Particle Filtering For Target Tracking

Sumakanth S. Kambhampati, Krishna V. Tangirala, Kamesh R. Namuduri and Sudharman K. Jayaweera
Department of Electrical and Computer Engineering
Wichita State University
Wichita, KS 67260, USA.
Email: sskambhampati@wichita.edu

***Abstract*— Particle filtering is a sequential Monte Carlo technique that recursively computes the posterior probability density function using the concept of "Importance Sampling". This paper considers the application of particle filtering technique to a target tracking application, in which a radar sends a signal towards a target and estimates the state (position and velocity) of the target using the observations (time delay and Doppler shift) from the reflected signal. State model and measurement model have been derived for the proposed target tracking problem. Effectiveness of particle filtering technique has been demonstrated by comparing the results with those obtained with Kalman filtering technique. The prediction error obtained by using particle filtering technique is found to be significantly less than that error obtained from Kalman filtering technique.**

***Keywords*— Kalman filtering, likelihood function, observation model, Particle filtering, posterior density, state model.**

I. Introduction

Over the past 40 years many techniques have been developed for target tracking in clutter environment, which include classical Kalman filtering [1], [2], unscented Kalman filtering [3] and extended Kalman filtering [3]. The tracking task presented in this paper is a state estimation problem consisting of state transition model which relates the target position and velocity at every time step and an observation model relating the current target state with the current target observations. This paper presents a solution to target tracking using particle filters. In recent years, particle filtering techniques have been used by several researchers for various applications in coding, communications and signal processing. Such applications include turbo coding [4], multiuser detection, blind detection and equalization [5].

The basic idea behind particle filtering [6] is to sample a the continuous posterior density function of interest into a set of weighted particles. If the weights are chosen appropriately, then these weighted set of particles represent the posterior density in a way that the posterior density function (pdf) can be made arbitrarily close to the equivalent set of weighted particles. The main task of particle filter is to assign appropriate weights and update the weights as time progresses. Problems dealing with particle filtering involve making inferences on the state vector $\mathbf{x}_t$, based on $\mathbf{z}_{0:t}$, the observations from time 0 to t. Using sequential importance sampling [7], particle filters can approximate the posterior density function i.e., $p(\mathbf{x}_t|\mathbf{z}_{0:t})$ regardless of the nature of the underlying model. A critical step in particle filtering is the choice of importance function [7]. Two standard choices of importance functions are the posterior and the prior. The posterior importance function is defined as

$$\pi(\mathbf{x}_t|\mathbf{x}_{0:t-1}^{(m)}, \mathbf{z}_{0:t}) \quad = \quad p(\mathbf{x}_t|\mathbf{x}_{0:t-1}^{(m)}, \mathbf{z}_{0:t}), \tag{1}$$

and the importance weights are given by

$$w_t^{(m)} \propto \; w_{t-1}^{(m)} \; p(\mathbf{z}_t|\mathbf{x}_{t-1}^{(m)}), \tag{2}$$

where the superscript (m) denotes the set of weights. The weights are given by $w_t = \frac{p(x)}{\pi(x)}$ [8]. The posterior importance function minimizes the variance of the importance weights [7]. However, the difficulty in using posterior importance function is that it involves complex high dimensional integrations to sample $p(\mathbf{x}_t|\mathbf{x}_{t-1}^m, \mathbf{z}_t)$. It is convenient to choose prior importance function due to its ease of imlementation. The prior importance function is defined as

$$\pi(\mathbf{x}_t|\mathbf{x}_{0:t}^{(m)}, \mathbf{z}_{0:t-1}) \quad = \quad p(\mathbf{x}_t|\mathbf{x}_{t-1}^{(m)}), \tag{3}$$

and the weights are defined as

$$w_t^{(m)} \;\propto\; w_{t-1}^{(m)} \; p(\mathbf{z}_t|\mathbf{x}_t^{(m)}). \tag{4}$$

Another important feature of particle filtering is resampling [9], in which the weights which have a very low value are eliminated and are replaced with the weights which have a higher value.

