Decentralized Detection of Stochastic Signals in Power-constrained Sensor Networks

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***Abstract*— Decentralized detection of a stochastic signal in a total average power constrained wireless sensor network is considered. Assuming amplify-and-relay local processing, the fusion performance is derived in closed-form under both Bayesian and Neyman-Pearson optimality, in the case of conditionally independent signal samples. An important observation is that the average fusion probability of error does not improve monotonically with the number of sensors unlike in the case of deterministic signal detection reported in [1]. In particular, there is an optimal number of sensors that minimizes the probability of error, which depends on both observation signal-to-noise ratio (SNR) as well channel SNR.**

***Index Terms*— Data fusion, decentralized detection, distributed detection, hypothesis testing, multi-sensor fusion, sensor networks.**

I. Introduction

In a distributed sensor network, relaying of local decisions, as opposed to sending the direct observations, to a central fusion center can significantly save system resources in terms of both required communication bandwidth and sensor power. Although this problem of decentralized decentralized detection and data fusion has been researched for more than two decades (see, for example, [2]–[6]), it is only recently that it has been considered in the specific context of resource-constrained wireless sensor networks [1], [7]–[9].

Although there is a considerable amount of previous work on the subject of distributed detection, most of them usually ignore the effect of noisy channels between the local sensors and data fusion center. Although this may be justified in wired systems without strict energy constraints, in the context of low-power wireless sensor networks one must consider them. The decentralized detection problem has only recently been considered under such system power and bandwidth constraints [1], [7], [9]. In particular, [1] considered the deterministic signal detection under a finite total power constraint on the sensor system. It was shown that in this case it is better to distribute the available energy among all nodes in the system. In other words, combining (fusing) as many noisy local decisions is better than combining a few very good local decisions. However their analysis considered only the decentralized detection of a deterministic signal. In [9] we showed that even if there is an additional bandwidth constraint on the system, still it is better to combine as many not-so-good local decisions as possible.

In this paper, we consider the problem of decentralized detection of a Gaussian stochastic signal under a total power constraint on the sensor system. Perhaps surprisingly, our analysis shows that in the case of stochastic signal detection, there is always an optimal number of sensors for any given power constraint. In particular, in a large sensor system if one were to allocate the total available power among all sensors equally it may lead to local decisions becoming useless at the fusion center.

The remainder of the paper is organized as follows: In Section II we formulate our problem. Next, in Section III we derive the optimal fusion center rule and analyze the resulting decentralized detection performance. In particular, we obtain a closed-form expression for the performance followed by a discussion of our analysis. Finally, in Section IV we conclude by summarizing our results.

II. Problem Formulation

A typical scenario in a (dense) low-power wireless sensor network is that a collection of ad-hoc distributed sensor nodes observe information on an event of interest. Each node performs some basic local processing and the resulting local decision is relayed to a processing center, named the fusion center, in order to make a collective decision on the event of interest. This problem of decentralized detection stems from the fact that, relaying of only the local decisions, as opposed to directly sending the complete sensor observations, to a central fusion center may significantly save system resources in terms of both communication bandwidth and sensor power consumption. Often, it is assumed that these local decisions are received at the fusion center perfectly. i.e. the assumption of a noise-less channel.

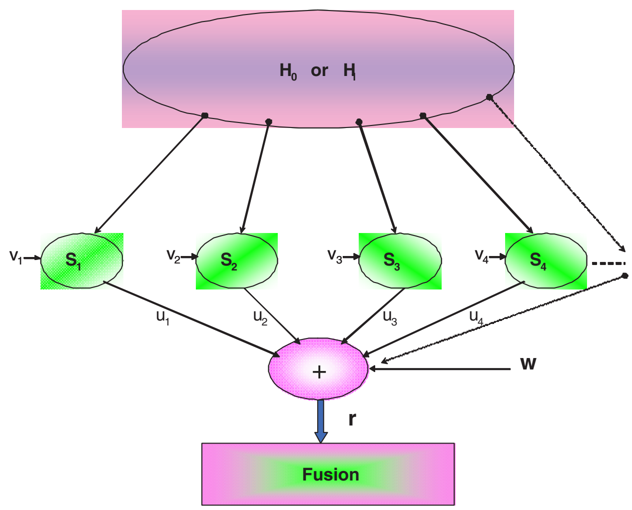

Fig. 1. Decentralized Detection in a Sensor Network Subjected to a Total Power Constraint.

