MIMO Capacity Results for Rician Fading Channels

Sudharman K. Jayaweera
Department of Electrical and Computer Engineering
Wichita State University
Wichita, KS 67260
Email: sudharman.jayaweera@wichita.edu

H. Vincent Poor
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544
Email: poor@princeton.edu

***Abstract*— The capacity of a multiple antenna system in Rician fading is considered under the assumption that channel state information (CSI) is available only at the receiver and that the transmitter has knowledge of statistical properties of the fading process though not the instantaneous CSI. Tight upper and lower bounds for the capacity of such a system are derived and a new input signalling scheme that significantly outperforms the existing choices is also proposed. It is shown that by exploiting the knowledge of *Rician-ness* at the transmitter, a much higher capacity can be achieved for a multiple antenna system in Rician fading. The derived capacity bounds have been evaluated explicitly to provide numerical results in some representative situations.**

I. Introduction

In recent years, there has been a significant interest in using dual antenna arrays in wireless communication systems due to the information theoretic results suggesting possible extraordinary capacity gains of multiple transmit and receive antenna systems [1]–[3].

The most common model employed in the analysis of wireless communications systems is the Rayleigh fading model in which the fading coefficients are assumed to be zero-mean, complex Gaussian distributed. The Rayleigh fading model is a reasonable assumption for many fading environments encountered in practical communications systems. However, in environments where there is a strong direct Line-Of-Sight (LOS) path between the transmitter and the receiver, the complex Gaussian distributed fading coefficient should be modelled as having a non-zero mean, giving rise to the Rician fading model. Thus, it is also of interest to investigate the capacity of multiple antenna communication systems in the presence of Rician fading. In fact, since both additive white Gaussian noise (AWGN) and Rayleigh fading channels may be considered to be limiting cases of the Rician model, the choice of a Rician model is a general approach to the analysis of wireless communications systems.

The Rician distribution is usually characterized by the Rice factor, κ (see Section II), which reflects the relative strength of the direct line of sight path component of the fading coefficient. When $\kappa = 0$ this model reduces to Rayleigh fading and as $\kappa \longrightarrow \infty$ the fading becomes deterministic giving rise to an AWGN channel. In both these cases if the transmitter knows whether the channel is Rayleigh or AWGN, we may interpret this as transmitter having the knowledge of the value of κ in a Rician channel but not the instantaneous fading coefficients. In a similar way, for the general Rician model we consider in this paper, it will be assumed that the transmitter has knowledge of the value of the Rice factor though not the exact CSI. The receiver is assumed to have perfect CSI. The capacity of a Rician channel with receiver CSI when the transmitter has no knowledge of even the Rice factor has been considered, for example, in [4], [5].

We investigate the capacity of a multiple antenna system in Rician fading subject to the assumption that the transmitter has knowledge of the value of the Rice factor and receiver has the knowledge of perfect CSI. Since the evaluation of the exact capacity of a multiple antenna Rician channel seems to be an intractable problem, we will content ourselves with the task of deriving tight upper and lower bounds for this capacity. An easy lower bound was obtained in [6] by considering the capacity of a system with input signals designed to be optimal for a Rayleigh fading channel or a Rician fading channel when the transmitter has no knowledge of even the Rice factor. By comparing this lower bound with an upper bound to the capacity, also derived in [6], we will see that there is a large capacity gap between the upper bound and the lower bound obtained with signals designed to be optimal for Rayleigh fading. This is not surprising since the transmitter is not exploiting the information provided by the knowledge of the Rice factor when it simply employs the Rayleigh-optimal input signals in a Rician channel. In order to better use the knowledge about the Rician-ness of the fading process provided by the knowledge of the Rice factor we propose a new input signalling scheme for a system having the value of κ at the transmitter though not the exact CSI. By analyzing, in terms of lower and upper bounds, the capacity of a multiple transmit antenna system employing this new input signal choice, we demonstrate that it indeed offers much higher capacity than that achieved by signals designed to be optimal for Rayleigh fading or for Rician fading when the transmitter has no knowledge of even the statistical properties of the fading.