This paper presents a particle filtering technique for target tracking [6] using a linear state model and a nonlinear measurement model with additive white Gaussian noise. Extensive experiments have been performed to analyze the behavior of the proposed model and the particle filtering method, specifically particle density, particle location and noise variance in the proposed model.

II. System Model

In this model, the target movement is modelled as a discrete-time, linear, state model perturbed by additive noise. The observation model is non-linear with additive white Gaussian noise.

Let $\mathbf{x}(t - \frac{\tau(t)}{2})$ be the state vector at time t which consists of the position and velocity of the moving target (i.e, $\mathbf{x}(t - \frac{\tau(t)}{2}) = [x(t - \frac{\tau(t)}{2}), y(t - \frac{\tau(t)}{2}), \dot{x}(t - \frac{\tau(t)}{2}), \dot{y}(t - \frac{\tau(t)}{2})]^T$) where superscript T denotes the transpose. We can then model the target movement as

$$\mathbf{x}\left(t - \frac{\tau(t)}{2} + T\right) = \mathbf{A}\mathbf{x}\left(t - \frac{\tau(t)}{2}\right) + \mathbf{u}\left(t - \frac{\tau(t)}{2}\right), \quad (5)$$

where $\mathbf{u}\left(t - \frac{\tau(t)}{2}\right)$ is the state noise which is assumed to be zero-mean with covariance matrix $\mathbf{Q}$ and matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

Suppose that a signal $E(t)$ is transmitted by the sensor at time t. The received signal reflected from the target is given by

$$s_r(t) = \Re[A_r E(t - \tau(t)) e^{j\omega_c(t - \tau(t))}], \quad (7)$$

where $A_r = \sqrt{2P_r}$, (P_r includes the transmit power and the two-way propagation and reflection processes [10]), ω_c is the carrier frequency and $\tau(t)$ is round trip delay of the transmitted wave between the sensor and the target.

The delay $\tau(t)$ is given by

$$\tau(t) = \tau_0 - \frac{2v_{rad}}{c}t, \quad (8)$$

where τ_0 is the reference delay and v_{rad} is the radial velocity of the target. Substituting $\tau(t)$ in (7), we get

$$s_r(t) = \Re\left[A_r E\left(t - \tau_0 + \frac{2v_{rad}}{c}t\right) e^{j\omega_c(t - \tau_0 + \frac{2v_{rad}}{c}t)}\right]. \quad (9)$$

