

Performance Analysis of Decorrelating Decision Feedback Detection for MIMO Systems in Rician Fading

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Abstract—The diversity gain of the joint error probability (JEP) of the decorrelating decision feedback detector (D-DFD) for multiple-input multiple-output (MIMO) systems in the presence of Rician fading is considered. Upper bounds on the JEP are derived for both M-ary Phase-Shift-Keying (PSK) and M-ary Quadrature Amplitude Modulation (QAM). It is shown that the diversity gain of the D-DFD receiver in both cases is equal to $N_R - N_T + 1$ where N_R and N_T are the number of transmit and receive antennas, respectively. As an aside, the Bartlett decomposition of a complex non-central Wishart matrix is also derived.

Index Terms—Bartlett decomposition, diversity, joint error probability, MIMO systems, non-central Wishart distribution, Rician fading, Wishart distribution.

I. INTRODUCTION

Recent results revealing the substantial capacity improvement that might be possible in fading channels if multiple antennas were employed at both transmitter and the receiver [1], [2] opens up a new solution to the problem of meeting the growing demand for high data rate cellular systems while maintaining the reliability. In [1], the Bell Labs Layered Space-time (BLAST) architecture was proposed as a way to exploit this enormous diversity gain offered by dual antenna array systems. The BLAST receiver is based on the decorrelating decision feedback detection, previously proposed in [3] and analyzed in [4] in the context of multi-user detection. However, the analysis in [4] is not sufficient in the case of MIMO channels since it does not take into account channel fading effects. Recently, [5] gave an analysis of the decorrelating decision feedback detection for MIMO systems in Rayleigh fading. It was shown in [5] that the diversity gain of the joint probability of error of the decorrelating decision feedback detector for QAM is equal to $N_R - N_T + 1$, where N_R and N_T are the number of receiver and transmitter antennas, respectively (assuming $N_R \geq N_T$) for a Rayleigh fading channel.

In this paper we analyze the performance of D-DFD receiver for multiple antenna systems in the presence of Rician fading for different modulation schemes. Upper bounds on the joint probability of error are derived for both M-ary PSK

and M-ary QAM. Based on these bounds, it is shown that the diversity gain of the JEP of the decorrelating decision feedback detector in Rician fading is equal to $N_R - N_T + 1$ for both these cases, similarly to that in Rayleigh channel.

The rest of this paper is organized as follows: First, we outline our system model and the assumptions. Next, we derive the Bartlett decomposition of a complex non-central Wishart matrix which will be essential to our probability of error analysis. We then analyze the joint probability of error of D-DFD receiver for MIMO systems in the presence of Rician fading. We derive upper bounds on the JEP for PSK and QAM signal constellations and show that the diversity gain in both modulation schemes is equal to $N_R - N_T + 1$. Finally, we finish by giving some concluding remarks.

Notation

The superscripts T and H denote the transpose and complex conjugate transpose, respectively. The notations $\text{tr}(\mathbf{A})$ and $|\mathbf{A}|$ denote the trace and determinant of the matrix \mathbf{A} respectively and $\text{etr}(\mathbf{A})$ stands for $\exp(\text{tr}(\mathbf{A}))$. Expectation with respect to the distribution of the random variable γ is denoted by $\mathcal{E}_\gamma\{\cdot\}$. Expectation with respect to the joint distribution of the entries of the random matrix \mathbf{A} is denoted by $\mathcal{E}_\mathbf{A}\{\cdot\}$.

II. SYSTEM MODEL DESCRIPTION

Consider a single user, uncoded, narrowband communications link in which the transmitter and receiver are equipped with N_T and N_R antennas, respectively. Throughout, we will assume that $N_R \geq N_T$. The discrete-time received signal in such a system can be written in matrix form as

$$\mathbf{y}(i) = \mathbf{H}(i)\mathbf{P}^{1/2}\mathbf{b}(i) + \mathbf{n}(i), \quad (1)$$

where $\mathbf{y}(i)$, $\mathbf{b}(i)$ and $\mathbf{n}(i)$ are the complex N_R -vector of received signals on the N_R receive antennas, the N_T -vector of transmitted signals on the N_T transmit antennas, and the complex N_R -vector of additive receiver noise, respectively, at symbol time i . The components of $\mathbf{n}(i)$ are independent,

zero-mean, circularly symmetric complex Gaussian with independent real and imaginary parts having equal variances. The noise is also assumed to be independent with respect to the time index, and $\mathcal{E}\{\mathbf{n}(i)\mathbf{n}(i)^H\} = N_0\mathbf{I}_{N_R}$, where \mathbf{I}_{N_R} denotes the $N_R \times N_R$ identity matrix.