We consider a binary hypothesis testing problem in an N_s-node wireless sensor network connected to a data fusion center via distributed parallel architecture [3]. Let us denote by H_0 and H_1 the null and alternative hypotheses, respectively, having corresponding prior probabilities $P(H_0) = p_0$ and $P(H_1) = p_1$. To be specific, we will consider that the observed stochastic process is either a desired Gaussian signal corrupted by additive white Gaussian noise or noise alone. The Gaussian signal of interest, denoted by X_n is completely characterized by its mean and the covariance function. For simplicity, we take the mean of the desired signal to be zero.

Under the two hypotheses the n-th local sensor observation z_n, for $n = 1, \cdots N_s$, can be written as

$$\begin{aligned} H_0 : \quad z_n &= v_n \\ H_1 : \quad z_n &= X_n + v_n \end{aligned} \tag{1}$$

where the observation noise v_n is assumed to be zero-mean Gaussian with the collection of noise samples having a covariance matrix Σ_v and $X_n \sim \mathcal{N}(0, \Sigma_x^2)$. In this paper, we assume that $\Sigma_v = \sigma_v^2 \mathbf{I}$. i.e. the noise samples at different sensors are independent of each other. Note that, we can pre-whiten the observation sequence z_n and still obtain a Gaussian stochastic signal detection problem with a modified signal covariance matrix.

Each local sensor processes its observation z_n independently to generate a local decision $u_n(z_n)$ which are sent to the fusion center. Let us denote by $\mathbf{r}(u_1(z_1), u_2(z_2), \cdots, u_{N_s}(z_{N_s}))$ the received signal at the fusion center. The fusion center makes a final decision based on the decision rule $u_0(\mathbf{r})$. The problem at hand is to choose $u_0(\mathbf{r}), u_1(z_1), u_2(z_2), \cdots, u_{N_s}(z_{N_s})$ so that a chosen performance metric is optimized.

Although, in general, the solution to this problem is known to be too complicated, it is known that if local observations are independent of each other conditioned on the true hypothesis, then all decision rules simplify to a set of likelihood ratio based decision rules at local sensors but with possibly coupled thresholds [2]. While optimal local processing schemes have been investigated, and have derived under certain special assumptions, a class of especially important local processers are those that simply amplify the observations before retransmission to the fusion center [1]. It has been shown that such simple amplify-and-relay local processing performs fairly well when the local observations are corrupted by additive noise, as in our formation [7]. Moreover, this type of amplify-and-relay local processing seems to be well-suited for low-power, tiny wireless sensor networks that are gaining popularity in various new applications. Thus, the local sensor decisions sent to the fusion center are given by

$$u_n = g z_n \quad \text{for } n = 1, \cdots N_s \tag{2}$$

where $g > 0$ is the analog relay amplifier gain at each node. The radiated power of node n is given by

$$\mathbb{E}\{|u_n|^2\} = g^2 \mathbb{E}\{|z_n|^2\} = g^2(p_1\sigma_x^2 + \sigma_v^2). \tag{3}$$

where we have assumed that $\mathbb{E}\{|X_n|^2\} = \sigma_x^2$ for all $n = 1, 2, \cdots, N_s$.

