Capacity of Multi-antenna Systems with Adaptive Transmission Techniques *

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Abstract

The capacity of multiple antenna systems in the presence of Rayleigh flat fading is considered under the assumption that channel state information (CSI) is available at both transmitter and receiver. The capacity expression for a general dual antenna array system of multiple transmitter and receiver antennae is derived together with an equation that determines the cut-off value for such a system. It is shown that, compared to the case in which there is only receiver CSI, large capacity gains are available with optimal power and rate adaptation scheme.

Keywords

Multiple antenna systems, capacity, adaptive transmission.

INTRODUCTION

The problem of determining the capacity of fading channels under various assumptions has received considerable attention over the years. the capacity of such channels of course varies depending on the assumptions one makes about the fading statistics and about the knowledge of fading coefficients.

When both transmitter and receiver have access to CSI, intuitively one would expect the transmitter to adjust its power and rate depending on the instantaneous value of the CSI. This results in adaptive transmission techniques. The capacity of fading channels with such adaptive transmission schemes has been treated previously in [3] for the case of single-antenna systems and in [1] for the case of receiver diversity. However, with the recent interest in multiple transmit antennae systems for wireless communications, it is also of interest to consider this problem in the context of multiple antennas at both transmitter and receiver. In this paper we investigate the capacity of such systems under adaptive transmission techniques.

We derive the capacity of optimal power and rate allocation scheme for such systems and evaluate this capacity for several representative situations showing that the capacity of such systems could be much larger than corresponding systems with only the receiver CSI. The increased capacity comes at the price of channel outage which may result in large delays.

The rest of this paper is organized as follows: In Section we outline our system model and the assumptions on the fading distribution. Section treats the capacity of a general system having multiple antennae at both transmitter and the receiver. We obtain the capacity of such systems under optimal power adaptation as well as the determining equation for the cut-off value of the optimal transmission scheme. In Section we numerically evaluate the derived capacity results for some representative situations. Finally, in Section we give some concluding remarks.

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SYSTEM MODEL DESCRIPTION

We consider a single user, flat fading communications link in which transmitter and receiver are equipped with N_T and N_R antennae, respectively. The discrete-time received signal in such a system can be written in matrix form as

$$\mathbf{y}(i) \quad = \quad \mathbf{H}(i)\mathbf{x}(i) + \mathbf{n}(i), \qquad (1)$$

where $\mathbf{y}(i)$, $\mathbf{x}(i)$ and $\mathbf{n}(i)$ are the complex N_R-vector of received signals at the N_R receive antennas, the (possibly) complex N_T-vector of transmit signals on the N_T transmit antennas, and the complex N_R-vector of additive receiver noise, respectively, at symbol time i. The components of $\mathbf{n}(i)$ are independent, zero-mean, circularly symmetric complex Gaussian with independent real and imaginary parts having equal variance. The noise is also assumed to be independent with respect to the time index, and $\mathrm{E}\{\mathbf{n}(i)\mathbf{n}(i)^H\} = \mathbf{I}_{N_R}$.

The matrix $\mathbf{H}(i)$ in (1) is the $N_R \times N_T$ matrix of complex fading coefficients which are assumed to be stationary and ergodic. The (n_R, n_T)-th element of the matrix $\mathbf{H}(i)$ represents the fading coefficient value at time i corresponding to the path between the n_R-th receiver antenna and the n_T-th transmitter antenna. We assume that elements of the matrix $\mathbf{H}(i)$ are independent, identically distributed (iid) complex Gaussian random variables with zero mean and $1/2$-variance per dimension (i.e. the Rayleigh fading channel model) and are known to both transmitter and the receiver. This is a reasonable assumption when the channel varies at a much slower rate compared to the data rate of the system.

As we will see shortly, the capacity will be dependent on the number of transmitter and receiver antennas only through the relative parameters defined as $n = \max\{N_R, N_T\}$ and $m = \min\{N_R, N_T\}$.