The (n, n) -th element in the $N_T \times N_T$ diagonal matrix $\mathbf{P}^{1/2}$ is equal to $\sqrt{P_n}$, where P_n is the average received power from transmitter n .

The matrix $\mathbf{H}(i)$ in the model (1) is the $N_R \times N_T$ matrix of complex fading coefficients. The (m, n) -th element of the matrix $\mathbf{H}(i)$ represents the fading coefficient value at time i between the m -th receiver antenna and the n -th transmitter antenna. The $\mathbf{H}(i)$ corresponding to each channel use is considered to be independent from that of other channel uses, i.e. $\mathbf{H}(i)$ and $\mathbf{H}(j)$ are independent whenever $i \neq j$. In this case, we may drop the explicit time index, i , in order to simplify notation.

A. Fading Model

We assume that the elements of \mathbf{H} are independent and identically distributed complex Gaussian random variables satisfying the normalization $\mathcal{E}\{|\mathbf{H}_{m,n}|^2\} = 1$ for $m = 1, \dots, N_R$ and $n = 1, \dots, N_T$. We further assume that real and imaginary parts of each element $(\mathbf{H})_{m,n}$, denoted respectively as $\text{Re}(\mathbf{H})_{m,n}$ and $\text{Im}(\mathbf{H})_{m,n}$, are independent and identically distributed. We assume a model in which $(\mathbf{H})_{m,1} \sim \mathcal{N}_c\left(\frac{\mu}{\sqrt{2}}(1+j), 2\sigma^2\right)$ for $m = 1, \dots, N_R$, and $(\mathbf{H})_{m,n} \sim \mathcal{N}(0, 2\sigma^2)$ for $m = 1, \dots, N_R$ and $n = 2, \dots, N_T$. The normalization criterion $\mathcal{E}\{|\mathbf{H}_{m,1}|^2\} = 1$ for $m = 1, \dots, N_R$ then requires that

$$\mu^2 + 2\sigma^2 = 1. \quad (2)$$

In practice, a model of this type may arise when there are direct Line-of-Sight (LOS) paths to the receiver antennas from one of the transmit antennas but not from the others. This specific model was introduced so that analytical expressions could be obtained for the probability or error bounds. We, also expect that performance bounds obtained for this particular fading model might serve as a guide to the more general situations.

Note that, with the above assumptions, the amplitudes $|\mathbf{H}_{m,n}|$, for $m = 1, \dots, N_R$ are Rician distributed when $n = 1$, and Rayleigh distributed when $n = 2, \dots, N_T$. In dealing with Rician distributed random variables it is convenient to introduce the so called *Rice factor*, κ , defined in this case as

$$\kappa = \frac{\mu^2}{2\sigma^2}. \quad (3)$$

Combining (2) and (3) we then have that $\mu^2 = \frac{\kappa}{1+\kappa}$ and $\sigma^2 = \frac{1}{2(1+\kappa)}$.

Define the $N_R \times N_T$ matrix \mathbf{M} as

$$\mathbf{M} = \mathcal{E}\{\mathbf{H}\} = \begin{bmatrix} | & | & \dots & | \\ \mathbf{m} & \mathbf{0} & \dots & \mathbf{0} \\ | & | & \dots & | \end{bmatrix}, \quad (4)$$

where \mathbf{m} is the N_R -vector given by

$$\mathbf{m} = \sqrt{\frac{\kappa}{2(1+\kappa)}}(1+j)\mathbf{e} \quad (5)$$

and \mathbf{e} is the N_R -vector of all ones.