Let us assume that the whole sensor system is subjected to a total power constraint of P. Then, the amplifier gain g is related to the size of the sensor system and the total available power P as

$$g = \sqrt{\frac{P}{N_s(p_1\sigma_x^2 + \sigma_v^2)}}. \tag{4}$$

The local decisions u_n's are transmitted to the fusion center over a noisy wireless channel. In this paper we assume that this sensor-to-fusion center communication is orthogonal. Hence, the received signal at the fusion center due to the n-th sensor node's transmission can be written as

$$r_n = g z_n + w_n, \tag{5}$$

where w_n is the fusion receiver noise. In vector notation,

$$\mathbf{r} = g\mathbf{z} + \mathbf{w} \tag{6}$$

where $\mathbf{r}$ and $\mathbf{w}$ are N_s-dimensional received vector and the receiver noise, respectively. We assume that the receiver noise is a white Gaussian noise process so that the noise vector $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{N_s})$. It is easily seen that the received signal $\mathbf{r}$ is Gaussian distributed such that

$$\begin{aligned} H_0: \quad & \mathbf{r} \sim \mathcal{N}(\mathbf{0}, \Sigma_0) \\ H_1: \quad & \mathbf{r} \sim \mathcal{N}(\mathbf{0}, \Sigma_1). \end{aligned} \tag{7}$$

We have that

$$\begin{aligned} \Sigma_0 &= g^2 \Sigma_v + \sigma_w^2 \mathbf{I}_{N_s} = (g^2 \sigma_v^2 + \sigma_w^2) \mathbf{I}_{N_s} \\ &= \sigma_n^2 \mathbf{I}_{N_s}, \end{aligned} \tag{8}$$

and

$$\Sigma_1 = g^2 (\Sigma_x + \Sigma_v) + \sigma_w^2 \mathbf{I}_{N_s} = g^2 \Sigma_x + \sigma_n^2 \mathbf{I}_{N_s}, \tag{9}$$

where we have defined $\sigma_n^2 = g^2 \sigma_v^2 + \sigma_w^2$.

III. Structure and the Performance Analysis of Optimal Fusion Rules

The detection problem at the fusion center is given by (7). It is well known that threshold tests in which the log-likelihood ratio (llr) is compared to a threshold are the optimal (for example, in the sense of either Bayesian or Neyman-Pearson optimality) tests for a problem of the type (7).

Computing the log-likelihood ratio, it can be shown that optimal fusion rules are of the form of

$$\delta_{opt}(\mathbf{r}) = \begin{cases} 1 & \\ & \text{if } T(\mathbf{r}) \overset{\geq}{<} \tau' \\ 0 & \end{cases}, \tag{10}$$

where the decision variable T is the quadratic form given by

$$\begin{aligned} T(\mathbf{r}) &= \mathbf{r}^T \left(\Sigma_0^{-1} - \Sigma_1^{-1} \right) \mathbf{r} \\ &= \frac{g^2}{\sigma_n^2} \mathbf{r}^T \Sigma_x \left(g^2 \Sigma_x + \sigma_n^2 \mathbf{I}_{N_s} \right)^{-1} \mathbf{r}, \end{aligned}$$

and

$$\tau' = \log \left(\tau^2 \frac{|\Sigma_1|}{|\Sigma_0|} \right) = \log \left(\tau^2 \sigma_n^{-2N_s} |\Sigma_1| \right), \tag{11}$$

with τ being the original threshold that depends on the specific optimality criteria. For example, in the case of Neyman-Pearson optimality at the fusion center, τ is chosen to minimize the miss probability P_m subject to an upper bound on the false-alarm probability P_f. On the other hand under Bayesian minimum probability of error optimality one would choose τ to minimize $P_e = p_0 P_f + p_1 P_m$. Evaluating the decentralized detection performance of the detector (10) requires the knowledge of the probability distribution of the decision statistic T. Following standard techniques it is straightforward to show that T can be written as

$$T(\mathbf{r}) = \sum_{k=1}^{N_s} |\bar{r}_k|^2 \tag{12}$$

where $\bar{r}_k$'s are a collection of independent (but, in general, not identical), Gaussian random variables having the following distributions under the two hypotheses:

$$\begin{aligned} H_0: \quad & \bar{r}_k \sim \mathcal{N}\left(0, \bar{\sigma}_{0,k}^2\right) \\ H_1: \quad & \bar{r}_k \sim \mathcal{N}\left(0, \bar{\sigma}_{1,k}^2\right). \end{aligned}$$