CAPACITY OF MULTI-ANTENNA SYSTEMS WITH CSI AT BOTH TRANSMITTER AND THE RECEIVER

In general we may decompose the fading coefficient matrix

$\mathbf{H}$ using the singular value decomposition:

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H , \quad (2)$$

where $\mathbf{U}$, $\mathbf{\Lambda}$ and $\mathbf{V}$ are matrices of dimension $N_R \times N_R$, $N_R \times N_T$ and $N_T \times N_T$, respectively. The matrices $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices satisfying $\mathbf{U}\mathbf{U}^H = \mathbf{U}^H\mathbf{U} = \mathbf{I}_{N_R}$ and $\mathbf{V}\mathbf{V}^H = \mathbf{V}^H\mathbf{V} = \mathbf{I}_{N_T}$. The matrix $\mathbf{\Lambda} = [\lambda_{i,j}]$ is a diagonal matrix with diagonal entries being equal to the non-negative square roots of the eigenvalues of either $\mathbf{H}\mathbf{H}^H$ or $\mathbf{H}^H\mathbf{H}$, and thus being uniquely determined. For later use, we may also define the following $m \times m$ matrix,

$$\mathbf{W} = \begin{cases} \mathbf{H}\mathbf{H}^H & \text{if } N_R \leq N_T \\ \mathbf{H}^H\mathbf{H} & \text{if } N_R > N_T \end{cases} . \quad (3)$$

Note that, $\mathbf{W}$ can have only m non-zero eigenvalues and thus correspondingly only m diagonal entries of the matrix $\mathbf{\Lambda}$ are non-zero. It is also worth mentioning that the distribution of the matrix $\mathbf{W}$ is given by the well-known Wishart distribution [5].

Defining the transformations $\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}$, $\tilde{\mathbf{x}} = \mathbf{V}^H\mathbf{x}$ and $\tilde{\mathbf{n}} = \mathbf{U}^H\mathbf{n}$, we see that the channel in (1) is equivalent to

$$\tilde{\mathbf{y}} = \mathbf{\Lambda}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}. \quad (4)$$

If the average transmit power is constrained as

$$E\{\mathbf{x}^H\mathbf{x}\} = \text{tr}\left[E\{\mathbf{x}\mathbf{x}^H\}\right] = P, \quad (5)$$

then we also have that

$$E\{\tilde{\mathbf{x}}^H\tilde{\mathbf{x}}\} = \text{tr}\left[E\{\tilde{\mathbf{x}}\tilde{\mathbf{x}}^H\}\right] = P. \quad (6)$$

Let us introduce the following notation:

$$\mathbf{\Lambda}^{'}(i) = \sqrt{\frac{P}{mN_0}}\mathbf{\Lambda}(i). \quad (7)$$

We may interpret each diagonal element of the matrix $\mathbf{\Lambda}^{'}(i)$ as a representation of the average Signal-to-Noise-Ratio (SNR) per mode.

We let the transmit power vary with the observed channel state information subject to the average power constraint P. If we define $\tilde{\mathbf{Q}} = \tilde{\mathbf{x}}\tilde{\mathbf{x}}^H$, then the instantaneous transmit power can be written as $\tilde{\mathbf{x}}^H\tilde{\mathbf{x}} = \text{tr}\left[\tilde{\mathbf{Q}}\right]$, and the average power constraint becomes $E\{\text{tr}\left[\tilde{\mathbf{Q}}\right]\} \leq P$. Hence in this case the adaptive transmission strategy based on the observed channel state information can be achieved by letting $\tilde{\mathbf{Q}}$ be a function of $\mathbf{\Lambda}^{'}(i)$. Thus, we denote the instantaneous value of $\tilde{\mathbf{Q}}(i)$ as $\tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'}(i))$. Then, we may define the average capacity of the vector, time-varying channel with adaptive transmission scheme as

$$C = \max_{\substack{\text{tr}(E\{\tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'})\})=P \\ \tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'})>0}} E_{\mathbf{\Lambda}^{'}}\left\{\log\det\left(\mathbf{I} + \mathbf{\Lambda}^{'}\frac{\tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'})}{(P/m)}\mathbf{\Lambda}^{'}\right)\right\} . \quad (8)$$