This paper is organized as follows: In Section II we introduce the multiple antenna system model and the assumptions on the fading process. In Section III we recall the upper and lower bounds for the capacity of a multiple antenna system in Rician fading with perfect receiver CSI and only the knowledge of κ at the transmitter derived in [6]. In Section IV we propose a new signalling scheme for a multiple antenna system with perfect-CSI at the receiver and only the knowledge of the value of the Rice factor at the transmitter. We derive tight upper and lower bounds for the capacity of a multiple transmit antenna system with this new input signal choice. By comparing these capacity bounds with that of the results obtained in Section III, we will show that the exploitation of the knowledge of the Rice factor at the transmitter can provide significant capacity gains. Finally, we finish with some concluding remarks in Section V.

II. Model Description

We consider a single user communication link in which the transmitter and receiver are equipped with N_T and N_R antennas, respectively. The discrete-time received signal in such a system can be written in matrix form as

$$\mathbf{y}(i) \;=\; \mathbf{H}(i)\mathbf{x}(i) + \mathbf{n}(i) \;, \tag{1}$$

where $\mathbf{y}(i)$, $\mathbf{x}(i)$ and $\mathbf{n}(i)$ are the complex N_R-vector of received signals on the N_R receive antennas, the (possibly) complex N_T-vector of transmitted signals on the N_T transmit antennas, and the complex N_R-vector of additive receiver noise, respectively, at symbol time i. The components of $\mathbf{n}(i)$ are independent, zero-mean, circularly symmetric complex Gaussian with independent real and imaginary parts having equal variances; i.e. $\mathbf{n}(i) \sim \mathcal{N}_c\left(\mathbf{0}, \mathbf{I}_{N_R}\right)$, where $\mathbf{I}_{N_R}$ denotes the $N_R \times N_R$ identity matrix. The noise is also assumed to be independent with respect to the time index.

The matrix $\mathbf{H}(i)$ in the model (1) is the $N_R \times N_T$ matrix of complex fading coefficients. The (n_R, n_T)-th element of the matrix $\mathbf{H}(i)$, denoted by $(\mathbf{H}(i))_{n_R,n_T}$, represents the fading coefficient value at time i between the n_R-th receiver antenna and the n_T-th transmitter antenna. The $\mathbf{H}(i)$ corresponding to each channel use is considered to be independent from that of other channel uses, i.e. $\mathbf{H}(i)$ and $\mathbf{H}(j)$ are independent whenever $i \neq j$. As noted in [2], this gives rise to a memoryless channel, and thus the capacity of the channel can be computed as the maximum mutual information between the input and output signal vectors $\mathbf{x}$ and $\mathbf{y}$. In this case, we may also drop the explicit time index, i, in order to simplify notation.

We assume that the elements of $\mathbf{H}$ are Gaussian with independent real and imaginary parts each distributed as $\mathcal{N}(\mu/\sqrt{2}, \sigma^2)$. Moreover, the elements of $\mathbf{H}$ are assumed to be independent of each other. Then the elements $(\mathbf{H})_{n_R,n_T}$ of $\mathbf{H}$ are independent and identically distributed complex Gaussian random variables $(\mathbf{H})_{n_R,n_T} \sim \mathcal{N}_c\left(\frac{\mu}{\sqrt{2}}(1+j), 2\sigma^2\right)$, for $n_R = 1, \ldots, N_R$ and $n_T = 1, \ldots, N_T$. Then, the distribution of the magnitudes of the elements of $\mathbf{H}$ is Rician.

In dealing with the Rician distribution, it is customary to introduce the parameter $\kappa = \frac{|\mu|^2}{2\sigma^2}$ usually referred to as the Rice factor. Also we introduce the normalization $|\mu|^2 + 2\sigma^2 = 1$. Note that the Rician distribution reduces to the Rayleigh distribution when $\kappa = 0$ (which implies that $\mu = 0$).

When the elements of $\mathbf{H}$ are distributed as described above we call that $\mathbf{H}$ is a complex normally distributed matrix, denoted as $\mathbf{H} \sim \mathcal{N}_c\left(\mathbf{M}, \mathbf{I}_{N_T} \otimes \mathbf{\Sigma}\right)$, with the probability density function (pdf) [7], [8]

$$f_{\mathbf{H}}(\mathbf{H}) = \frac{1}{(\pi)^{N_T N_R}|\mathbf{\Sigma}|^{N_T}} e^{-\mathrm{tr}\left[\mathbf{\Sigma}^{-1}(\mathbf{H}-\mathbf{M})(\mathbf{H}-\mathbf{M})^H\right]}, \tag{2}$$

where tr denote the trace of a matrix, $\mathbf{\Sigma}$ is the Hermitian covariance matrix of the columns (assumed to be the same for all columns) of $\mathbf{H}$ and $\mathbf{M} = E\{\mathbf{H}\}$. For the assumed model,

$$\mathbf{\Sigma} \;=\; 2\sigma^2 \mathbf{I}_{N_R}, \tag{3}$$

and

$$\mathbf{M} \;=\; \frac{\mu}{\sqrt{2}}(1+j)\mathbf{\Psi}_{N_R,N_T}, \tag{4}$$

where $\mathbf{\Psi}_{N_R,N_T}$ is defined as the $N_R \times N_T$ matrix of all ones.