From (9), we note that the doppler frequency shift f_D of the received signal is

$$f_D = \frac{2f_c}{c}v_{rad}. \quad (10)$$

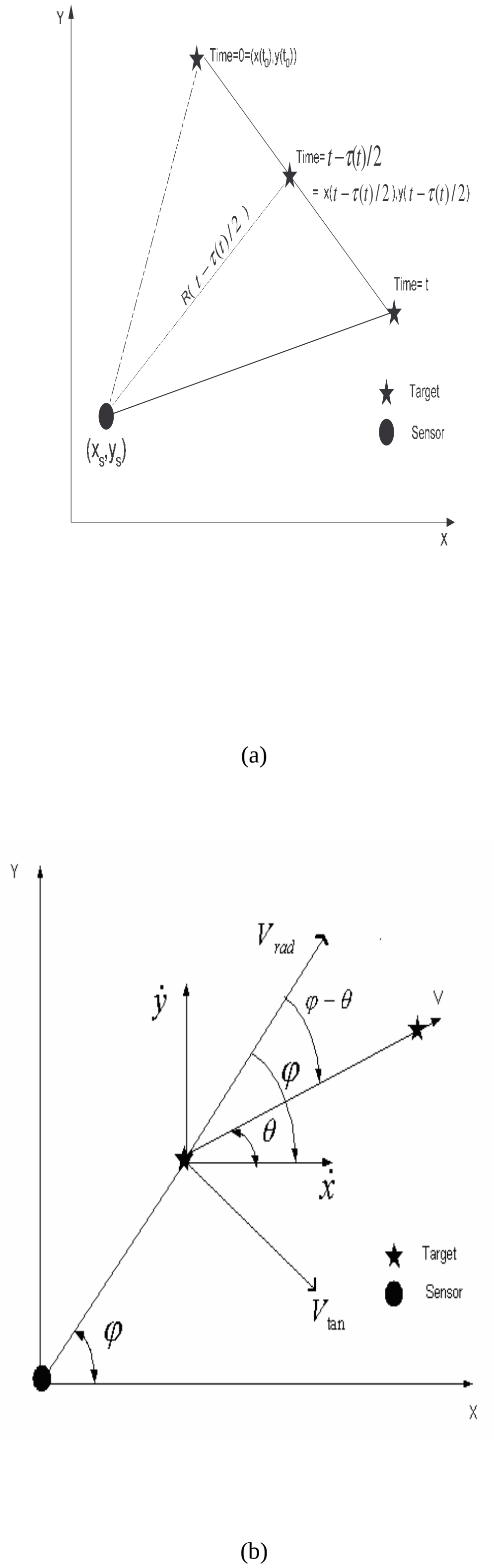


Fig. 1. (a) The Orientation of sensor and target for computing delay and doppler. (b) Velocity components of the target.

Referring to Fig.1(a), the time delay $\tau(t)$ is given by

$$\tau(t) = \frac{2}{c}\sqrt{\left(x\left(t-\frac{\tau(t)}{2}\right)-x_s\right)^2+\left(y\left(t-\frac{\tau(t)}{2}\right)-y_s\right)^2},$$

where (x_s, y_s) represents the sensor location.

Suppose that the target is moving with a velocity $\mathbf{v}$ as shown in Fig. 1(b). Then, the radial component of the velocity is given by

$$v_{rad} = \dot{x}\left(t-\frac{\tau(t)}{2}\right)\cos(\varphi)+\dot{y}\left(t-\frac{\tau(t)}{2}\right)\sin(\varphi), \quad (11)$$

where angle φ represents the orientation of the line connecting the target and sensor.

The observation vector is defined as $\mathbf{z}\left(t-\frac{\tau(t)}{2}\right) = [\tau(t), f_D(t)]^T$, where $[\tau(t), f_D(t)]^T$ are given by (8) and (10). v_{rad} component is the nonlinear term in (8) and (10). One approach to target tracking is to approximate v_{rad} into a linear term and apply the classical Kalman filtering technique [11]. Particle filtering provides a solution without requiring any such approximations.

III. Tracking Algorithm Using Particle Filtering

The first step in particle filtering is the choice of the importance sampling function [8], [7], using which we sample the continuous density function $p(x)$ into a set of discrete particles with weights assigned to each sample or particle. Sampling from the continuous density function might be intractable. Therefore, to sample this continuous density function a function $\pi(x)$, termed as importance function and has same support as that of $p(x)$, for which sampling is tractable is used. Then the sampled approximation to the continuous density is given by

$$p(x) \ = \ \sum_{m=1}^{M} w^{(m)}\delta(x-x^{(m)}), \quad (12)$$

where

$$w^{(m)} \ = \ \frac{p(x)}{\pi(x)}, \quad (13)$$

(m) denotes the set of particles and *M* denotes the total number of particles. If the posterior density function $p(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1})$ is sampled and the samples, $\mathbf{x}^{m}_{0:t-1}$ were drawn from the importance function $\pi(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1})$, then weights defined in equation (13) become

$$w^{(m)}_{t-1} \ = \ \frac{p(\mathbf{x}^{(m)}_{0:t-1}|\mathbf{z}_{0:t-1})}{\pi(\mathbf{x}^{(m)}_{0:t-1}|\mathbf{z}_{0:t-1})}. \quad (14)$$