Next, denote by \mathbf{h}_m^T the m -th row of \mathbf{H} . If we denote by $\mathbf{\Sigma}$ the $N_T \times N_T$ diagonal covariance matrix of the vector \mathbf{h}_m (same for all $m = 1, \dots, N_R$), it can be shown that

$$\mathbf{\Sigma} = \begin{bmatrix} \frac{1}{1+\kappa} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}. \quad (6)$$

With these definitions, \mathbf{H} is said to have a complex normal distribution $\mathbf{H} \sim \mathcal{N}_c(\mathbf{M}, \mathbf{I}_{N_R} \otimes \mathbf{\Sigma})$ with the probability density function (pdf)

$$f_{\mathbf{H}}(\mathbf{H}) = \frac{1}{\pi^{N_R N_T} (1+\kappa)^{-N_R}} \text{etr}\left(-\mathbf{\Sigma}^{-1}(\mathbf{H} - \mathbf{M})^H(\mathbf{H} - \mathbf{M})\right) (d\mathbf{H}),$$

where we have also included the differential volume element $(d\mathbf{H})$ for the sake of clarity.

As we will soon see the performance of the multiple antenna system will be determined by the properties of the $N_T \times N_T$ matrix \mathbf{W} defined as

$$\mathbf{W} = \mathbf{H}^H \mathbf{H} \quad (7)$$

When \mathbf{H} is complex Gaussian distributed as above, and $N_R \geq N_T$, it is known that the matrix \mathbf{W} is non-central Wishart distributed with the pdf

$$f_{\mathbf{W}}(\mathbf{W}) = \frac{1}{\tilde{\Gamma}_{N_T}(N_R)(1+\kappa)^{-N_R}} \text{etr}\left(-\mathbf{\Sigma}^{-1}\mathbf{W}\right) |\mathbf{W}|^{N_R - N_T} \times \text{etr}\left(-\mathbf{\Omega}\right) {}_0\tilde{F}_1(N_R; \mathbf{\Omega}; \mathbf{\Sigma}^{-1}\mathbf{W})(d\mathbf{W}), \quad (8)$$

where the complex multivariate gamma function $\tilde{\Gamma}_m(a)$ can be written as [6]

$$\tilde{\Gamma}_m(a) = \pi^{\frac{1}{2}m(m-1)} \prod_{k=1}^m \Gamma(a - (k-1)), \quad (9)$$

and we have let $\mathbf{\Omega} = \mathbf{\Sigma}^{-1}\mathbf{M}^H\mathbf{M}$. Note that the Bessel function ${}_0\tilde{F}_1(m; \mathbf{A})$ of an $n \times n$ Hermitian matrix \mathbf{A} in (8) is defined as [7], [6]

$${}_0\tilde{F}_1(m; HH^H) = \int_{\mathcal{U}(m)} e^{\text{tr}(HU_1 + \overline{HU_1})} (dU), \quad (10)$$

where H is an $n \times m$ complex matrix with $n \leq m$, $U = [U_1, U_2]$ with U_1 being an $n \times m$ complex matrix and $U_1^H U_1 = I_n$. In (10) \overline{H} denotes the complex conjugate of the matrix H and (dU) is the normalized invariant measure on the unitary group $\mathcal{U}(n)$. It can be shown that for a scalar a , (10) reduces to

$${}_0\tilde{F}_1(m; a^2) = \Gamma(m) a^{-(m-1)} I_{m-1}(2a) \quad (11)$$

where $I_\nu(a)$ is the ν -th order modified Bessel function of the first kind.

We will use the notation $\mathbf{W} \sim \mathcal{W}_c(N_T, N_R, \mathbf{\Sigma}, \mathbf{\Omega})$ to denote that matrix \mathbf{W} has the complex non-central Wishart distribution given by (8).

III. BARTLETT DECOMPOSITION OF A COMPLEX NON-CENTRAL WISHART MATRIX

Since \mathbf{W} is an $N_T \times N_T$ Hermitian matrix it can be represented as $\mathbf{W} = \mathbf{T}^H \mathbf{T}$, where \mathbf{T} is an $N_T \times N_T$ complex, upper-triangular matrix with positive diagonal elements. In the case where \mathbf{W} is real symmetric Wishart, the distribution of the elements of \mathbf{T} are well known and is referred to as the Bartlett decomposition [8], [9], [10]. Bartlett decomposition of a real non-central Wishart matrix can also be found in, for example, [11], [12]. Although the derivation of the Bartlett decomposition for a complex non-central Wishart matrix is not that difficult, the result at least is not readily available in the published literature and thus here we derive it explicitly for the above model.