It can be shown that the above maximization is achieved by a diagonal $\tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'})$ and that the diagonal entries are given by a matrix water filling formula to be, for $i = 1, \ldots, m$,

$$\frac{\tilde{\mathbf{Q}}_{i,i}}{(P/m)} = \begin{cases} \frac{1}{\gamma_{i,0}} - \frac{1}{\gamma_i} & \text{if } \gamma_i \geq \gamma_{i,0} \\ 0 & \text{if } \gamma_i \leq \gamma_{i,0} \end{cases} \quad (9)$$

where γ_i is defined, for $i = 1, \ldots, m$, as

$$\gamma_i = \bar{\gamma}\lambda_i , \quad (10)$$

where λ_i are the eigenvalues of the Wishart matrix $\mathbf{W}$ and we have defined $\bar{\gamma}$ as

$$\bar{\gamma} = \frac{P}{mN_0}. \quad (11)$$

The cut-off values $\gamma_{i,0}$ in (9) are chosen to satisfy the power constraint,

$$\begin{aligned} P &= \text{tr}(E\{\tilde{\mathbf{Q}}(\mathbf{\Lambda}^{'})\}) \\ &= \frac{P}{m}\sum_{i=1}^{m}\int_{\gamma_{i,0}}^{\infty}\left(\frac{1}{\gamma_{i,0}} - \frac{1}{\gamma_i}\right) f_{\gamma_i}(\gamma_i)\, d\gamma_i . \quad (12) \end{aligned}$$

where $f_{\gamma_i}(\gamma_i)$ denotes the pdf of the i-th non-zero eigenvalue of the Wishart matrix $\mathbf{W}$. If we let $f_\gamma(\gamma)$ denotes the pdf of any γ_i, for $i = 1, \ldots, m$, then (12) leads to

$$\int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_\gamma(\gamma)\, d\gamma = 1, \quad (13)$$

where γ_0 is the cut off transmission value corresponding to any eigenvalue.

The probability distribution function $p_\lambda(\lambda)$ of an un-ordered eigenvalue of a Wishart distributed matrix can be written as [6]

$$p_\lambda(\lambda) = \frac{e^{-\lambda}\lambda^{n-m}}{m}\sum_{k=1}^{m}\frac{(k-1)!}{(n-m+k-1)!}\left[L_{k-1}^{n-m}(\lambda)\right]^2 , \quad (14)$$

where the associated Laguerre polynomial of order k, $L_k^{n-m}(\lambda)$, is defined as [2, 4],

$$L_k^a(\lambda) = \sum_{p=0}^{k}(-1)^p\binom{k+a}{k-p}\frac{\lambda^p}{p!} , \quad (15)$$

with the binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Then from the definition in (10) we have that $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}}p_\lambda(\frac{\gamma}{\bar{\gamma}})$, and substituting this into (13), the equation that must be satisfied by the cut off becomes

$$\sum_{k=1}^{m}\frac{(k-1)!}{(n-m+k-1)!}\int_{\mu}^{\infty}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)e^{-\gamma}\gamma^{n-m}\left[L_{k-1}^{n-m}(\gamma)\right]^2 d\gamma = m\bar{\gamma}$$

where μ is defined as

$$\mu = \frac{\gamma_0}{\bar{\gamma}}. \quad (17)$$

In the next section we show that for any $\bar{\gamma} > 0$, the above cut off equation (16) has a unique solution μ.

Uniqueness of the Cut-off Value

Intuitively one would expect (16) to have a unique solution μ. In fact, by studying the properties of (16) we may show that this indeed holds true.

For convenience let us define the integrand in (16) to be

$$f_{n-m,k}(\gamma, z) = \left(\frac{1}{z} - \frac{1}{\gamma}\right)e^{-\gamma}\gamma^{n-m}\left[L_{k-1}^{n-m}(\gamma)\right]^2 \quad (18)$$

Next, define the function $F(z)$ as,

$$F(z) = \sum_{k=1}^{m} \frac{(k-1)!}{(n-m+k-1)!} \int_{z}^{\infty} f_{n-m,k}(\gamma, z) d\gamma - m\bar{\gamma}. \quad (19)$$