If the importance density function can be factorized as

$$\pi(\mathbf{x}_{0:t}|\mathbf{z}_{0:t}) = \pi(\mathbf{x}_t|\mathbf{x}_{0:t-1},\mathbf{z}_{0:t})\pi(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1},) \quad (15)$$

and if,

$$\mathbf{x}^{(m)}_{0:t-1} \sim \pi(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1}) \quad (16)$$

then by augmenting the existing samples $\mathbf{x}^{(m)}_{0:t-1} \sim \pi(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1})$ with the new state $\mathbf{x}^{(m)}_t \sim \pi(\mathbf{x}_t|x_{0:t-1},\mathbf{z}_{0:t})$, one can obtain samples $\mathbf{x}^{(m)}_{0:t} \sim \pi(\mathbf{x}_{0:t}|\mathbf{z}_{0:t})$. Using Bayes rule [12], the weight update equation $p(\mathbf{x}_{0:t}|\mathbf{z}_{0:t})$ is computed as follows:

$$\begin{aligned} p(\mathbf{x}_{0:t}|\mathbf{z}_{0:t}) &= \frac{p(\mathbf{z}_t|\mathbf{x}_{0:t},\mathbf{z}_{0:t-1})p(\mathbf{x}_{0:t}|\mathbf{z}_{0:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{0:t-1})} \\ &= \frac{p(\mathbf{z}_t|\mathbf{x}_{0:t},\mathbf{z}_{0:t-1})p(\mathbf{x}_t|\mathbf{x}_{0:t-1},\mathbf{z}_{0:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{0:t-1})} \\ &\quad \mathbf{x}\ p(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1}) \quad (17) \\ &= \frac{p(\mathbf{z}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1})}{p(\mathbf{z}_t|\mathbf{z}_{0:t-1})}p(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1}) \end{aligned}$$

Therefore,

$$p(\mathbf{x}_{0:t}|\mathbf{z}_{0:t}) \propto \ p(\mathbf{z}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1})p(\mathbf{x}_{0:t-1}|\mathbf{z}_{0:t-1}). \quad (18)$$

The denominator can be ignored since it is a normalizing constant. By substituting (15) and (18) into (14), the updated weights are obtained as follows

$$\begin{aligned} w^{(m)}_t &\propto \frac{p(\mathbf{z}_t|\mathbf{x}^{(m)}_t)p(\mathbf{x}^{(m)}_t|\mathbf{x}^{(m)}_{t-1})p(\mathbf{x}^{(m)}_{0:t-1}|\mathbf{z}_{0:t-1})}{\pi(\mathbf{x}^{(m)}_t|\mathbf{x}^{(m)}_{0:t-1},\mathbf{z}_{0:t})\pi(\mathbf{x}^{(m)}_{0:t-1}|\mathbf{z}_{0:t-1})} \\ &= w^{(m)}_{t-1}\frac{p(\mathbf{z}_t|\mathbf{x}^{(m)}_t)p(\mathbf{x}^{(m)}_t|\mathbf{x}^{(m)}_{t-1})}{\pi(\mathbf{x}^{(m)}_t|\mathbf{x}^{(m)}_{0:t-1},\mathbf{z}_{0:t})}. \quad (19) \end{aligned}$$

The two most frequently used importance functions are the prior importance function given by $p(\mathbf{x}_t|\mathbf{x}^{(m)}_{t-1})$ with which the updated weights from (19) become

$$w^{(m)}_t \ \propto \ w^{(m)}_{t-1}\ p(\mathbf{z}_t|\mathbf{x}^{(m)}_t) \quad (20)$$

and the posterior or optimal importance function given by $p(\mathbf{x}_t|\mathbf{x}^{(m)}_{0:t-1},\mathbf{z}_{0:t})$ with which the updated weights from (19) become

$$w^{(m)}_t \ \propto \ w^{(m)}_{t-1}\ p(\mathbf{z}_t|\mathbf{x}^{(m)}_{t-1}). \quad (21)$$