It can be shown that, for $j = 0, 1$

$$\bar{\sigma}_{j,k}^2 = \begin{cases} \frac{g^2 \lambda_k}{\sigma_n^2 + g^2 \lambda_k} & \text{if } j = 0 \\ \frac{g^2 \lambda_k}{\sigma_n^2} & \text{if } j = 1 \end{cases}, \tag{13}$$

where λ_k's are the eigenvalues of the signal covariance matrix Σ_x. It is known that for a general covariance matrix Σ_x, the distribution of T in (12) may not be determined in closed form. However, it turns out that in the special case of $\Sigma_x = \sigma_x^2 \mathbf{I}$, the decision variable T is a Gamma random variable of the form of $T \sim G\left(\frac{N_s}{2}, \frac{1}{2\bar{\sigma}_j^2}\right)$ under the hypotheses H_j where, for $j = 0, 1$

$$\bar{\sigma}_j^2 = \begin{cases} \frac{g^2 \sigma_x^2}{\sigma_w^2 + g^2(\sigma_x^2 + \sigma_v^2)} & \text{if } j = 0 \\ \frac{g^2 \sigma_x^2}{\sigma_w^2 + g^2 \sigma_v^2} & \text{if } j = 1 \end{cases}. \tag{14}$$

For this independent and identical sensor observations case, it can be shown that the false-alarm P_f and the miss P_m probabilities of the detector (10) are given by

$$P_f = P(H_1|H_0) = 1 - \frac{\Gamma\left(\frac{N_s}{2}; \frac{\tau'}{2\bar{\sigma}_0^2}\right)}{\Gamma\left(\frac{N_s}{2}\right)}, \tag{15}$$

and

$$P_m = P(H_0|H_1) = \frac{\Gamma\left(\frac{N_s}{2}; \frac{\tau'}{2\bar{\sigma}_1^2}\right)}{\Gamma\left(\frac{N_s}{2}\right)}, \tag{16}$$

where $\bar{\sigma}_0^2$ and $\bar{\sigma}_1^2$ are given by (14), $\Gamma(a) = \int_0^\infty e^{-y} y^{a-1} dy$ is the Gamma function and $\Gamma(a,t) = \int_0^t e^{-y} y^{a-1} dy$ is the incomplete Gamma function. In addition, the threshold τ' in (11) simplifies to $\tau' = N_s \log\left(\tau^2\left(1+\frac{g^2\sigma_x^2}{\sigma_n^2}\right)\right)$.

Note that (15) and (16) that characterize the decentralized detection performance in a wireless sensor network subjected to a total average power constraint look very similar to well-known centralized system case. However, as we will show now the actual performance in this case is different from that case. In particular, in a centralized system, usually increasing the number of observations improves the final performance. This was also shown to be true in a power-constrained decentralized detection system in [1] in the case of deterministic signal detection. However, this trend does not hold in the case of stochastic signal detection. To show this fact, let us consider minimum probability of error criterion with equal priors so that $\tau' = \tau'_m = N_s \log\left(1+\frac{g^2\sigma_x^2}{\sigma_n^2}\right)$. Then,

$$\tau'_{m,\infty} \;=\; \lim_{N_s \longrightarrow \infty} \tau'_m \;=\; \frac{P/\sigma_w^2}{0.5+(\sigma_x^2/\sigma_v^2)^{-1}}, \quad (17)$$

and thus

$$P_{e,\infty} \;=\; \lim_{N_s \longrightarrow \infty} P_e \;=\; 0.5. \quad (18)$$