Next, let us define

$$n \;=\; \max\{N_R, N_T\} \quad \text{and} \quad m \;=\; \min\{N_R, N_T\},$$

and

$$\mathbf{W} \;=\; \begin{cases} \mathbf{H}\mathbf{H}^H & \text{if } N_R < N_T \\ \mathbf{H}^H\mathbf{H} & \text{if } N_R \geq N_T \end{cases} . \tag{5}$$

Then, $\mathbf{W}$ is always an $m \times m$ matrix. It is known that when $\mathbf{H}$ is distributed as in (2), the distribution of $\mathbf{W}$ is given by the non-central Wishart distribution [7]–[9] with pdf

$$f_{\mathbf{W}}(\mathbf{W}) = e^{-\mathrm{tr}\left[\mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{M}^H\right]} {}_0\tilde{F}_1(n; \mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{M}^H\mathbf{\Sigma}^{-1}\mathbf{W}) f^0_{\mathbf{W}}(\mathbf{W}), \tag{6}$$

where in (6) $f^0_{\mathbf{W}}(\mathbf{W})$ denotes the (central) Wishart pdf:

$$f^0_{\mathbf{W}}(\mathbf{W}) \;=\; \frac{1}{\tilde{\Gamma}_m(n)|\mathbf{\Sigma}|^n} e^{-\mathrm{tr}\mathbf{\Sigma}^{-1}\mathbf{W}} |\mathbf{W}|^{n-m} \tag{7}$$

which results when the elements of $\mathbf{H}$ are independent and identical distributed (iid) zero mean Gaussian random variables, and the Bessel function of matrix argument ${}_0\tilde{F}_1$ in (6) is defined as [8], [10]

$${}_0\tilde{F}_1(n; HH^H) \;=\; \int_{\mathcal{U}(n)} e^{\mathrm{tr}(HU_1 + \overline{HU_1})}(dU), \tag{8}$$

where H is an $m \times n$ complex matrix with $m \leq n$, $U = [U_1, U_2]$ with U_1 being an $n \times m$ complex matrix and $U_1^H U_1 = I_m$. Note that, in (8) $\overline{H}$ and (dU) denote the complex conjugate of the matrix H and the normalized invariant measure on the unitary group $\mathcal{U}(n)$, respectively. It can be shown that for a scalar r, (8) reduces to

$${}_0\tilde{F}_1(n; r^2) \;=\; \Gamma(n) r^{-(n-1)} I_{n-1}(2r) \tag{9}$$

where $I_\nu(z)$ is the ν-th order modified Bessel function of the first kind (ν an integer) [11].

The complex multivariate gamma function, $\tilde{\Gamma}_m(a)$, in (6) is defined as [8]

$$\tilde{\Gamma}_m(a) \;=\; \pi^{\frac{1}{2}m(m-1)} \prod_{k=1}^{m} \Gamma\left(a - (k-1)\right). \tag{10}$$

Note that, it follows from (10) that $\tilde{\Gamma}_1(a) = \Gamma(a)$, where $\Gamma(a)$ is the usual (scalar) gamma function.

Note also that in (6) we have assumed, without loss of generality, that $\mathbf{W} = \mathbf{H}\mathbf{H}^H$. We will continue to use this case throughout unless stated otherwise. We use the short hand notations $\mathbf{W} \sim \mathcal{W}_m\left(n, \mathbf{\Sigma}\right)$ and $\mathbf{W} \sim \mathcal{W}_m\left(n, \mathbf{\Sigma}, \mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{M}^H\right)$ to denote that $\mathbf{W}$ is Wishart distributed with pdf (7) and that $\mathbf{W}$ is non-central Wishart distributed with pdf (6), respectively.