For simplicity reasons, prior importance function is taken in the simulations. The main objective in particle filtering is to estimate the posterior density function i.e., the set of weights $w^{(m)}_t$. In order to compute the posterior density function, it is necessary to have the knowledge of the prior density function i.e., $w^{(m)}_{t-1}$ and the nature of the likelihood function i.e., $p(\mathbf{z}_t|\mathbf{x}_{t-1})$.

A. *Selection of Prior*

Initially, the prior probability density function is unknown. Hence, we start with an assumption that it is a Gaussian. The initial weights for the prior are computed for each particle assuming the following:

$$p(\mathbf{x}_{t-1}^{(m)}|\mathbf{z_{0:t-1}}) \sim \mathcal{N}(\underline{\mu}, \mathbf{R}), \quad (22)$$

where $\underline{\mu} = [\mu_x, \mu_y, \mu_{\dot{x}}, \mu_{\dot{y}}]$ is the mean and **R** is the covariance matrix with diagonal elements representing the variances of the positions (x and y) and the velocity components ($\dot{x}$ and $\dot{y}$).

B. Computation of Likelihood

Once the prior is computed, the next step is to compute the likelihood function. The observation model which is nonlinear, consists of two observations i.e., $[\tau(t), f_D(t)]^T$, given by, $z_t = [z_1(t), z_2(t)]^T + \mathbf{n}(t)$ where

$$z_1(t) = \frac{2v_{rad}}{c} * t \quad (23)$$

$$z_2(t) = \frac{2f_c}{c} * v_{rad} \quad (24)$$

where **n(t)** is the additive Gaussian noise with zero mean and a covariance matrix **Q**. The radial velocity v_{rad} obtained from Fig. (1) is given by

$$v_{rad} = \dot{x}\cos(\varphi) + \dot{y}\sin(\varphi), \quad (25)$$

where

$$\varphi = \arctan(\frac{y}{x}). \quad (26)$$

The joint density of the two observations given $\mathbf{x}_t$, at time instant t, for each particle is given by

$$p(\mathbf{z}_t|\mathbf{x}_t^m) = \frac{1}{2\pi|R|^{\frac{1}{2}}} \exp -((\mathbf{z}_t - \mu_{\mathbf{z}_t})^T R^{-1}(\mathbf{z}_t - \mu_{\mathbf{z}_t})), \quad (27)$$

where $\mu_{\mathbf{z}_t}$ is the mean of $\mathbf{z}_t$ and **R** is the covariance matrix. The joint conditional density $p(\mathbf{z}_t|\mathbf{x}_t^m)$ is Gaussian because **n**(t) is Gaussian.

C. Computation of Mean for the Likelihood

Given $\mathbf{x}_t$, the mean and covariance of $\mathbf{z}_t$ can be computed from (23) and (24). The mean $\mu_{\mathbf{z}_t}$ is shifted by a constant value obtained from that of **n**(t). The covariance of $\mathbf{z}_t$ is same as that of **n**(t). The continuous density function which was sampled into particles is used to compute the means of the observation vector and is given by

$$\mu_{z_1(t)} = \frac{2v_{rad}}{c} * t \quad (28)$$

$$\mu_{z_2(t)} = \frac{2f_c}{c} * v_{rad} \quad (29)$$

where v_{rad} is given by

$$v_{rad} = \dot{x}^{(m)}\cos(\varphi) + \dot{y}^{(m)}\sin(\varphi), \quad (30)$$

and

$$\varphi = \arctan\left(\frac{y^{(m)}}{x^{(m)}}\right). \quad (31)$$

D. Weight Update

An important step in particle filtering technique is to update the weights or the posterior density as time progresses. In order to compute the weights the prior and likelihood have to be computed as explained in the previous sections. To start with, we select a density function with an arbitrary mean and variance, sample the pdf to get a set of particles, use them in the prior i.e., (22) and compute the prior for each particle. Then, we compute the likelihood using (27) for each particle, multiply the likelihood with the prior at a given time instant. This process is repeated recursively at each time step.