Note that (16) is then equivalent to the case of $F(z) = 0$. Differentiating (19) with respect to z gives

$$F'(z) = -\sum_{k=1}^{m} \frac{(k-1)!}{(n-m+k-1)!} \frac{1}{z^2} \int_{z}^{\infty} e^{-\gamma}\gamma^{n-m} \left[L_{k-1}^{n-m}(\gamma)\right]^2 d\gamma, \quad (20)$$

and we immediately notice that, since the integrand in (20) is positive,

$$F'(z) \quad < \quad 0 \text{ for } z > 0. \quad (21)$$

Similarly, one can also show that $F''(z) > 0$ for $z > 0$.
Next, either relying on the normalization property of a pdf or by explicitly recalling the integral equation 7.414.9 of [4] we have that,

$$\lim_{z \longrightarrow 0^+} \int_{z}^{\infty} e^{-\gamma}\gamma^{n-m} \left[L_{k-1}^{n-m}(\gamma)\right]^2 d\gamma = \frac{(n-m+k-1)!}{(k-1)!} \quad \text{for } n-m \geq 0. \quad (22)$$

Using equation 7.414.12 of [4], for $n-m > 0$, we also have that, for $n-m > 0$

$$\lim_{z \longrightarrow 0^+} \int_{z}^{\infty} e^{-\gamma}\gamma^{n-m-1} \left[L_{k-1}^{n-m}(\gamma)\right]^2 d\gamma = \frac{\Gamma(n-m)\Gamma(n-m+k)}{\Gamma(n-m+1)[(k-1)!]^2} \times \frac{d^{k-1}}{dh^{k-1}} \left[\frac{F(\frac{n-m}{2}, \frac{n-m}{2} + \frac{1}{2}; n-m+1; \frac{4h}{(1+h)^2})}{(1-h)(1+h)^{n-m}} \right]_{h=0}, \quad (23)$$

where $F(a, b; c; x)$ is the hypergeometric function defined as [2, 4]

$$F(a, b; c; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!}, \quad (24)$$

with hypergeometric coefficient $(a)_k$ defined as the product

$$(a)_k = a(a+1)\ldots(a+k-1), \quad (25)$$

with $(a)_0 = 1$.
Applying a transformation formula for a hypergeometric function (equation 9.134.2 of [4]) to (23) we have, for $n - m > 0$,

$$\lim_{z \longrightarrow 0^+} \int_{z}^{\infty} e^{-\gamma}\gamma^{n-m-1} \left[L_{k-1}^{n-m}(\gamma)\right]^2 d\gamma = \frac{(n-m+k-1)!}{(n-m)(k-1)!}. \quad (26)$$

Substitution of (22) and (26) into (19) shows that, for $n-m > 0$,

$$\lim_{z \longrightarrow 0^+} F(z) = +\infty \quad \text{for } n-m > 0. \quad (27)$$

Similarly, for $n-m = 0$,

$$\lim_{z \longrightarrow 0^+} F(z) = \sum_{k=1}^{m} \frac{(k-1)!}{(n-m+k-1)!} \lim_{z \longrightarrow 0^+} \left[\frac{1}{z} - E_1(z)\right], = +\infty \quad \text{for } n-m = 0, \quad (28)$$

where E is the Euler's constant, $E_1(z)$ is the exponential integral function [2, 4] defined as

$$E_1(z) = \int_{z}^{\infty} \frac{e^{-t}}{t} dt, \quad (29)$$

and we have also made use of the fact that

$$\lim_{z \longrightarrow 0} z \log(z) = 0.$$

Also from (19) it is easily seen that

$$\lim_{z \longrightarrow +\infty} F(z) = -m\bar{\gamma} \quad \text{for } n-m \geq 0. \quad (30)$$

Thus, from (21), (27), (28) and (30) it follows that for $z > 0$, the function $F(z)$ has a unique zero for all $n-m \geq 0$. From (17) then we see that for any $\bar{\gamma} > 0$ there exists a unique cut-off value γ_0 for any $n-m \geq 0$ which satisfies (16), as we expected.