III. Capacity of the Multiple Antenna Rician Fading Channel

As argued, for example in [6], the capacity achieving transmit signal distribution $\mathbf{x}$ for a multiple antenna system subjected to the average total transmit power constraint $E\{\mathbf{x}^H\mathbf{x}\} \leq P$ is circularly symmetric, zero-mean complex Gaussian with $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{Q}$, where $\mathrm{tr}\{\mathbf{Q}\} \leq P$, independent of the actual fading distribution, as long as the receiver knows the exact channel fading coefficients, but not the transmitter. The capacity of the multiple antenna system is then given by [1]

$$C_{N_T,N_R} = \max_{\substack{\mathrm{tr}\ \mathbf{Q} \leq P \\ \mathbf{Q} \geq 0}} E_{\mathbf{H}}\{\log\det(\mathbf{H}\mathbf{Q}\mathbf{H}^H + \mathbf{I}_{N_R})\}. \quad (11)$$

Thus, in order to evaluate the capacity of the Rician channel (1) with perfect receiver CSI and only the value of κ at the transmitter all we need to know is the optimal choice of the covariance matrix $\mathbf{Q}$ subject to the trace requirement imposed by the average power constraint.

As $\kappa \longrightarrow \infty$, the fading becomes deterministic and the matrix $\mathbf{H}$ has all its elements equal to unity. In this case the so called water-filling algorithm [12] specifies the optimal covariance matrix structure of the capacity achieving Gaussian distribution to be of the form [2] $\mathbf{Q}^\infty = \frac{P}{N_T}\mathbf{\Psi}_{N_T}$ where $\mathbf{\Psi}_{N_T}$ is an $N_T \times N_T$ matrix of all ones.

On the other hand, when $\kappa = 0$ (Rayleigh fading) the capacity achieving distribution has a scaled identity covariance matrix of the form [1], [2] $\mathbf{Q}^0 = \frac{P}{N_T}\mathbf{I}_{N_T}$.

Thus, as argued in [6], for a channel with Rician distributed fading having a general value of κ, which is known to the transmitter, one would expect the covariance matrix $\mathbf{Q}$ of the capacity achieving distribution to lie in between these two extremes.

Applying Jensen's inequality to (11), the following upper bound for the capacity of a MIMO Rician channel with only receiver CSI and transmitter known Rice factor was derived in [6]:

$$C_{N_T,N_R} \leq \begin{cases} \log \ \ 1 + (N_T-1)\kappa + \frac{N_R}{N_T}\frac{1+N_T\kappa}{1+\kappa}P \ \ + & \\ (N_T-1)\log \ \ 1 - \frac{\kappa}{1+N_T\kappa} + \frac{N_R}{N_T}\frac{1}{1+\kappa}P & \text{for } 0 \leq \kappa < \infty \\ \log(1 + N_T N_R P) & \text{for } \kappa \longrightarrow \infty \end{cases}$$

In [6], a lower bound for this capacity was also obtained by considering a the capacity of a system designed to be optimal for Rayleigh fading (i.e. the capacity of a scheme that completely ignores the available knowledge of κ in designing input signals).

In the special case of $m = 1$, this lower bound was shown to be

$$C_{N_T,N_R} \geq \int_0^\infty \log\left(1 + \frac{P}{N_T}W\right) f_{\mathbf{W}}(W)dW, \quad (12)$$

where $f_{\mathbf{W}}(W)$ is given by

$$f_{\mathbf{W}}(W) = \left(\frac{e^{-\kappa}}{2\sigma^2}\right)^n e^{-\frac{W}{2\sigma^2}} |W|^{\frac{n-1}{2}} \omega^{-(n-1)} I_{n-1}\left(2\omega\sqrt{W}\right), \quad (13)$$

with ω defined as

$$\omega^2 = n\frac{\mu^2}{(2\sigma^2)^2}, = n\kappa(1+\kappa). \quad (14)$$

In Fig. 1 we have shown these bounds for a multiple antenna system with $N_R = 1$, against the number of transmit antennas for $\kappa = 1$ and $\kappa = 10$. Included on the same graphs are the corresponding capacity curves for the Rayleigh fading channel ($\kappa = 0$ case). We observe that, with the scaled identity covariance matrix choice, the Rician channel has a slightly larger capacity than that of the Rayleigh channel and the capacity gap is more prominent for larger values of κ. However, from Fig. 1 it is observed that the two capacities converge to the same value as $N_T \longrightarrow \infty$.