Applying the QR decomposition we may write

$$\mathbf{H} = \mathbf{U}\mathbf{T}, \quad (12)$$

where \mathbf{U} is an $N_R \times N_T$ matrix with orthonormal columns and \mathbf{T} is an $N_T \times N_T$ complex, upper-triangular matrix. Since the elements of \mathbf{H} are independent it can be shown that \mathbf{H} has rank N_T with probability one and thus the diagonal elements of \mathbf{T} are positive with probability one. Combining (7) and (12) we have that $\mathbf{W} = \mathbf{T}^H \mathbf{T}$, and thus

$$|\mathbf{W}| = |\mathbf{T}|^2 = \prod_{i=1}^{N_T} t_{i,i}^2, \quad (13)$$

and

$$\text{tr}(\boldsymbol{\Sigma}^{-1} \mathbf{W}) = (1 + \kappa) t_{1,1}^2 + \sum_{i=2}^{N_T} t_{i,i}^2 + \sum_{i=2}^{N_T} \sum_{j=i+1}^{N_T} |t_{i,j}|^2, \quad (14)$$

where $t_{i,j}$ denotes the (i, j) -th element of \mathbf{T} . Furthermore, one can show that the only non-zero eigenvalue of $\boldsymbol{\Omega} \boldsymbol{\Sigma}^{-1} \mathbf{W}$ is $s_1 = \omega^2 t_{1,1}^2$ where we have defined $\omega = \sqrt{N_R \kappa (1 + \kappa)}$. It then follows (using (11)) that in this case

$${}_0\tilde{F}_1(N_R; \boldsymbol{\Omega} \boldsymbol{\Sigma}^{-1} \mathbf{W}) = \frac{\Gamma(N_R)}{(\omega t_{1,1})^{N_R-1}} I_{N_R-1}(2\omega t_{1,1}), \quad (15)$$

Substituting (13), (14) and (15) into (8), and using the Jacobian of the transformation given by $(d\mathbf{W}) = 2^{N_T} \prod_{i=1}^{N_T} t_{i,i}^{2N_T-2i+1} (d\mathbf{T})$ one can obtain the following pdf for \mathbf{T} :

$$f_{\mathbf{T}}(\mathbf{T}) = \left(\frac{e^{-N_R \kappa}}{(1 + \kappa)^{-N_R}} \left(\frac{t_{1,1}}{\omega} \right)^{N_R-1} e^{-(1+\kappa)t_{1,1}^2} I_{N_R-1}(2\omega t_{1,1}) dt_{1,1}^2 \right) \left(\prod_{i=2}^{N_T} \frac{e^{-t_{i,i}^2} (t_{i,i})^{(N_R-i)}}{\Gamma(N_R-i+1)} dt_{i,i}^2 \right) \left(\prod_{i=1}^{N_T} \prod_{j=i+1}^{N_T} \frac{e^{-|t_{i,j}|^2}}{\pi} dt_{i,j} \right) \quad (16)$$

Note that, in obtaining (16) we have made use of the observation that from (9) we may write $\tilde{\Gamma}_{N_T}(N_R)$ as $\tilde{\Gamma}_{N_T}(N_R) = \prod_{i=1}^{N_T} \left(\Gamma(N_R-i+1) \prod_{j=i+1}^{N_T} \pi \right)$ and that for an upper-triangular matrix \mathbf{T} the differential volume element is given by $(d\mathbf{T}) = \prod_{i=1}^{N_T} \prod_{j=i}^{N_T} dt_{i,j}$.

Theorem 1: (Bartlett Decomposition of a Complex Non-central Wishart Matrix):

