Large System Performance of Power-constrained Distributed Detection with Analog Local Processing

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***Abstract*— The problem of decentralized detection in a sensor network subjected to a total average power constraint is investigated assuming that local sensor decisions are based on analog relay amplifier processing. For both deterministic and stochastic Gaussian signals, asymptotic performance in a large sensor system is analyzed by deriving the error exponents. The large system analysis shows that, in the case of deterministic signals when the whole system is subjected to a total average power constraint it is better to combine as many not-so-good local decisions as possible rather than relying on a few very-good local decisions. However, in the case of stochastic signals there is an optimal number of local sensor decisions that should be combined at the fusion center in order to obtain the best performance. While up to this optimal limit it is better to divide the power among many not-so-good local decisions, beyond this value the performance starts to degrade as we increase the number of sensors. In other words, in the case of distributed detection of a stochastic signal there is a minimum power level that each node needs to maintain if they are to make a useful contribution to the final fusion decision.**

I. Introduction

In this paper we consider decentralized detection in a power-constrained large wireless sensor network in the presence of a noisy communication channel. Although there is a considerable amount of previous work on the subject of distributed detection and data fusion, mostly they ignore the effect of noise in the channel between the local sensors and data fusion center. However, recently there has been a great interest in distributed detection under communication constraints in the context of wireless sensor networks [1]–[4].

An important design objective in low-power wireless sensor systems is to extend the whole network lifetime. Hence, a sensible constraint on the sensor system is a finite total power as proposed in [2]. This assumption implies that as number of nodes in the network increases, the power available to individual nodes decreases allowing to trade off individual node power against the number of nodes in the network and vice versa. In [2], the performance of distributed detection of a deterministic signal was considered in such a power-constrained wireless sensor network. Assuming the same power constraint and the deterministic signal model it was shown in [4] that even if the communication channel is band limited, it is better to combine as many local decisions as possible at the fusion center rather than dividing the available power among only a few nodes.

In this paper we consider a wireless sensor network subjected to a total power constraint and investigate the distributed detection of both deterministic as well as stochastic Gaussian signals. We first analyze the finite-size system performance. However, our main contribution is to characterize the large system asymptotic performance via error exponents. We derive error exponents for both Bayesian and Neyman-Pearson distributed detection in generally correlated noise. One of the important conclusions that follow from the error exponent analysis is that although in the case of deterministic signals it is better to divide the available power among as many nodes as possible this is not the case with stochastic signals. In this case, there is an optimal number of sensor nodes among which one should divide the total available power. Up to this optimal value it is better to divide the available power among many nodes. However, once we attempt to divide power among nodes beyond that limit the performance again starts to degrade.

The remainder of the paper is organized as follows: In Section II we present the sensor system model and formulate the decentralized detection and fusion problem. Next, in Section III we analyze the performance in the case of deterministic signals and derive the large system error exponents. Section IV derives the corresponding large system error exponent expressions in the case of stochastic signals. In Sections III and IV we also present numerical examples to validate our error exponent analysis and discuss design guidelines that emerge from our analysis. Finally, in Section V we summarize our findings.

II. System Model Description

We consider a binary hypothesis testing problem in an n-node wireless sensor network connected to a data fusion center via distributed parallel architecture. Let us denote by H_0 and H_1 the null and alternative hypotheses, respectively, having corresponding prior probabilities $P(H_0) = p_0$ and $P(H_1) = p_1$. Under the alternative hypothesis the observed stochastic process consists of a Gaussian signal, denoted by X_j, for $j = 1, \cdots n$, corrupted by additive Gaussian noise. The j-th local sensor observation z_j, for $j = 1, \cdots n$, can be written as

$$\begin{aligned} H_0: \quad & z_j = v_j \\ H_1: \quad & z_j = X_j + v_j \end{aligned} \qquad (1)$$

where observation noise v_j and the desired signal X_j are both assumed to be Gaussian with the collection of noise samples