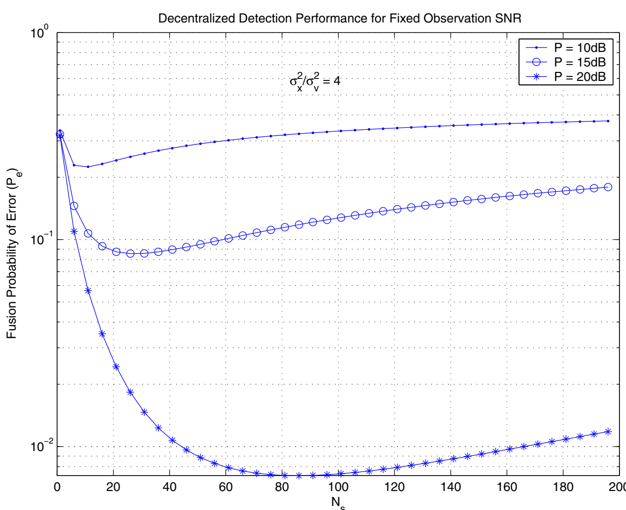

Fig. 2. Decentralized Detection Performance of a Stochastic Gaussian Signal in a Sensor Network Subjected to a Total Power Constraint for a Fixed Observation SNR $\frac{\sigma_x^2}{\sigma_v^2}$.

Figure 2 shows the fusion center probability of error P_e as a function of the sensor network size N_s for a fixed observation signal-to-noise ratio $\frac{\sigma_x^2}{\sigma_v^2}$. It is clear from Fig. 2 that there is an optimal number of sensor nodes for which the error probability is minimized for a given power constraint P. In particular, as predicted by (18) the error probability goes to 0.5 for asymptotically large values of N_s (note that, although this is not clear from Fig. 2 for large values of P, if we increase N_s significantly then P_e indeed tends to 0.5). Moreover, the optimal number of sensors is a function of the total average power constraint P. As P increases, the optimal number of sensors increases and the minimum achievable fusion error probability decreases. In other words, there is a minimum received SNR requirement for each node for it to make a useful contribution at the fusion center. When P is large we can divide the available power among a larger number of sensors still maintaining this minimum SNR requirement. However, once we attempt to increase the number of sensors beyond this point, the performance again starts to degrade since the observations at the fusion center is not of sufficient quality.

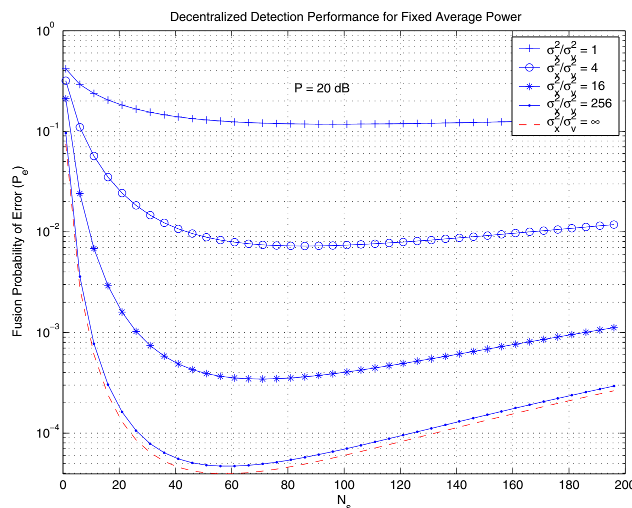

Fig. 3. Decentralized Detection Performance of a Stochastic Gaussian Signal in a Sensor Network Subjected to a Fixed Total Power Constraint P.

In Fig. 3, we have shown the fusion center probability of error for a fixed power constraint P but for different observation SNR values. Figure 3 shows that for a fixed total power constraint, the performance in general is improved as the observation SNR $\frac{\sigma_x^2}{\sigma_v^2}$ improves. This is to be expected intuitively. However, Fig. 3 also shows that for each observation SNR value again there is an optimal number of sensors. In particular, as the observation SNR improves the value of this optimal number of sensors decreases (compare this with fixed observation SNR and increasing total power constraints in Fig. 2).