More importantly, we observe from Fig. 1 that the upper and lower bounds used above have a large capacity gap. Since the lower bound above assumes a signal distribution which is optimal for the Rayleigh fading channel or for a system in which transmitter does not know the value of κ, Fig. 1 strongly suggests that the scaled identity matrix might not be the form of the covariance matrix of the capacity achieving input signal distribution for a multiple antenna Rician channel when transmitter knows κ. With better signal choices that exploit the *Rician-ness* inherent in the fading distribution, we may be able to obtain higher capacities.

IV. A New Signalling Scheme for Multiple Transmit Antenna Systems in Rician Fading

We propose the following choice for the covariance matrix of the zero-mean complex Gaussian distributed input signal $\mathbf{x}$.

$$\mathbf{Q}^\kappa = \frac{P}{N_T(1+\kappa)}\left(\mathbf{I}_{N_T} + \kappa\mathbf{\Psi}_{N_T}\right) \quad (15)$$

where, as before, $\mathbf{\Psi}_{N_T}$ is an $N_T \times N_T$ matrix of all ones. Note that, when $\kappa = 0$ the second term in (15) is zero and thus it becomes the optimal covariance for the Rayleigh fading channel. On the other hand as $\kappa \longrightarrow \infty$ it is easily seen that $\mathbf{Q}^\kappa \longrightarrow \mathbf{Q}^\infty$ which is the optimal covariance for the AWGN multiple transmit antenna channel. Thus, $\mathbf{Q}^\kappa$ reduces to the optimal covariance matrix at the two extreme ends of the Rician fading.

With this choice for the covariance matrix $\mathbf{Q}$, the capacity of the multiple antennae system becomes

$$C^\kappa_{N_T,N_R} = E_{\mathbf{H}} \ \ \log\det \ \ \frac{P}{N_T(1+\kappa)}\mathbf{H} \ \ \mathbf{I}_{N_T} + \kappa\mathbf{\Psi}_{N_T} \ \ \mathbf{H}^H + \mathbf{I}_{N_R} \quad .$$

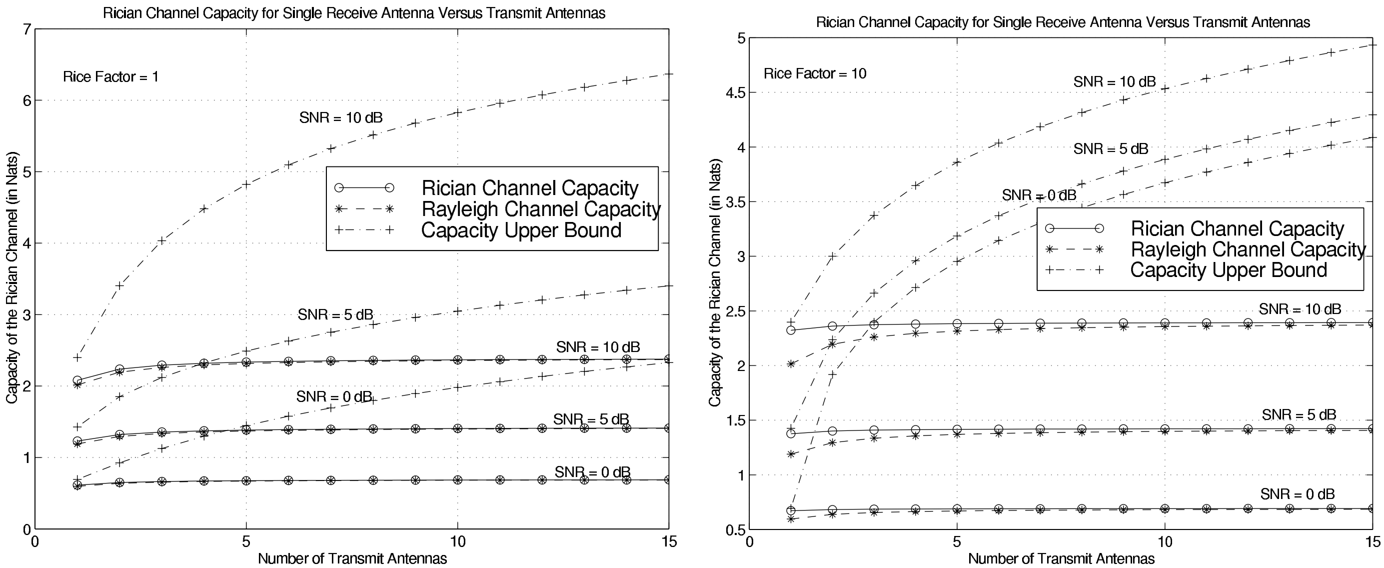
Note that we may write $\mathbf{\Psi}_{N_T} = \mathbf{e}\mathbf{e}^T$ where $\mathbf{e}$ denotes the N_T-vector of all ones. Since the matrix $\mathbf{H}$ reduces to an N_T length row vector when $N_R = 1$, the capacity of the multiple antenna system in this case can be written as