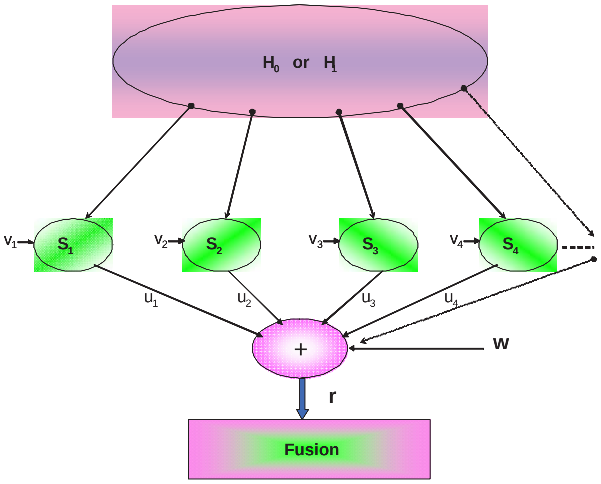


Fig. 1. Decentralized Detection and Data Fusion in a Sensor Network Subjected to a Total Power Constraint in a Noisy Channel

distributed as $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \Sigma_v)$. The collection of desired signal samples $\mathbf{X}$ is in general modeled as $\mathbf{X} \sim \mathcal{N}(\mathbf{m}, \Sigma_x)$ where $\mathbf{m} = m[1, 1, \cdots, 1]^T = m\mathbf{e}$ with $\mathbf{e}$ being an n-vector of all ones.

Each local sensor processes its observation z_j independently to generate a local decision $u_j(z_j)$ which are sent to a fusion center. Let us denote by $\mathbf{r}(u_1(z_1), u_2(z_2), \cdots, u_n(z_n))$ the received signal at the fusion center. The fusion center makes a final decision based on the decision rule $u_0(\mathbf{r})$. The problem at hand is to choose $u_0(\mathbf{r}), u_1(z_1), u_2(z_2), \cdots, u_n(z_n)$ so that a chosen performance metric is optimized.

The solution to this problem is known to be too complicated under the most general conditions [5]. While optimal local processing schemes have been derived in certain special cases, a class of especially important local processers are those that simply amplify the observations before retransmission to the fusion center [2]. In this case the local sensor decisions sent to the fusion center are given by,

$$u_j \quad = \quad g_j z_j \qquad \text{for } j = 1, \cdots n$$

where $g_j > 0$ is the analog relay amplifier gain at the j-th node that depends on the total power constraint P on the whole sensor system. It has been shown that such simple amplify-and-relay local processing performs fairly well when the local observations are corrupted by additive noise, as in our formation [1]. Moreover, this type of amplify-and-relay local processing seems to be well-suited for low-power wireless sensor networks that are becoming popular.

We assume that these analog relay amplifier processed local decisions u_j's are transmitted to the fusion center over a noisy wireless channel. Assuming, for simplicity, that $g_j = g$, for all j and the sensor-to-fusion center communication is orthogonal, the received signal from all the nodes can be written in vector notation as

$$\mathbf{r} \quad = \quad g\mathbf{z} + \mathbf{w} \tag{2}$$

where $\mathbf{r}$ and $\mathbf{w}$ are n-dimensional received vector and the receiver noise, respectively. The receiver noise is assumed to be a white Gaussian noise process so that the noise vector $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}_n)$.

The fusion problem can be reduced to the following hypothesis testing problem:

$$\begin{array}{lccc} H_0: & \mathbf{r} & \sim & p_0(\mathbf{r}) \\ H_1: & \mathbf{r} & \sim & p_1(\mathbf{r}) \end{array}$$

where p_i is the density of $\mathbf{r}$ under the hypothesis H_i, for $i = 0$ and $i = 1$. It is well known that the optimal decision rules for the above fusion center design should then be based on the likelihood ratio $\mathcal{L}(\mathbf{r}) = \frac{p_1(\mathbf{r})}{p_0(\mathbf{r})}$.

When considering correlated observations (either due to noise or signal correlations) we always assume that the correlations are a function of distance between the nodes. For simplicity, we consider the one dimensional sensor network model as in [2]. According to this model the sensors are equi-spaced along a straight line with distance between any two adjacent nodes being d as shown in Fig. 2.

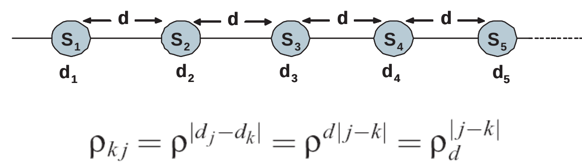


Fig. 2. One-dimensional Sensor Network Model. $\rho_d = \rho^d$

If we denote the sensor locations as $d_1, d_2, \cdots, d_n$, then $d_j = (j-1)d$. The spatial correlation coefficient ρ_{kj} between the k-th and j-th sensor nodes is then given by

$$\rho_{kj} \quad = \quad \rho^{|d_j - d_k|} \quad = \quad \rho^{d|j-k|} \quad = \quad \rho_d^{|j-k|} \tag{3}$$

where we have defined $\rho_d = \rho^d$. With this model the normalized correlation matrix can be written as

$$R \quad = \quad \begin{bmatrix} 1 & \rho_d & \rho_d^2 & \cdots & \rho_d^{n-1} \\ \rho_d & 1 & \rho_d & \cdots & \rho_d^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_d^{n-1} & \rho_d^{n-2} & \rho_d^{n-3} & \cdots & 1 \end{bmatrix}. \tag{4}$$