Let the $N_R \times N_T$ ($N_R \geq N_T$) matrix $\mathbf{H} \sim \mathcal{N}_c(\mathbf{M}, \mathbf{I}_{N_R} \otimes \boldsymbol{\Sigma})$, where \mathbf{M} and $\boldsymbol{\Sigma}$ are as defined in (4) and (6), respectively, so that $\mathbf{W} = \mathbf{H}^H \mathbf{H}$ has the non-central Wishart distribution $\mathcal{W}_c(N_T, N_R, \boldsymbol{\Sigma}, \boldsymbol{\Omega})$. Let $\mathbf{W} = \mathbf{T}^H \mathbf{T}$ where \mathbf{T} is a complex, upper-triangular matrix with positive diagonal elements. Then, the elements $t_{i,j}$ are all independent for $i = 1, \dots, N_T$, $j = i, \dots, N_T$, $t_{i,j} \sim \mathcal{N}_c(0, 1)$ for $1 \leq i < j \leq N_T$, $2t_{i,i}^2 \sim \mathcal{X}_{2(N_R-i+1)}^2$ for $i = 2, \dots, N_T$ and $2(1 + \kappa)t_{1,1}^2 \sim \mathcal{X}_{2N_R}^2(\delta)$ with $\delta = 2N_R \kappa$. Here $X \sim \mathcal{X}_n^2$ and $Y \sim \mathcal{X}_n^2(\delta)$ mean that X has the \mathcal{X}^2 distribution with n degrees of freedom and that Y has the non-central \mathcal{X}^2 distribution with n degrees of freedom and non-centrality parameter δ .

Proof: Recall that if $X \sim \mathcal{X}_n^2$, $Y \sim \mathcal{X}_n^2(\delta)$ and $Z \sim \mathcal{N}_c(0, 1)$, and if $f_X(x)$, $f_Y(y)$ and $f_Z(z)$ denote their pdfs, respectively, then

$$f_X(x) = \frac{1}{2^{n/2} \Gamma(n/2)} e^{-x/2} x^{n/2-1} dx,$$

$$f_Y(y) = e^{-\frac{\delta}{2}} {}_0\tilde{F}_1\left(\frac{n}{2}; \frac{1}{4} \delta y\right) \frac{1}{2^{n/2} \Gamma(n/2)} e^{-y/2} y^{n/2-1} dy,$$

and

$$f_Z(z) = \frac{1}{\pi} e^{-|z|^2} dz.$$

Letting $x_{1,1} = 2(1 + \kappa)t_{1,1}^2$ and $x_{i,i} = 2t_{i,i}^2$, for $i = 2, \dots, N_T$, one can manipulate (16) to obtain

$$f_{\mathbf{T}}(\mathbf{T}) = \left(e^{-N_R \kappa} {}_0\tilde{F}_1\left(N_R; \frac{1}{4} (2N_R \kappa) x_{1,1}\right) \frac{e^{-x_{1,1}/2} x_{1,1}^{N_R-1}}{2^{N_R} \Gamma(N_R)} dx_{1,1} \right) \left(\prod_{i=2}^{N_T} \frac{e^{-x_{i,i}/2} x_{i,i}^{N_R-i+1}}{2^{N_R-i+1} \Gamma(N_R-i+1)} dx_{i,i} \right) \left(\prod_{i=1}^{N_T} \prod_{j=i+1}^{N_T} \frac{e^{-|t_{i,j}|^2}}{\pi} dt_{i,j} \right) \quad (17)$$

from which the theorem follows immediately. \blacksquare

IV. JOINT ERROR PROBABILITY ANALYSIS OF D-DED RECEIVER IN RICIAN FADING

A. D-DFD Receiver for MIMO Systems

Using the QR decomposition of \mathbf{H} given in (12) we may transform the received signal in (1) to obtain

$$\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{y} = \mathbf{T}\mathbf{P}^{1/2} \mathbf{b} + \boldsymbol{\eta}, \quad (18)$$

where we have let $\boldsymbol{\eta} = \mathbf{U}^H \mathbf{n}$. It is easy to see that $\boldsymbol{\eta} \sim \mathcal{N}_c(\mathbf{0}, N_0 \mathbf{I}_{N_T})$ and has independent real and imaginary parts.

Since \mathbf{T} is upper triangular it is clear from (18) that $\tilde{\mathbf{y}}_k$, the k -th element of $\tilde{\mathbf{y}}$, only depends on symbols b_n for $n = k, \dots, N_T$. Denote the output decision of the receiver for symbol b_n by \hat{b}_n , for $n = 1, \dots, N_T$. Then, the decorrelating decision feedback detector decision statistic for the symbol b_k is given, for $k = N_T, N_T - 1, \dots, 1$, by

$$\mathbf{z}_k = \tilde{\mathbf{y}}_k - \sum_{n=k+1}^{N_T} \sqrt{P_n} t_{k,n} \hat{b}_n$$

$$= \sqrt{P_k} t_{k,k} b_k + \sum_{n=k+1}^{N_T} \sqrt{P_n} t_{k,n} \tilde{b}_n + \eta_k, \quad (19)$$

where $\eta_k \sim \mathcal{N}_c(0, N_0)$ denotes the k -th element of the noise vector $\boldsymbol{\eta}$ and we have let $\tilde{b}_n = b_n - \hat{b}_n$.