$$C^\kappa_{N_T,1} = E_{Z,W}\left\{\log\left(\frac{P}{N_T(1+\kappa)}\left(W + \kappa|Z|^2\right) + 1\right)\right\}, \quad (16)$$

where as usual $W = \mathbf{H}\mathbf{H}^H = \sum_{n_T=1}^{N_T} |\,(\mathbf{H})_{1,n_T}\,|^2$ and we have defined $Z = \mathbf{e}^T\mathbf{H}^H = \sum_{n_T=1}^{N_T} (\mathbf{H})_{1,n_T}$.

Since each $(\mathbf{H})_{1,n_R}$ is an independent complex Gaussian random variable of the form $\mathcal{N}_c\left(\frac{\mu}{\sqrt{2}}(1+j), 2\sigma^2\right)$, it is seen that $Z \sim \mathcal{N}_c\left(N_T\frac{\mu}{\sqrt{2}}(1+j), 2N_T\sigma^2\right)$. Hence with our earlier notation it can easily be shown that $|Z|^2 \sim \mathcal{W}_1\left(1, \frac{N_T}{1+\kappa}, N_T\kappa\right)$ and $W \sim \mathcal{W}_1\left(N_T, \frac{1}{1+\kappa}, N_T\kappa\right)$.

It is clear that these two random variables Z and W are not independent and so we do not know their joint distribution, which is required for evaluating (16), although we have their marginal



(a) (b)

Fig. 1. Rician Channel Capacity for Single Receiver Antenna Versus Transmit Antennas. (a) $\kappa = 1$. (b) $\kappa = 10$.

distributions. Thus we resort to capacity bounds and derive both upper and lower bounds to the capacity of the multiple antenna system with the proposed covariance matrix choice. In particular, we obtain a tight lower bound on the capacity which shows that the proposed choice of the covariance matrix is superior to the scaled identity covariance matrix for any non-zero κ (and, as remarked earlier, is the same as that capacity when $\kappa = 0$).

A. Upper Bound for $C^{\kappa}_{N_T,1}$

Applying Jensen's inequality to (16), and noting that $E\left\{|Z|^2\right\} = \frac{N_T}{1+\kappa}\left(1 + N_T\kappa\right)$ and $E\{W\} = N_T$, we have the following upper bound for the capacity of a multiple transmit and single receiver antenna system in Rician fading with the proposed covariance matrix:

$$C^{\kappa}_{N_T,1} \leq \log\left(1 + \frac{P}{1+\kappa}\left(1 + \frac{\kappa}{1+\kappa} + \frac{N_T\kappa^2}{1+\kappa}\right)\right). \quad (17)$$

It can be shown that in the special cases of $\kappa = 0$ and $\kappa \longrightarrow \infty$, the upper bound (17) reduces respectively to,

$$C^{\kappa=0}_{N_T,1} \leq \log\left(1 + P\right), \quad (18)$$

and

$$C^{\kappa=\infty}_{N_T,1} \leq \log\left(1 + N_T P\right). \quad (19)$$

From the results in [2], the right hand side of (18) is in fact the exact capacity of the system in this case (i.e. Rayleigh fading) as $N_T \longrightarrow \infty$. Hence, in the case of $\kappa = 0$ the upper bound (17) is achieved as $N_T \longrightarrow \infty$. Also, from [2], [6], the right hand side of (19) is the exact capacity of the system in this case of $\kappa \longrightarrow \infty$ for any value of N_T. Hence, when $\kappa \longrightarrow \infty$ the upper bound (17) is achieved for any value of N_T.