Evaluation of Cut-off Value

Substituting the polynomial representation (15) of $L_k^a(\mu)$ in (16) we obtain

$$\sum_{k=1}^{m} \frac{(k-1)!}{(n-m+k-1)!} \sum_{p=0}^{k-1}\sum_{q=0}^{k-1} \binom{n-m+k-1}{k-1-p} \times \binom{n-m+k-1}{k-1-q} \frac{(-1)^{p+q}}{p!q!} G_{p,q}(\mu) = m\bar{\gamma}, \quad (31)$$

where, for $p+q = 0, 1, \ldots, 2(m-1)$, we have defined the integral $G_{p,q}(\mu)$ to be

$$G_{p,q}(\mu) = \int_{\mu}^{\infty} \left(\frac{1}{\mu} - \frac{1}{\gamma}\right) e^{-\gamma}\gamma^{n-m+p+q} d\gamma. \quad (32)$$

Next, we consider the two cases of $n-m > 0$ and $n-m = 0$ separately in order to obtain an explicit solution to (31).

Case 1: $n-m > 0$
Note that, when $n-m > 0$, for $p+q = 0, \ldots, 2(m-1)$, we have that $n-m+p+q-1 \geq 0$ and $n-m+p+q \geq 1 > 0$. Then we easily have that

$$G_{p,q}(\mu) = \frac{\Gamma(n-m+p+q+1, \mu)}{\mu} - \Gamma(n-m+p+q, \mu) \quad \text{for } p+q = 0, 1, \ldots, 2(m-1) \quad \text{and} \quad n-m > 0 \quad (33)$$

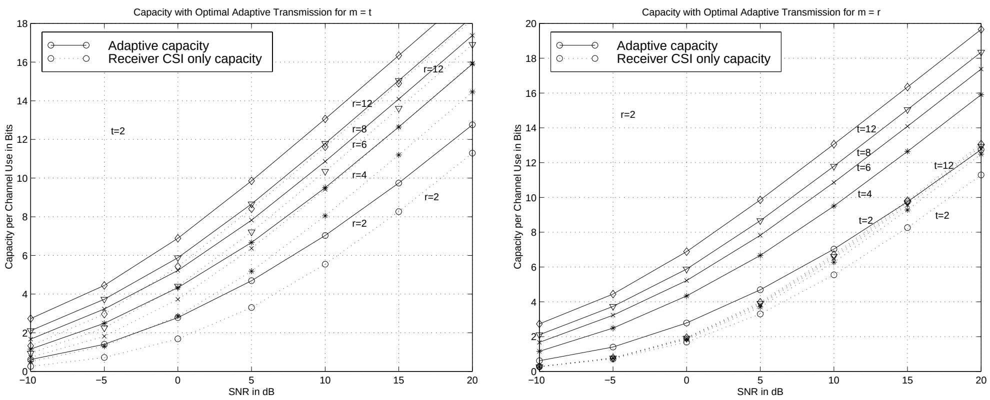
where, for $Re\{a\} > 0$, the complementary incomplete gamma function, $\Gamma(a, x)$, is defined as the integral,

$$\Gamma(a, x) = \int_{x}^{\infty} e^{-t}x^{a-1} dt, \quad (34)$$

and we have also made use of the integral identity

$$\int_{\mu}^{\infty} e^{-\gamma}\gamma^{n} d\gamma = n! e^{-\mu} \sum_{j=0}^{n} \frac{\mu^j}{j!} \quad \text{for } n \geq 0, \quad (35)$$

which can be verified straightforwardly via repeated application of integration by parts.
Substituting (33) into (31) we obtain a closed form equation that can be solved for a unique μ (which is known to exist by the previous section), in general, via numerical root finding.



(a) (b)

Figure 1. Capacity of the Multiple Antenna System with Optimal Adaptive Transmission Versus SNR (in dB). $m = 2$**. (a)** $N_T = m$ **in the Receiver CSI only System. (b)** $N_R = m$ **in the Receiver CSI only System.**