B. Joint Error Probability Analysis

Let C_k and C_k^g denote the events that the D-DFD and its genie-aided version correctly detect the k -th symbol b_k , where by the genie-aided D-DFD we mean a detector which has perfect feedback. Let C denotes the event that all symbols are detected correctly by the D-DFD. Of course, we have that $C = \bigcap_{k=1}^{N_T} C_k$. The event E that at least one symbol is detected erroneously is then given by $E = C^c = \bigcup_{k=1}^{N_T} E_k$, where C^c denotes the complement of the event C and $E_k = C_k^c$ denotes the event that the k -th symbol is detected erroneously by the D-DFD. Then, the joint error probability that at least one of the symbols are detected erroneously is given by $\text{JEP} = \text{P}(E) = \text{P}\left(\bigcup_{k=1}^{N_T} E_k\right)$. Since for any decision feedback detector it can be shown that $\bigcap_{k=1}^{N_T} C_k = \bigcap_{k=1}^{N_T} C_k^g$, JEP can also be written as

$$\text{JEP} = \text{P}\left(\bigcup_{k=1}^{N_T} E_k^g\right). \quad (20)$$

From (20) we may obtain the following simple bounds on the JEP:

$$\text{JEP} \geq \max_{1 \leq k \leq N_T} \text{P}(E_k^g) \quad (21)$$

and

$$\text{JEP} \leq \sum_{k=1}^{N_T} \text{P}(E_k^g) \leq N_T \max_{1 \leq k \leq N_T} \text{P}(E_k^g). \quad (22)$$

Note that if we denote by $\text{P}(E_k^g|\mathbf{T})$ the probability that the genie-aided D-DFD makes an error for the k -th symbol decision given \mathbf{T} (or \mathbf{H}), then for $k = 1, \dots, N_T$,

$$\text{P}(E_k^g) = \mathcal{E}_{\mathbf{T}} \{\text{P}(E_k^g|\mathbf{T})\}. \quad (23)$$

It is easily seen from (19) that for the genie-aided version of the D-DFD the second term on the right hand side of (19) vanishes and thus the decision statistic for the k -th symbol is simply given by

$$\mathbf{z}_k^g = \sqrt{P_k} t_{k,k} b_k + \eta_k. \quad (24)$$

1) M-ary Quadrature Amplitude Modulation (QAM):

From (24), $\text{P}(E_k^g|\mathbf{T})$ in this case nothing more than the probability of error of M-ary QAM in an additive white Gaussian noise (AWGN) channel. Assuming a rectangular signal constellation \mathcal{B}_k of size $|\mathcal{B}_k|$, one can upper bound $\text{P}(E_k^g|\mathbf{T})$ as [13],

$$\text{P}(E_k^g|\mathbf{T}) \leq 4Q\left(\sqrt{2\alpha_k t_{k,k}^2}\right), \quad (25)$$

where $\alpha_k = \frac{3P_k}{2N_0(|\mathcal{B}_k|-1)}$ and Q -function is the integral $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$. Using the standard Gaussian tail

function bound $Q(\sqrt{x}) \leq \frac{1}{2} \exp(-\frac{x}{2})$ for $x \geq 0$, we may write

$$\text{P}(E_k^g|\mathbf{T}) \leq 2 \exp(-\alpha_k t_{k,k}^2). \quad (26)$$

Hence, from (23) and (26) we have the following upper bound

$$\text{P}(E_k^g) \leq 2\mathcal{E}_{t_{k,k}} \left\{ \exp(-\alpha_k t_{k,k}^2) \right\}. \quad (27)$$