B. Lower Bound for $C^{\kappa}_{N_T,1}$

Since both W and Z are non-negative random variables we may lower bound the capacity in (16) as

$$C^{\kappa}_{N_T,1} \geq E_Z\left\{\log\left(\frac{P\kappa}{N_T(1+\kappa)}|Z|^2 + 1\right)\right\}. \quad (20)$$

Substituting the pdf of $|Z|^2$,

$$f_{|Z|^2}(z^2) = \frac{(1+\kappa)\exp(-N_T\kappa)}{N_T}\exp\left(-\frac{1+\kappa}{N_T}z^2\right){}_0\tilde{F}_1(1;\kappa(1+\kappa)z^2)$$

into the right hand side of (20), and using (9) we obtain the following lower bound for the capacity of a multiple transmit and single receiver antenna system in Rician fading with the proposed covariance matrix:

$$C^{\kappa}_{N_T,1} \geq \frac{1+\kappa}{e^{N_T\kappa}}\int_0^{\infty}\log\left(\frac{P\kappa}{1+\kappa}z + 1\right)\frac{I_0\left(2\omega\sqrt{z}\right)}{e^{(1+\kappa)z}}dz, \quad (21)$$

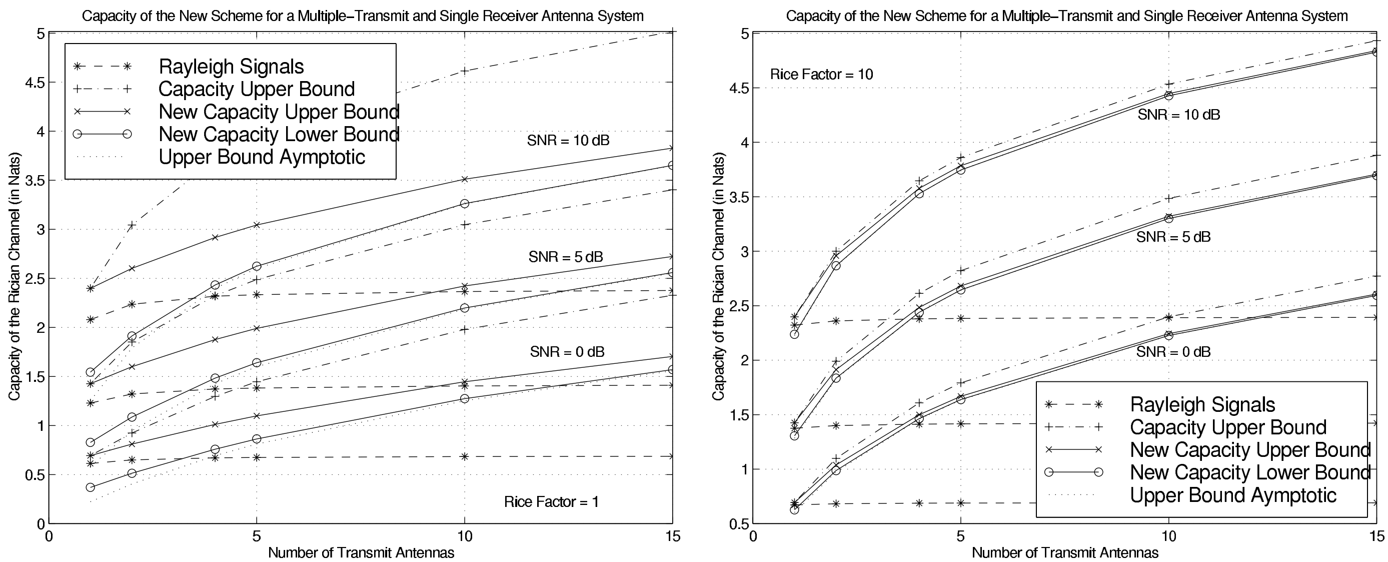
where ω is as defined in (14) (with n replaced by N_T).

Figure 2 shows the derived bounds for the capacity of a multiple-transmit antenna system along with the capacity corresponding to the scaled identity covariance matrix. From the lower bounds shown on these figures it is clear that the proposed signalling scheme achieves much higher capacity than that of the scaled identity covariance matrix for sufficiently large values of κ or N_T. It is also observed that the difference between the upper and lower bounds decreases as N_T increases. From Fig. 2b we note that when κ is large the upper and lower bounds are almost the same unless the number of transmit antennas is very small. Thus, for large κ, a reasonable approximation to the capacity of the proposed scheme can be obtained by taking the large κ asymptotic of the upper bound (17), which is seen to be

$$C^{\kappa}_{N_T,1} \approx \log\left(1 + \frac{PN_T\kappa^2}{(1+\kappa)^2}\right) \quad \text{for} \quad \kappa \gg 1. \quad (22)$$

Figure 2 also shows this large κ approximation to the upper bound of the capacity. We observe that indeed (22) is a very good approximation to the capacity even for relatively small values of κ. Note also that (22) gives the exact capacity in the cases of $\kappa = 0$ and $\kappa \longrightarrow \infty$.