Case 2: $n - m = 0$

When $n-m = 0$, for $p+q = 0, \ldots, 2(m-1)$, we still have that $n - m + p + q \geq 0$. However, in this case $n - m + p + q - 1 \geq -1$. For $n - m = 0$ (31) reduces to

$$\sum_{k=1}^{m}\sum_{p=0}^{k-1}\sum_{q=0}^{k-1} \frac{(-1)^{p+q}}{p!q!}\binom{k-1}{k-1-p}\binom{k-1}{k-1-q}G_{p,q}(\mu) = m\bar{\gamma},$$

and similarly, for $p+q = 0, 1, \ldots, 2(m-1)$, integral $G_{p,q}(\mu)$ in (36) becomes

$$G_{p,q}(\mu) \;=\; \int_{\mu}^{\infty}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)e^{-\gamma}\gamma^{p+q}d\gamma. \qquad (36)$$

Then, we can easily show that

$$G_{p,q}(\mu) \;=\; \begin{cases} \frac{\Gamma(1,\mu)}{\mu} - E_1(\mu) & \text{if } p+q = 0 \\ \frac{\Gamma(p+q+1,\mu)}{\mu} - \Gamma(p+q,\mu) & \text{if } p+q > 0 \end{cases},(37)$$

where $E_1(\mu)$ is the exponential integral function defined in (29).

Substituting (37) into (31) again we may obtain a closed form equation in μ that can be solved for a unique solution. It is also easily verified that this general equation reduces to the corresponding equation given in [1] for the case of $r = t = 1$. It may be observed via numerical evaluation of the cut off value, that γ_0 lies in the range $0 \leq \gamma_0 \leq 1$, and specifically $\gamma_0 \longrightarrow 1$ as $\bar{\gamma} \longrightarrow \infty$. This was previously observed for single transmitter antenna systems by Alouini in [1].

Evaluation of Capacity

Substituting (9) into (8) we can show that the capacity of the multiple antenna system is

$$C \;=\; m\int_{\gamma_0}^{\infty}\log\left(\frac{\gamma}{\gamma_0}\right)\, f_{\gamma}(\gamma)\, d\gamma, \qquad (38)$$

where γ_0 is the cut off transmission value corresponding to any eigenvalue derived in the previous section and $f_{\gamma}(\gamma)$ is the pdf of any scaled, un-ordered eigenvalue given in earlier. Using the explicit form of the pdf $f_{\gamma}(\gamma)$ and the representation of Laguerre polynomial given in (15), we can write (38) as

$$C \;=\; \sum_{k=1}^{m}\frac{(k-1)!}{(n-m+k-1)!}\sum_{p=0}^{k-1}\sum_{q=0}^{k-1}\binom{n-m+k-1}{k-1-p}\times \binom{n-m+k-1}{k-1-q}\frac{(-1)^{p+q}}{p!q!}\mathcal{J}_{n-m+p+q+1}(\mu) \quad (39)$$

where $\mathcal{J}_p(\mu)$, for $p = 1, 2, \ldots$, is an integral function that can be evaluated in closed form to be [1]

$$\mathcal{J}_p(\mu) \;=\; (p-1)!\left[E_1(\mu) + \sum_{j=1}^{p-1}\frac{\mathcal{P}_j(\mu)}{j}\right], \quad (40)$$

with the Poisson sum $\mathcal{P}_k(\mu)$ given by

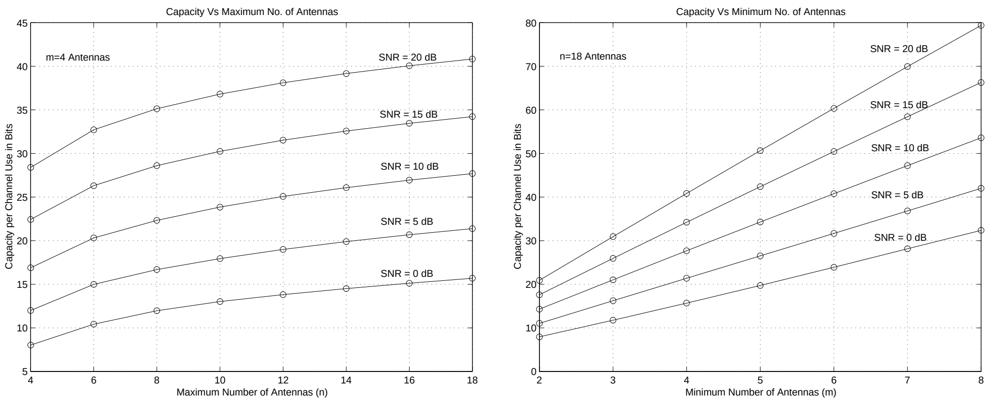
$$\mathcal{P}_k(\mu) \;=\; e^{-\mu}\sum_{j=0}^{k-1}\frac{\mu^j}{j!}. \qquad (41)$$