III. Distributed Detection of a Deterministic Signal in a Large Sensor System

Suppose that the signal to be detected is a deterministic signal as in [2]. Specifically, let $X_j = m$ under H_1, so that

$$\begin{array}{lccc} H_0: & p_0(\mathbf{r}) & = & \mathcal{N}(\mathbf{0}, \Sigma_n) \\ H_1: & p_1(\mathbf{r}) & = & \mathcal{N}(gm\mathbf{e}, \Sigma_n) \end{array}$$

where $\Sigma_n = g^2 \Sigma_v + \sigma_w^2 \mathbf{I}$ is the noise covariance at the fusion center. We can easily show that in this case the total power constraint $P = \sum_{j=1}^{n} \mathbb{E}\left\{|u_j|^2\right\}$ leads to

$$g^2 \quad = \quad \frac{P}{n\left(\sigma_v^2 + \frac{m^2}{2}\right)}, \tag{5}$$

where σ_v^2 is the variance of the noise sample v_j, for $j = 1, \cdots, n$. The optimal tests are then of the form of

$$\delta_{opt}(\mathbf{r}) = \begin{cases} 1 & \geq \\ \quad \text{if} \quad m\mathbf{e}^T\Sigma_n^{-1}\mathbf{r} & \tau' \\ 0 & < \end{cases},$$

where $\tau' = \frac{\log(\tau)}{g} + \frac{gm^2}{2}\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}$ and τ is the original threshold that will depend on the exact optimality criteria. For example, $\tau = 1$ for minimum probability of error Bayes detection (equal priors), and in an ϵ-level Neyman-Pearson design it is determined by ϵ.

The performance of a binary hypothesis testing problem can be specified via the false-alarm probability α_n and the miss probability β_n. For the above decentralized detection of a deterministic signal in noise they become:

$$\alpha_n = Q\left(\frac{\log(\tau) + \frac{g^2m^2}{2}\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}}{g\sqrt{\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}}}\right), \quad (6)$$

$$\beta_n = Q\left(\frac{\frac{g^2m^2}{2}\mathbf{e}^T\Sigma_n^{-1}\mathbf{e} - \log(\tau)}{g\sqrt{\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}}}\right). \quad (7)$$

While it is possible to evaluate the performance for specific systems via (6) and (7), it is rather difficult to draw general conclusions from them regarding the design of decentralized detection systems. In order to facilitate this we investigate the asymptotic behavior of the above performance via error exponents. The required error exponent for Bayesian detection is the Chernoff information as given by the following theorem [6]:

Theorem 1 (Chernoff Information): The best error exponent achievable in Bayesian testing is given by the negative of the Chernoff information and is defined as

$$\mu^* = \min_{s\in[0,1]} \log \mathbb{E}\left\{\mathcal{L}^s(\mathbf{r})|H_0\right\}.$$

For the deterministic signal above, we can show that

$$\mu^* = -\frac{g^2m^2}{8}\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}. \quad (8)$$

Similarly, in the case of Neyman-Pearson hypothesis testing the asymptotic error exponent of the miss probability is given by the Stein's lemma [6]:

Theorem 2 (Stein's Lemma): The best achievable error exponent β_n^ϵ if $\alpha_n \leq \epsilon$ is

$$\lim_{\epsilon\longrightarrow 0}\lim_{n\longrightarrow\infty} \frac{1}{n}\log\beta_n^\epsilon = -D\left(p_0||p_1\right)$$

where $D\left(p_0||p_1\right)$ is the Kulback-Leibler measure or the relative entropy of p_0 and p_1.

It is easily seen that for the above deterministic signal

$$D\left(p_0||p_1\right) = \frac{g^2m^2}{2}\mathbf{e}^T\Sigma_n^{-1}\mathbf{e}. \quad (9)$$

As we see from (8) and (9), in the case of deterministic signals the Chernoff information and the KL measure are proportional to each other. Thus, in investigating the asymptotic distributed detection performance in a large sensor system we only consider one of them.

