P71 2-7 Exclusive-OR gates

Tuesday
9-17-2002

1/5

Definitions

XOR: $X \oplus Y = X\bar{Y} + \bar{X}Y$

Truth Table:

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

~~Enc~~ Encrption.

Notice that: $(X \oplus Y) \oplus Y = X$ from:

X		$X \oplus Y$	Y	$(X \oplus Y) \oplus Y$
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

same.

If: X represents a background pixel,
Y represents a foreground pixel.
$X \oplus Y$ "places" Y on X,
while $(X \oplus Y) \oplus Y$ removes Y and restores X

Its complement:

$$\overline{X \oplus Y} = (X\bar{Y} + \bar{X}Y)'$$
$$= (X' + Y)(X + Y')$$
$$= \underbrace{\cancel{X'X}}_{0} + X'Y' + YX + \underbrace{\cancel{YY'}}_{0}$$
$$= X'Y' + YX$$

which is only true if X is the same as Y:

$X \backslash Y$	0	1
0	1	0
1	0	1

$\overline{X \oplus Y}$

Note diagonal pattern

$\oplus$ is commutative: $A \oplus B = B \oplus A$

$\oplus$ is associative: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
$= A \oplus B \oplus C$

$\oplus$ K-maps occupy alternating tiles:

$X \oplus Y \oplus Z$

$X \backslash YZ$	00	01	11	10
0		1		1
1	1		1	

1 when the number of 1s in XYZ is odd.

$X \oplus Y \oplus Z$

Identities:

$X \oplus 0 = X$ $\quad$ $X \oplus 1 = \bar{X}$

$X \oplus X = 0$ $\quad$ $X \oplus \bar{X} = 1$

$X \oplus \bar{Y} = \overline{X \oplus Y}$ $\quad$ $\bar{X} \oplus Y = \overline{X \oplus Y}$

- 4-Variable map obeys the same principle: It has 1s when the number of 1s in ABCD is odd.

AB \ CD	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

$A \oplus B \oplus C \oplus D$

This property makes XOR ideal for <u>parity generation</u> and <u>parity checking</u>

<u>Parity Generation</u>: For a given number of bits, attach a new bit of 0 or 1 to ensure that the total number of 1s is either even or odd.

- Eg: odd parity bit generation.

101 has 2 1s.

(a) Attach 1 at the end to get: 1011 which has 3 1s and odd-parity

(b) Attach 0 at the end to get: 1010 which has 2 1s and even-parity

- The logical function f which returns 1, thus making the total number of 1s in its argument even is called the odd-function. It can be realized using XOR:

$$f(A,B) = A \oplus B$$

$$f(A,B,C) = A \oplus B \oplus C$$

$$f(A,B,C,D) = A \oplus B \oplus C \oplus D$$

$$\vdots$$

$$f(X_1, \ldots, X_n) = X_1 \oplus X_2 \oplus \cdots \oplus X_n$$

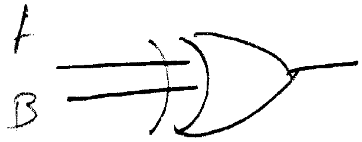
- To generate ~~even~~ even parity bit for 7-bits: $f(x_1, \dots, x_7) = x_1 \oplus \cdots \oplus x_7$.

For ~~even~~ odd parity, simply complement:

$$f'(x_1, \dots, x_7) = \overline{(x_1 \oplus \cdots \oplus x_7)},$$

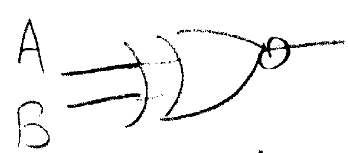
or implement using exclusive-NOR gates

- XOR GATES



$A \oplus B$

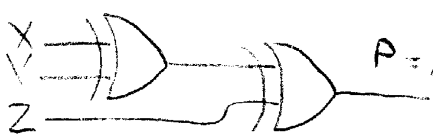
X-NOR GATES



$\overline{A \oplus B}$

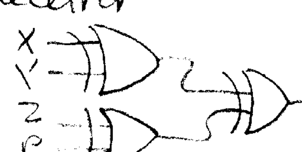
Even parity generation & checking (use odd functions)

Sender



$P = X \oplus Y \oplus Z$

Receiver



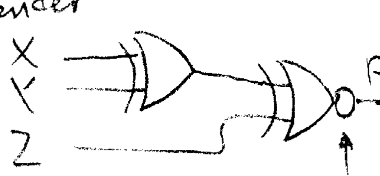
$C = X \oplus Y \oplus Z \oplus P$

$C = 1 \rightarrow$ error

number of 1's odd

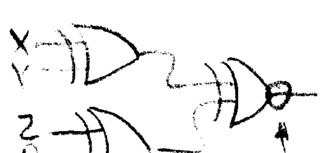
Odd parity generation & checking (use even functions)

Sender



$P = \overline{X \oplus Y \oplus Z}$

Receiver



$C = \overline{X \oplus Y \oplus Z \oplus P}$

$C = 1$: error.

# of 1's even

Ignore

Plotting XOR K-maps

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11				
10	1	1	1	1

$A \oplus B$

$A \oplus B = 1$

when $\begin{cases} A=1,\ B=0 \\ A=0,\ B=1 \end{cases}$

$A' \oplus B = 1$

when $\begin{cases} A'=0,\ B=1 \\ A'=1,\ B=0 \end{cases}$

or $A=1,\ B=1$

$A=0,\ B=0$

AB \ CD	00	01	11	10
00	1	1	1	1
01				
11	1	1	1	1
10				

$A' \oplus B = 1$

$A' \oplus B \oplus C = 1$

AB \ CD	00	01	11	10
00	1	1		
01			1	1
11	1	1		
10			1	1

when we have odd # of 1s:

$A'=1,\ B=0,\ C=0,\ (A=0)$

$A'=1,\ B=1,\ C=1,\ (A=0)$

$A'=0,\ B=1,\ C=0\ \ (A=1)$

$A'=0,\ B=0,\ C=1\ \ (A=1)$