Substituting (40) into (39) we obtain the capacity of the multiple antenna system, in Bits per Channel Use, to be

$$C \;=\; \log_2(e)\sum_{k=1}^{m}\frac{(k-1)!}{(n-m+k-1)!}\sum_{p=0}^{k-1}\sum_{q=0}^{k-1}\frac{(-1)^{p+q}}{p!q!}\binom{n-m+k-1}{k-1-p}\times \binom{n-m+k-1}{k-1-q}(n-m+p+q)!\left[E_1(z) + \sum_{j=1}^{n-m+p+q}\frac{\mathcal{P}_j(z)}{j}\right] (42)$$

NUMERICAL RESULTS

Figure 1 plots the capacity of a multiple antenna system for $m = 2$ with different values of n versus the SNR. Shown on the same figure is the capacity of the corresponding multiple antenna system with only receiver CSI obtained in [6]. While capacity of a multiple antenna system with CSI at both transmitter and receiver is invariant to which end of the link has the larger number of antennas, this is not the case with only receiver CSI. Thus, Fig. 1 (a) specifically corresponds



(a) (b)

Figure 2. Dependence of Multiple Antenna System Capacity on the Relative Values os the Number of Antennas. (a) Versus Maximum Number of Antennas (n). (b) Versus Minimum Number of Antennas (m). $n = 18$.

to the case when the receiver CSI system has $N_T = m$ and $N_R = n$, while 1 (b) corresponds to the case when the receiver CSI only system has $N_T = n$ and $N_R = m$. It is clear from Fig. 1 (a) that large capacity improvements can be achieved with adaptive power and rate allocation when CSI is available at both ends of the system as compared to the case when there is only receiver CSI available.

Note that, the receiver CSI only system has a lower capacity in the case of Fig. 1 (b) than in the case of Fig. 1 (a) resulting in a larger capacity gap compared to our adaptive transmission system. In fact, it was shown in [6] that the asymptotic capacity of the receiver CSI only system in the Fig. 1 (a) case is $\log(1 + N_R \frac{P}{N_0})$ while in the Fig. 1 (b) system it is $\log(1 + \frac{P}{N_0})$. However, the capacity of the adaptive transmission scheme is invariant under the swapping of the transmitter and receiver antennae and also larger than either of the cases in receiver CSI only system.

Next, in Fig. 2 we plot the dependence of the multiple antenna system capacity on the relative values of the number of antennas, namely m and n. Figure 2 (a) plots the capacity of multiple antenna systems versus the maximum number of antennae n for a fixed minimum number of antennas $m = 4$. This figure shows that once the minimum number of antennae m is fixed, increasing n beyond some large value returns diminishing capacity gains. In other words, capacity is more dependent on the minimum number of antennae m at either end, as long as the maximum number of antennae n is sufficiently large. This, in fact, is similar to the case when CSI is available only at the receiver end of the communications link [6].

Finally, Fig. 2 (b) shows the capacity against the minimum number of antennae m at one of the ends of the link against a fixed but large maximum number of antennae n at the other end of the link. Figure 2 (b) corresponds to $n = 18$. As observed in the case of CSI available only at the receiver in [6], from Fig. 2 (b) we see that again the capacity is almost linear in the minimum number of antennae m.

CONCLUSIONS

In this paper we have derived the capacity of multiple antenna systems in the presence of Rayleigh flat fading under the assumption that CSI is available at both transmitter and receiver. We obtained the optimal power and rate adaptation scheme for such a system with the determining equation for the associated cut-off transmission value. By numerically evaluating the derived capacity expressions it was shown that large capacity gains are available with optimal power and rate adaptation scheme when CSI is available at both ends, compared to the receiver CSI only case.

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