We may use the results developed in the previous section to evaluate the upper bound (27) on $\text{P}(E_k^g)$. In fact, using the results of theorem 1 and (16) in (22), we can obtain the following union bound for rectangular M-ary QAM joint error probability:

$$\text{JEP} \leq 2 \left(\frac{\exp\left(\frac{-N_R \kappa}{1 + \frac{1+\kappa}{\alpha_1}}\right)}{\left(1 + \frac{\alpha_1}{1+\kappa}\right)^{N_R}} + \sum_{k=2}^{N_T} \frac{1}{(1 + \alpha_k)^{N_R - k + 1}} \right). \quad (28)$$

Note that if the signal constellations were such that $|\mathcal{B}_k| = 2^{2p}$, for some integer p , then the bound (25) can further be tightened by a factor of $\left(1 - \frac{1}{\sqrt{|\mathcal{B}_k|}}\right)$. The resulting expression for JEP can be obtained by straightforward modification of (28).

Theorem 2: The diversity gain of the joint error probability of the D-DFD receiver for the MIMO Rician fading channel with QAM is $N_R - N_T + 1$.

Proof: It can be shown that

$$\frac{\exp\left(\frac{-N_R \kappa}{1 + \frac{1+\kappa}{\alpha_1}}\right)}{\left(1 + \frac{\alpha_1}{1+\kappa}\right)^{N_R}} - \frac{1}{(1 + \alpha_1)^{N_R}} \leq 0 \quad \text{for } 0 \leq \alpha_1 < \infty.$$

Hence, $\text{P}(E_1^g)$ decays at least as $\alpha_1^{-N_R}$ for large values of SNR. On the other hand, from the exact expressions derived in [5] for the Rayleigh channel, it can be shown that for large values of SNR $\text{P}(E_k^g)$, for $k = 2, \dots, N_T$ decays as $N_R - k + 1$. Thus, for large SNR, (21) becomes

$$\text{JEP} \geq \text{P}(E_{N_T}^g).$$

Hence, the diversity gain of the JEP of the D-DFD is upper bounded by the diversity order of $\text{P}(E_{N_T}^g)$, which is equal to $N_R - N_T + 1$. But, from (22) it is clear that the diversity gain is also lower bounded by the diversity order of $\text{P}(E_{N_T}^g)$. Thus, the diversity gain of the JEP of the D-DFD receiver for QAM in Rician fading is $N_R - N_T + 1$. ■

2) *M-ary PSK Modulation:* Using the approximation to the symbol error probability of a single-user M-ary PSK system [13], we have that,

$$\text{P}(E_k^g|\mathbf{T}) \approx 2Q\left(\sqrt{2\beta_k t_{k,k}^2}\right), \quad (29)$$

where $\beta_k = \frac{2}{3}\alpha_k(|\mathcal{B}_k|-1)\sin^2\left(\frac{\pi}{|\mathcal{B}_k|}\right)$. Comparing (29) with (25), we see that the JEP for the M-ary PSK modulation can be obtained by replacing α_k by β_k and then multiplying the results by a factor of two. We omit the details here. It can

