 3, 2, 1 < 10$$

with the smallest possible digit being 0,
while the largest possible digit being $9 = 10 - 1$.

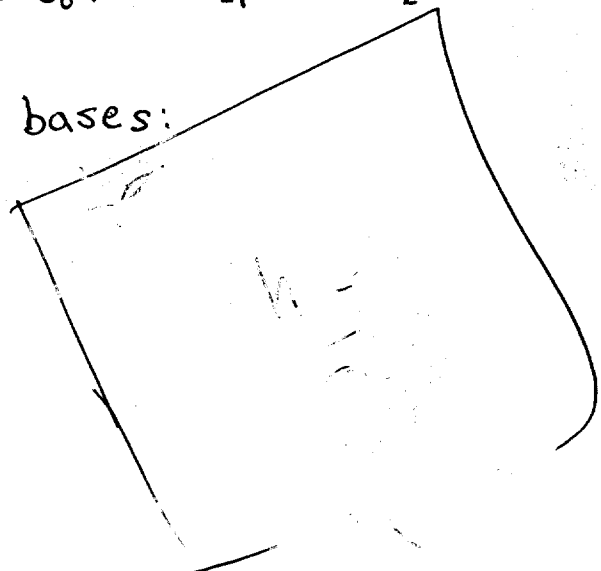
We generalize our notation for arbitrary base - b .

Let $c_N c_{N-1} \dots c_1 c_0 \cdot c_{-1} c_{-2} \dots c_{-M}$
the decimal point
represent:

$$c_N \cdot b^N + c_{N-1} \cdot b^{N-1} + \dots + c_1 \cdot b^1 + c_0 \cdot b^0 + c_{-1} \cdot b^{-1} + c_{-2} \cdot b^{-2} + \dots + c_{-M} \cdot b^{-M}$$

We are interested in ~~4~~ bases:

- base-10, $b = 10$
- base-2, $b = 2$
- base-8, $b = 8$
- base-16, $b = 16$



For each base, we adopt the following conventions:

- base-10, digits: 0, 1, 2, ..., 9
- base-2, digits: 0, 1
- base-8, digits: 0, 1, 2, 3, ..., 7
- base-16, digits: 0, 1, 2, 3, ..., 7, 8, 9, A, B, C, D, E, F

What is "new" is that:

- A represents 10
- B represents 11
- C represents 12
- D represents 13
- E represents 14
- F represents 15

for the base-16 system.

We give "special names" to our bases of interest

- base-10 is the decimal system
- base-2 is the binary system
- base-8 is the octal system
- base-16 is the hexadecimal system.

From now on, we must specify the number system of interest in order to specify a number.

Example:

What does 543.21 represent in

(a) hexadecimal?

(b) octal?

Solution

(a) $543.21 = ?$

$\begin{array}{c} \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \text{2nd} \quad \text{1st} \quad \text{0th digit} \quad \text{-1th digit} \\ \text{-2 digit} \end{array}$

The order of the digit can be used to determine the number:

$$(543.21)_{16} = 5 \cdot 16^2 + 4 \cdot 16^1 + 3 \cdot 16^0 + 2 \cdot 16^{-1} + 1 \cdot 16^{-2}$$

$$= 1347.12890625$$

(b) For octal, we write:

$$(543.21)_8 = 5 \cdot 8^2 + 4 \cdot 8^1 + 3 \cdot 8^0 + 2 \cdot \left(\frac{1}{8}\right) + 1 \cdot \left(\frac{1}{64}\right)$$

$$= 355.265625$$