A. Deterministic Signal in Uncorrelated Noise

From (9) it follows straightforwardly that when the noise is uncorrelated $D\left(p_0||p_1\right) = \gamma_0\left(\frac{2}{n} + \frac{2+\gamma_0}{\gamma_c}\right)^{-1}$, where we have defined observation SNR as $\gamma_0 \triangleq \frac{m^2}{\sigma_v^2}$ and channel SNR as $\gamma_c \triangleq \frac{P}{\sigma_w^2}$. For completeness, the asymptotic performance of the deterministic signal detection under a total power constraint is summarized in the following proposition (the proof is omitted since it is a result of simple limit properties):

Proposition 1: The error exponent for the decentralized detection of a deterministic signal in uncorrelated noise has following asymptotic properties:

$$\lim_{n\longrightarrow\infty} D\left(p_0||p_1\right) = \frac{\gamma_c}{1 + \frac{2}{\gamma_0}}$$

$$\lim_{\gamma_0\longrightarrow\infty} D\left(p_0||p_1\right) = \gamma_c,$$

$$\lim_{\gamma_c\longrightarrow\infty} D\left(p_0||p_1\right) = \frac{n}{2}\gamma_0.$$

B. Deterministic Signal in Correlated Noise

Next we consider the situation with correlated observation noise. We assume that the noise $\mathbf{v}$ has a covariance matrix $\Sigma_v = \sigma_v^2 R$. Since $\Sigma_n = g^2\Sigma_v + \sigma_w^2\mathbf{I}$, in this case it is not directly clear from (9) how the asymptotic performance depends on the parameters ρ_d, γ_0, γ_c and the sensor network size n. However, as we show below, an elegant representation of the asymptotic error exponent can be obtained once we consider a large sensor network. Note that, due to the assumed spatial correlation structure of the noise, Σ_v is a Hermitian and toeplitz matrix. Then, Σ_n is also Hermitian and toeplitz. It is well known that for large n, the Hermitian, Toeplitz matrix Σ_n is uniquely determined by the sequence $t_j = g^2\sigma_v^2\left\{1 + \frac{\sigma_w^2}{g^2\sigma_v^2}, \rho_d, \rho_d^2, \rho_d^3, \cdots,\right\}$. Equivalently, for large n we may define $\Sigma_n = \Sigma_n(f)$ via the function f defined as $f(\omega) = \sum_{j=-\infty}^{\infty} t_j e^{ij\omega}$. The following theorem establishes the equivalence of a toeplitz matrix to a circulant matrix when the matrix size is large [7].

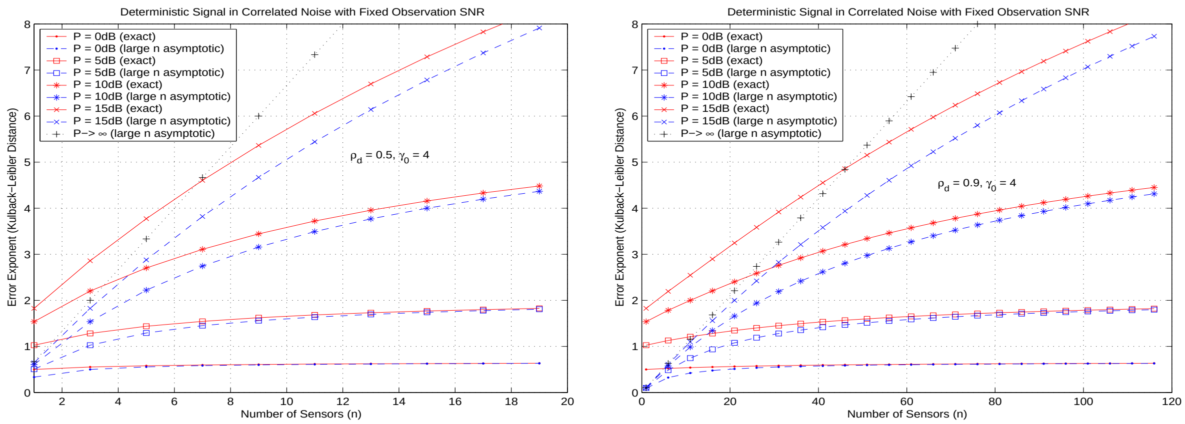
Theorem 3: For large n, $\Sigma_n(f)$ is equivalent (in the weak norm) to a circulant $C_n(f)$ matrix where the entries $c_j^{(n)}$ of $C_n(f)$ are given by $c_j^{(n)} = \frac{1}{n}\sum_{k=0}^{n-1}\lambda_{n,k}e^{i\frac{2\pi j}{n}k}$, and the eigenvalues $\lambda_{n,j}$ of $C_n(f)$ are equal to $\lambda_{n,j} = f\left(\frac{2\pi}{n}j\right)$.