Once the new weights are computed for each particle, the expected value is computed as follows

$$E(\mathbf{x}) = \sum_i \mathbf{x}\ p(\mathbf{x}). \quad (32)$$

In the next time step, the particles are chosen around the expected value and are updated by

$$\mathbf{x}_t^m = \mathbf{A}\mathbf{x}_{t-1}^m. \quad (33)$$

IV. Simulation Results

Simulations have been performed by choosing 2500 to 10000 particles, and by varying the target locations, keeping the velocity range constant. The carrier frequency is chosen as 10^9 and the initial time delay is chosen to be 0.01 sec. The true initial location and velocity of the target are assumed to be [5,5,1,1] for all simulations. The range for the location and the velocity are selected in such a way that they include the true location and extends beyond the true location. It is assumed that the noise is uncorrelated and its correlation matrix is given by

$$\mathbf{Q_{cov}} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}. \quad (34)$$

As time varies, the correlation matrix changes randomly. The error in estimating the range ($E_{\Delta R}$) is given by

$$E_{\Delta R} = \sqrt{(\Delta x^2 + \Delta y^2)}$$

where Δx and Δy are the errors in estimating the locations x and y. Figures 2 (a), 2 (b) and 2 (c) illustrates the error in range estimation for a set of particles that are selected within the range [4,8], which includes the true location of the target and [6,10], which extend beyond the true location. Comparing Figure 2 (b) and Figure 2 (c), it can be observed that by increasing the number of time steps in the simulations better results are obtained. It can also be observed that the error obtained using particle filtering technique is less than the error obtained using Kalman filtering.

V. Conclusions

In this paper, we present a particle filtering technique for target tracking using a linear state model and a non-linear measurement model with additive white Gaussian noise. The results suggest that particle filtering technique performs much better than Kalman filtering technique. Future work includes applying particle filtering technique to a non-linear state model and by incorporating non-Gaussian noise.

References

[1] K.V. Ramachandra, *Kalman Filtering Techniques for Radar Tracking*, Marcel Dekker, Inc, New York, USA, 2000.

[2] H. Vincent Poor, *An Introduction to Signal Detection and Estimation, Second Edition*, Springer-Verlag, New York, USA, 1994.

[3] S. Haykin, "Kalman Filtering and Neural Networks," Wiley-Interscience, September 2001.

[4] B. Dong, X. Wang and A. Doucet, "A New Class of Soft MIMO Demodulation Algorithms," IEEE *Trans. Signal Processing*, vol. 51, no. 11, pp. 2752-2763, 2003.

[5] Petar M. Djurić, Jayesh H. Kotecha, Jianqui Zhang, Yufei Huang, Tadesse Ghirmai, Mónica F. Bugallo, and Joaquín Míguez, "Particle Filtering," Signal Processing Magazine, IEEE, vol.20, Issue: 5 , Sep 2003

[6] C. Hue, J. Le Carde, P. Perez, "Tracking multiple objects with particle filtering," IEEE *Trans. Aerospace and Elect. Systems*, vol 38, no.3, July 2002.

[7] A. Doucet, S. J. Godsill, and C. Andrieu, "On Sequential Monte Carlo sampling methods for Bayesian filtering," *Statist. Comput. Commun.*, vol.3, pp. 8-27, July 2002.