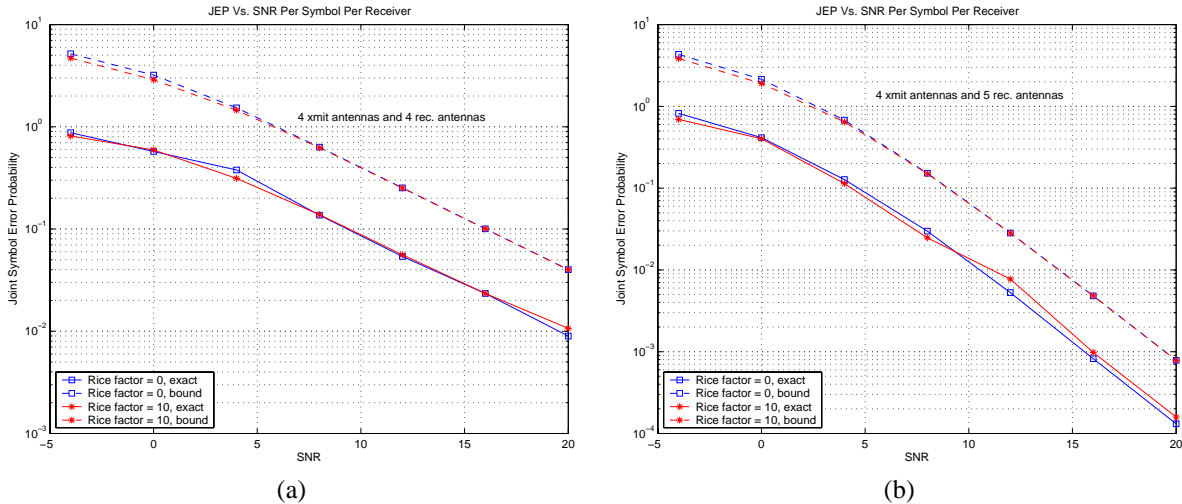


Fig. 1. Joint Error Probability of the Decorrelating Decision Feedback Detector for Multiple Antenna Systems in Rician Fading. Uncoded QPSK Modulation on each Transmitter Antenna with $N_T = 4$. (a) $N_R = 4$. (b) $N_R = 5$.

be shown, in a similar way, that the diversity gain of the JEP of D-DED in PSK system is also equal to $N_R - N_T + 1$.

Theorem 3: The diversity gain of the joint error probability of the D-DFD receiver for MIMO Rician fading channel with PSK modulation is $N_R - N_T + 1$.

V. NUMERICAL EXAMPLES

Figure 1 shows the JEP of the D-DFD for a 4-transmit-antenna MIMO system with $N_R = 4$ and $N_R = 5$. In Fig. 1 it is assumed that each antenna transmits uncoded symbols using QPSK (or 4-ary QAM) modulation thus resulting in a total bit rate of 8 bits per channel use. From Fig. 1 it is clear that the slope of the JEP in Rician fading ($\kappa = 10$ in the figure) is almost the same as the slope of the JEP in Rayleigh fading ($\kappa = 0$), thus confirming our results on the same diversity gain in the two fading environments.

VI. CONCLUSIONS

The diversity gain of the joint probability of error of the decorrelating decision feedback detector for multiple-input multiple-output systems has been derived in the presence of Rician fading. Upper bounds for joint error probability have been derived for both QAM and PSK constellations. It has been shown that the diversity gain of D-DFD receiver in both cases is equal to $N_R - N_T + 1$. We have also derived the Bartlett decomposition of a complex non-central Wishart matrix which was essential to our joint error probability analysis.

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REFERENCES

- [1] G. J. Foschini, "Layered space-time architecture for wireless communication in a flat fading environment when using multi-element antennas," *Bell Labs Tech. Journ.*, vol. 1, no. 2, pp. 41–59, 1996.
- [2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.
- [3] A. Duel-Hallen, "Decorrelating decision feedback multiuser detector for synchronous code-division multiple-access channel," *IEEE Trans. Commun.*, vol. 41, no. 2, pp. 285–290, Feb. 1993.
- [4] M. K. Varanasi, "Decision feedback multiuser detection: A systematic approach," *IEEE Trans. Inform. Theory*, vol. 45, no. 1, pp. 219–240, Jan. 1999.
- [5] N. Prasad and M. K. Varanasi, "Analysis of decision feedback detection for MIMO Rayleigh fading channels and optimum power/rate allocation," *IEEE Trans. Inform. Theory*, submitted.
- [6] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Stat.*, vol. 35, pp. 475–501, June 1964.
- [7] C. S. Herz, "Bessel functions of matrix argument," *Annals of Math.*, vol. 61, no. 3, pp. 474–423, May 1955.
- [8] P. C. Mahalanobis, R. C. Bose, and S. N. Roy, "Normalization of variates and the use of rectangular co-ordinates in the theory of sampling distributions," *Sankhyā*, vol. 3, pp. 1–40, 1937.
- [9] R. A. Wijsman, "Random orthogonal transformations and their use in some classical distribution problems in multivariate analysis," *Ann. Math. Stat.*, vol. 28, pp. 415–422, 1957.
- [10] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*. New York: Wiley, 1982.
- [11] A. M. Kshirsagar, "Effect of non-centrality on the Bartlett decomposition of a Wishart matrix," *Ann. Inst. Stat. Math.*, vol. 14, pp. 217–228, 1963.
- [12] D. G. Kabe, "A note on the Bartlett decomposition of a Wishart distribution," *Journ. Royal Stat. Soc.*, vol. 26, pp. 270–273, 1964.
- [13] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 2001.