In order to obtain the error exponents of a power-constrained distributed detection system in correlated observation noise we first note that with the assumed noise model

$$f(\omega) = g^2\sigma_v^2 + \sigma_w^2 + 2g^2\sigma_v^2\rho_d\frac{\cos\omega - \rho_d}{1 - 2\rho_d\cos\omega + \rho_d^2}.$$

In the following proposition we summarize the large system error exponent and the asymptotic performance in the presence of correlated observation noise.

Proposition 2: The large system error exponent for the decentralized detection of a deterministic signal in correlated



(a) (b)

Fig. 3. Error Exponent for the Decentralized Detection of a Deterministic Signal in Correlated Noise

noise is given by

$$D\left(p_0||p_1\right) \sim \frac{P\gamma_0}{2f(0)\left(1+0.5\gamma_0\right)}$$
$$= \quad 0.5\gamma_0\left(\left(1+0.5\gamma_0\right)\left(\frac{1}{\gamma_c}+\frac{1}{n\left(1+0.5\gamma_0\right)}\frac{1+\rho_d}{1-\rho_d}\right)\right)^{-1}.$$

The above $D\left(p_0||p_1\right)$ has the following asymptotic properties:

$$\lim_{n\longrightarrow\infty} D\left(p_0||p_1\right) = \frac{\gamma_c}{1+\frac{2}{\gamma_0}}$$
$$\lim_{\gamma_0\longrightarrow\infty} D\left(p_0||p_1\right) = \gamma_c$$
$$\lim_{\gamma_c\longrightarrow\infty} D\left(p_0||p_1\right) = \frac{n}{2}\frac{1-\rho_d}{1+\rho_d}\gamma_0.$$

Proof: See Appendix I.

Essentially, both propositions 1 and 2 say that it is better to divide the available total power among as many nodes as possible. For a given power constraint P and observation noise SNR, the performance monotonically improves with the sensor system size n regardless of the fact that observations are correlated or uncorrelated. This behavior is illustrated in Fig. 3. It also shows that the large system analysis based error exponent provides a very good approximation to the exact error exponent of a finite size system unless n is very small. More importantly, however, Fig. 3 shows that for a fixed number of nodes the performance is limited by correlation ρ_d. However, as can be seen by comparing Figs. 3a and 3b, any performance level achieved by a system with less correlated observations can also be achieved in a system with highly correlated observations by increasing sensor network size n.

Propositions 1 and 2 also assert that ultimately the asymptotic performance in a large system will be limited by channel and observation signal qualities. When the channel is very reliable the performance is monotonic in both n and observation quality γ_0. On the other hand, when the local decisions are very good, the final performance will be limited only by the channel quality γ_c.

The key conclusion that follows from the above large system analysis is that in the case of deterministic signals the available power should be divided among as many nodes as possible no matter whether the observations are correlated or not.

IV. Distributed Detection of a Stochastic Signal in a Large Sensor System

Next, let us assume that the signal X_j, for $j = 1, \cdots, n$, to be detected is a zero-mean Gaussian signal such that $\mathbf{X} \sim \mathcal{N}(\mathbf{0}, \Sigma_x)$ and that the collection of noise samples is distributed as $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \sigma_v^2\mathbf{I})$. Again we assume that the sensors are d-spaced in a 1-D network, and that the signal field correlations are a function of distance leading to a signal covariance matrix of the form $\Sigma_x = \sigma_x^2 R$, where, as before, $\rho_d = \rho^d$.

With orthogonal communication, the detection problem at the fusion center is then equivalent to:

$$\begin{aligned} H_0: \quad p_0(\mathbf{r}) &= \mathcal{N}\left(\mathbf{0}, \Sigma_n\right) \\ H_1: \quad p_1(\mathbf{r}) &= \mathcal{N}\left(\mathbf{0}, g^2\Sigma_x + \Sigma_n\right) \end{aligned} \qquad (10)$$

where $\Sigma_n = g^2\Sigma_v + \sigma_w^2\mathbf{I} = (g^2\sigma_v^2 + \sigma_w^2)\mathbf{I} = \sigma_n^2\mathbf{I}$, and now

$$g^2 = \frac{P}{n\left(\sigma_v^2 + \frac{\sigma_x^2}{2}\right)}.$$

For this case we re-define the observation SNR as $\gamma_0 = \frac{\sigma_x^2}{\sigma_v^2}$. It follows that the optimal procedures for the problem (10) should be based on the following likelihood ratio:

$$\mathcal{L}(\mathbf{r}) = \frac{1}{2}\log\frac{|\Sigma_n|}{|\Sigma_1|}\exp\left(\frac{1}{2}\mathbf{r}^T\left(\Sigma_n^{-1} - \Sigma_1^{-1}\right)\mathbf{r}\right),$$

where, for brevity, we have defined $\Sigma_1 = g^2\Sigma_x + \Sigma_n$.