Finally, it is worth noting that in these figures we have also included the capacity upper bound for a Rician channel with receiver CSI derived in Section III. From Fig. 2a we observe



(a) (b)

Fig. 2. Capacity of a Multiple-Transmit Antenna System in Rician Fading with Proposed New Signalling Scheme. $N_R = 1$. (a) $\kappa = 1$. (b) $\kappa = 10$.

that for relatively small values of κ there is still a significant gap between the general upper bound for the Rician channel in this case and the upper bound on the capacity of the proposed new design given by (17). However, as κ increases we observe from Fig. 2b that this difference also becomes smaller, although the proposed scheme still does not achieve the upper bound given in Section III.

V. Conclusions

In this paper we have investigated the capacity of multiple antenna systems in Rician fading when the receiver has access to the CSI, but not the transmitter. We have derived both upper and lower bounds on the capacity of such systems in the case in which the transmitter has knowledge of the Rice factor κ though not the exact value of CSI. In some special cases we have numerically evaluated these capacity bounds and demonstrated that the choice of a scaled identity covariance matrix does not exploit the advantage offered by the Rician-ness.

In order to better exploit the Rician-ness inherent in the fading, a new signalling scheme has been proposed for the case when the transmitter has knowledge of the value of κ, though not the exact CSI. We have analyzed the capacity of this new scheme, in terms of lower and upper bounds, for a multiple transmit antenna system and demonstrated that it offers much higher capacity than that of a scaled identity matrix scheme. We have also derived a simple approximation to the capacity of this scheme for sufficiently large values of κ.

Our results suggest that Rician fading could in fact improve the capacity of a multiple antenna system, if the transmitter knows the value of κ. This indicates that it is worth investigating the design of signals for multiple antenna systems taking into account the *Rician-ness* inherent in the channel fading process, when that knowledge is available at the transmitter.

Acknowledgment

This research was supported in part by the National Science Foundation under Grant $CCR-99-79441$, and in part by the New Jersey Center for Wireless Telecommunications.

References

[1] G. J. Foschini and M. J. Gans, “On limits of wireless communications in a fading environment when using multiple antennas,” *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[2] I. E. Telatar, “Capacity of multi-antenna Gaussian channels,” *Eur. Trans. Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.

[3] S. K. Jayaweera and H. V. Poor, “Capacity of multi-antenna systems with adaptive transmission techniques,” in *Proc. 5^{th} Intl. Symp. On Wireless Personal Multimedia Commun*, Honolulu, HI, October 2002, pp. 392–396.

[4] F. R. Farokhi, G. J. Foschini, A. Lozano, and R. A. Valenzuela, “Link-optimal space-time processing with multiple transmit and receiver antennas,” *IEEE Commun. Lett.*, vol. 5, pp. 85–87, Mar. 2001.

[5] P. F. Driessen and G. J. Foschini, “On the capacity formula for multiple-input multiple-output wireless channels: A geometric interpretation,” *IEEE Trans. Commun.*, vol. 2, pp. 173–176, Feb. 1999.

[6] S. K. Jayaweera and H. V. Poor, “On the capacity of multi-antenna systems in the presence of Rician fading,” in *Proc. IEEE Veh. Technol. Conf.*, Vancouver, BC, Canada, September 2002, pp. 1963–1967.

[7] A. G. Constantine, “Some non-central distribution problems in multivariate analysis,” *Ann. Math. Stat.*, vol. 34, pp. 1270–1285, Dec. 1963.

[8] A. T. James, “Distributions of matric variates and latent roots derived from normal samples,” *Ann. Math. Stat.*, vol. 35, pp. 475–501, June 1964.

[9] ——, “The non-central Wishart distribution,” in *Proc. Royal Soc. London. Series A., Math. and Phys. Sci.*, vol. 229, May 1955, pp. 364–366.

[10] C. S. Herz, “Bessel functions of matrix argument,” *Annals of Math.*, vol. 61, no. 3, pp. 474–423, May 1955.

[11] N. W. McLachlan, *Bessel Functions for Engineers*. Oxford, UK: Oxford Univ. Press., 1955.

[12] R. G. Gallager, *Information Theory and Reliable Communication*. New York: Wiley, 1968.