Although in general the performance improves with improving observation SNR, Fig. 3 further shows that ultimately the performance for asymptotically large observation SNR values is limited by the total power constraint. In particular, we can show that in the Bayesian minimum probability of error case with equal priors, as $\frac{\sigma_x^2}{\sigma_v^2} \longrightarrow \infty$, the fusion center error performance is given by

$$\begin{aligned}\tilde{P}_e &= \lim_{\frac{\sigma_x^2}{\sigma_v^2}\longrightarrow\infty} P_e \\ &= \frac{1}{2}\left(1 - \frac{\Gamma\left(\frac{N_s}{2};t_0\right) - \Gamma\left(\frac{N_s}{2};t_1\right)}{\Gamma\left(\frac{N_s}{2}\right)}\right), \quad (19)\end{aligned}$$

where $t_0 = t_1 + \frac{N_s}{2}\log\left(1+\frac{P/\sigma_w^2}{N_s/2}\right)$ and

$$t_1 = \frac{N_s^2}{4P/\sigma_w^2}\log\left(1+\frac{P/\sigma_w^2}{N_s/2}\right). \quad (20)$$

Figure 3 includes this large observation SNR error probability asymptotic given by (19) as well. Indeed from Fig. 3 we can see that although for large observation SNR the performance improves asymptotically it finally converges to (19). As a result the optimal number of sensors nodes also converges to the value of N_s which minimizes (19).

IV. Conclusions

In this paper we analyzed the performance of decentralized detection of a stochastic signal in a total power constrained wireless sensor network. Assuming amplify-and-relay local processing, the fusion performance is derived in closed-form under both Bayesian and Neyman-Pearson optimality, in the case of conditionally independent signal samples. An important observation is that the average fusion probability of error does not improve monotonically with the number of sensors as in the case of deterministic signal detection reported in [1]. In particular, there is an optimal number of sensors that minimizes the probability of error, which depends on both observation signal-to-noise ratio (SNR) as well as channel SNR. For a fixed observation SNR, this optimal number of nodes increases with increasing total available power. In other words, there is a minimum received SNR requirement for a node to make a useful contribution at the fusion center. However, for a fixed total available power constraint the optimal number of sensor nodes decreases with increasing observation SNR and converge to a value that is a function of P. This can be understood as when local decisions (amplified observations) are of good quality it is better to send a smaller number of those good decisions to the fusion center with a high degree of accuracy.

References

[1] J. Chamberland and V. V. Veeravalli, "Decentralized detection in wireless sensor systems with dependent observations," in *Proc. 2nd Intl. Conf. on Computing, Commun. and Contrl. Technologies (CCCT04)*, Austin, TX, USA, Aug. 2004.

[2] R. R. Tenney and N. R. Sandell Jr., "Detection with distributed sensors," *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-17, no. 4, pp. 501–510, July 1981.

[3] P. K. Varshney, *Distributed Detection and Data Fusion*. New York, NY: Springer-Verlag, 1996.

[4] Z. Chair and P. K. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-22, pp. 98–101, Jan. 1986.

[5] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors: Part I - Fundamentals," *Proc. of the IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.

[6] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors: Part II - Advanced topics," *Proc. of the IEEE*, vol. 85, no. 1, pp. 64–79, Jan. 1997.

[7] J. Chamberland and V. V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE Journal on Select. Areas in Commun.*, vol. 22, no. 6, pp. 1007 – 1015, Aug. 2004.

[8] S. Appadwedula, V. V. Veeravalli, and D. L. Jones, "Energy-efficient detection in sensor networks," *IEEE Journ. Select. Areas in Commun.*, vol. 23, pp. 693–702, April 2005.

[9] S. K. Jayaweera, "Large sensor system performance of decentralized detection in noisy, bandlimited channels," in *Proc. IEEE Veh. Technol. Conf. (VTC05 Spring)*, Stockholm, Sweden, May. 2005, available at http://webs.wichita.edu/?u=cspgroup&p=/publication/.