In the case of correlated stochastic signals it is well-known that we cannot evaluate the exact error probabilities in closed form [8]. However, we can obtain easy to evaluate error exponents in the case of $\Sigma_x = \sigma_x^2 R$ via large system analysis. In the following proposition we summarize the large system Kulback-Leibler distance based error exponent for the Neyman-Pearson detection of a stochastic Gaussian signal under a power constraint.

Proposition 3: The best error exponent achievable in distributed Neyman-Pearson detection of a correlated stochastic signal is given by

$$D\left(p_0 || p_1\right) = \frac{n}{2}\left[K_0 - \sigma_1^2\left(\left(\sigma_1^2 + \frac{(1-\rho_d)}{(1+\rho_d)}\right)\left(\sigma_1^2 + \frac{(1+\rho_d)}{(1-\rho_d)}\right)\right)^{-1/2}\right]$$

where $\sigma_1^2 = \gamma_0\left[1 + n\left(1 + 0.5\gamma_0\right)/\gamma_c\right]^{-1}$ and $K_0 = \frac{1}{2\pi}\int_0^{2\pi} \log\left(1 + \frac{(1-\rho_d^2)\sigma_1^2}{1-2\rho_d\cos\omega+\rho_d^2}\right) d\omega$. In particular,

$$\lim_{n\longrightarrow\infty} D\left(p_0 || p_1\right) = 0 \quad \text{for} \quad |\rho_d| < 1$$

Proof: See Appendix I.

We may also obtain the large system error exponent for Bayesian distributed detection of a stochastic signal as below:

Proposition 4: The best error exponent achievable in distributed Bayesian detection of a correlated stochastic signal is given by

$$\mu^* = \frac{n}{2}\left[\frac{1}{2\pi}\int_0^{2\pi} \log \frac{1 - 2\rho_d\cos\omega + \rho_d^2 + (1-\rho_d^2)\sigma_1^2}{1 - 2\rho_d\cos\omega + \rho_d^2 + (1-s_0)(1-\rho_d^2)\sigma_1^2} d\omega - s_0 K_0\right]$$

where $\sigma_1^2 = \gamma_0\left[1 + n\left(1 + 0.5\gamma_0\right)/\gamma_c\right]^{-1}$ and

$$s_0 = 1 + \left(\frac{1+\rho_d^2}{1-\rho_d^2}\right)\frac{1}{\sigma_1^2} - \sqrt{\frac{4\rho_d^2}{(1-\rho_d^2)^2\sigma_1^4} + \frac{1}{K_0^2}}$$

with $K_0 = \frac{1}{2\pi}\int_0^{2\pi} \log\left(1 + \frac{(1-\rho_d^2)\sigma_1^2}{1-2\rho_d\cos\omega+\rho_d^2}\right) d\omega$.

Proof: The key steps of the proof involves essentially the same technical details as in that of proposition 3. Hence, we omit the proof which can be found in [9].

We may use propositions 3 and 4 to obtain the corresponding error exponents for the special case of uncorrelated stochastic signal for which one can compute the exact performance as shown in [9]. In this case, a considerable simplification can be achieved leading to the following corollary:

Corollary 1: From propositions 3 and 4, the best error exponent achievable in distributed Bayesian detection of an uncorrelated stochastic signal is given by

$$\mu^* = \frac{n}{2}\left[\log\frac{1+\sigma_1^2}{1+(1-s_0)\sigma_1^2} - s_0\log\left(1+\sigma_1^2\right)\right],$$

where $\sigma_1^2 = \gamma_0\left[1 + n\left(1 + 0.5\gamma_0\right)/\gamma_c\right]^{-1}$ and $s_0 = 1 + \frac{1}{\sigma_1^2} - \frac{1}{\log\left(1+\sigma_1^2\right)}$. Similarly, the best error exponent achievable in Neyman-Pearson detection is given by

$$D\left(p_0 || p_1\right) = \frac{n}{2}\left[\log\left(1+\sigma_1^2\right) - \frac{\sigma_1^2}{1+\sigma_1^2}\right].$$