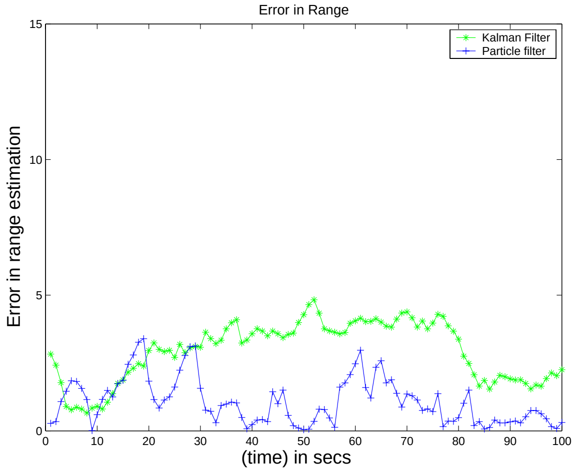
[8] S. Arulampam, S. Maskell, N. Gordon, and T. Clapp, " A tutorial on particle filters for on-line non-linear/non-Gaussian Bayesian tracking," IEEE *Trans. Sig. Proc.*, vol 50, pp. 174-188, Feb 2002.

[9] Petar M. Djurić, Jayesh H. Kotecha, Jianqui Zhang, Yufei Huang, Tadesse Ghirmai, Mónica F. Bugallo, and Joaquín Míguez, "Applications of particle filtering to selected problems in communications: A review and new developments," Signal Processing Magazine, 2003

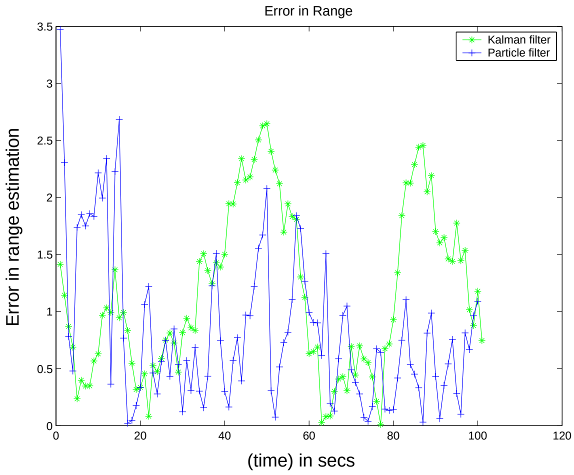
[10] M. I. Skolnik, *Introduction to Radar Systems, Second Edition*, McGraw-Hill Book Company, New York, USA, 1980.

[11] S. Balasubramanian, I. Elangovan, S. K. Jayaweera and K. R. Namuduri, "Energy-aware distributed colloborative tracking in ad-hoc Wireless sensor networks." Wireless Commun. and Networking Conf. (WCNC 04), Mar 2004, Atlanta, GA.

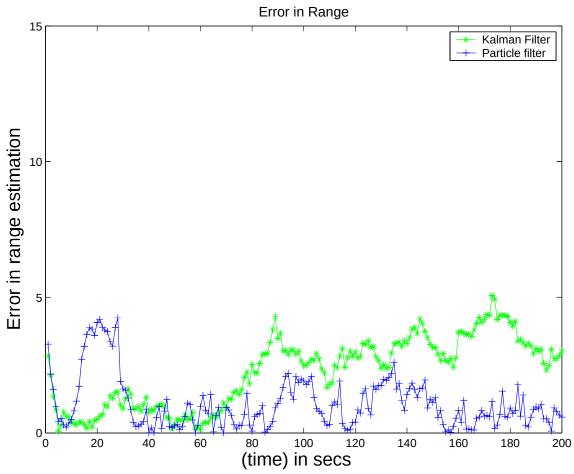
[12] Alberto Leon-Garcia, "Probability and Random Processes for Electrical Engineering," Addison-Wesley Publishing Company, May 1989.



(a)



(b)



(c)

Fig. 2. (a) The Error in range when the position range is within [4,8] and velocity range is [1,3]. (b) The Error in range for 100 time steps when the position range is within [6,10] and velocity range is [1,3]. (c) The Error in range for 200 time steps when the position range is within [6,10] and velocity range is [1,3].