The decentralized detection performance characterization given in corollary 1 for a wireless sensor network subjected to a total average power constraint looks very similar to well-known centralized system case. However, a closer look shows that the actual performance in this case is very different from that of the centralized case. In particular, in a centralized system, usually increasing the number of observations improves the final performance. This was also true in a power-constrained decentralized detection system in in the case of deterministic signal detection considered in [2] and as we saw earlier via large system error exponents. Surprisingly this trend does not hold in the case of stochastic signals. This has already been observed in [9] as well. In fact, [9] showed that for a fixed total power constraint, when the system size n is very large the fusion center performance approaches 0.5. In other words, the detection system becomes useless.

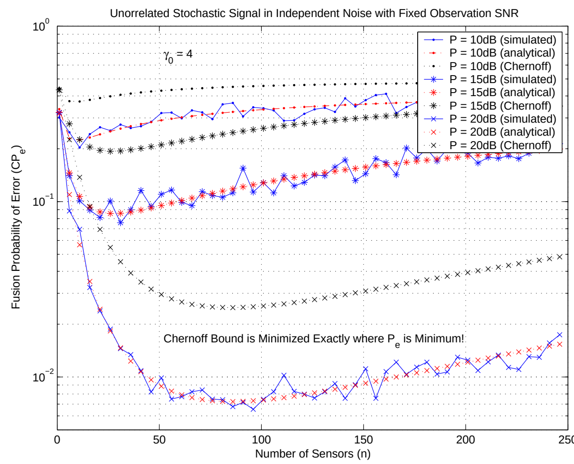


Fig. 4. Minimum Achievable Probability of Error for Distributed Detection of an Uncorrelated Gaussian Signal under a Total Power Constraint. $\gamma_0 = 4$

Figure 4 shows the fusion center probability of error P_e as a function of the sensor network size n for a fixed observation SNR γ_0. It is clear from Fig. 4 that there is an optimal number of sensor nodes for which the error probability is minimized for a given power constraint P. In particular, as predicted in [9], the error probability goes to 0.5 for asymptotically large values of n. The optimal number of sensors is a function of the total power constraint P. As P increases, the optimal number of sensors increases and the minimum achievable error probability decreases. In other words, there is a minimum received SNR requirement for each node for it to make a useful contribution at the fusion center. When P is large we can divide the available power among a larger number of sensors while maintaining this minimum SNR requirement. However, once we increase the number of sensors beyond this point the performance again starts to degrade since the quality of observations at the fusion center is not sufficient.

As a verification of the general performance predicted in proposition 3 by the large system analysis, and its special case in corollary 1, Fig. 4 also includes the Chernoff information based bound for the fusion center error probability. Note that, the large system analysis based error exponent follows the same performance trend as the exact error probability. In particular, the error exponent is also optimized for a specific value of n. In fact, a close investigation of Fig. 4 shows that the large system analysis based error exponent in Fig. 4 is optimized exactly at the same n value at which the exact probability of error is minimized. This provides a useful way to design power-constrained sensor systems by optimizing the error exponent performance.

The fusion center probability of error for a fixed power constraint P but for different observation SNR values is shown in Fig. 5. While Fig. 5 shows that for a fixed total

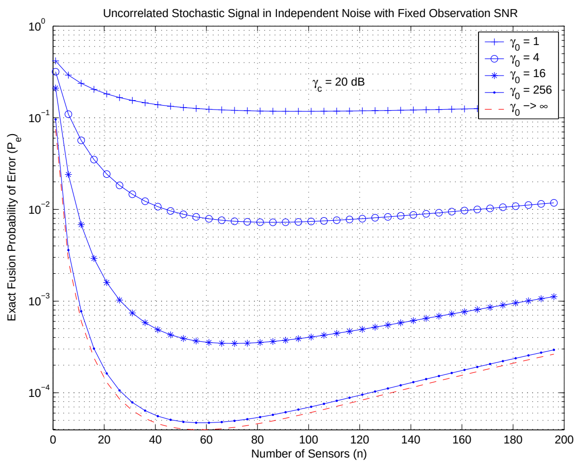


Fig. 5. Minimum Achievable Probability of Error for Distributed Detection of an Uncorrelated Gaussian Signal under a Total Power Constraint. $\gamma_c = 20dB$

power constraint the performance in general is improved as the observation SNR γ_0 improves, it also shows that for each observation SNR value again there is an optimal number of sensors. In particular, as the observation SNR improves the value of this optimal number of sensors decreases. This behavior implies that when local decisions are more reliable, we can achieve a given level of performance by dividing the available total power among a less number of sensor nodes. Figure 5 also shows that ultimately the performance for asymptotically large observation SNR values is limited by the total power constraint. In particular, for reasonably large γ_0 values P_e indeed converges to its asymptotic value.

V. Conclusions

We analyzed the decentralized detection performance of a total power constrained wireless sensor network in a noisy channel assuming analog relay amplifier local processing. For both deterministic and stochastic Gaussian signals, asymptotic performance in a large sensor system is analyzed by deriving the error exponents. Our large system analysis showed that in the case of deterministic signals it is better to combine as many not-so-good local decisions as possible rather than relying on a few very-good local decisions. However, in the case of stochastic signals we showed that there is an optimal number of local sensor decisions that should be combined at the fusion center in order to obtain the best performance. In other words, in the case of distributed detection of a stochastic signal there is a minimum power level that each node needs to maintain if they are to make a useful contribution to the final fusion decision.

Appendix I

A. Outline of Proof of Proposition 2

Proof: Since $\Sigma_n(f)$ is equivalent to a circulant $C_n(f)$, we also have that $\Sigma_n^{-1}(f)$ is equivalent to the circulant $C_n^{-1}(f)$. The eigenvalues of $C_n^{-1}(f)$ are then given by $\lambda'_{n,j} = \lambda_{n,j}^{-1} = 1/f\left(\frac{2\pi}{n}j\right)$. If the entries of $C_n^{-1}(f)$ are denoted by $d_j^{(n)}$, then the relative entropy is:

$$\begin{aligned} D\left(p_0||p_1\right) &\sim \frac{g^2m^2}{2}\mathbf{e}^T C_n^{-1}(f)\mathbf{e} \\ &= \frac{ng^2m^2}{2}\sum_{j=0}^{n-1} d_j^{(n)} = \frac{ng^2m^2}{2}\sum_{j=0}^{n-1}\left(\frac{1}{n}\sum_{k=0}^{n-1}\lambda'_{n,k}e^{i\frac{2\pi j}{n}k}\right) \\ &= \frac{g^2m^2}{2}\sum_{k=0}^{n-1}\lambda_{n,k}^{-1}\sum_{j=0}^{n-1}e^{i\frac{2\pi j}{n}k} = \frac{ng^2m^2}{2f(0)}. \end{aligned}$$

The rest of the proof follows by substitutions and taking limits.

B. Outline of Proof of Proposition 3

Proof: In this case, $\Sigma_n = \sigma_n^2\mathbf{I}_n$. Let us define $\Sigma_1 = g^2\Sigma_x + \Sigma_n$ and denote by λ_k, for $k = 1,\cdots,n$, the eigenvalues of Σ_1. Then, we have

$$\begin{aligned} D\left(p_0||p_1\right) &= \frac{1}{2}\left(\log\left(\frac{|\Sigma_1|}{|\Sigma_n|}\right) + \text{tr}\left(\Sigma_n\Sigma_1^{-1}\right) - n\right) \\ &= \frac{1}{2}\left(\sum_{k=1}^{n}\log\left(1+\frac{\lambda_k}{\sigma_n^2}\right) - \sum_{k=1}^{n}\frac{\frac{\lambda_k}{\sigma_n^2}}{1+\frac{\lambda_k}{\sigma_n^2}}\right). \quad (11) \end{aligned}$$

But since Σ_1 is a Hermitian and toeplitz matrix with a bounded norm, then it follows that for large n it can equivalently be characterized by the function $g(\omega) = \sum_{j=-\infty}^{\infty} t_j e^{ij\omega} = \frac{g^2\sigma_x^2(1-\rho_d)^2}{1-2\rho_d\cos\omega+\rho_d^2}$. Applying Szego's theorem on the distribution of eigenvalues of large toeplitz matrices to $\Sigma_1 = \Sigma_1(g)$, then for large n we can write (11) as

$$D\left(p_0||p_1\right) \sim \frac{n}{4\pi}\left(\int_0^{2\pi}\log\left(1+\frac{g(\omega)}{\sigma_n^2}\right)d\omega - \int_0^{2\pi}\frac{\frac{g(\omega)}{\sigma_n^2}}{1+\frac{g(\omega)}{\sigma_n^2}}d\omega\right).$$

We evaluate the second integral using $\int_0^{2\pi}\frac{d\omega}{a+b\cos\omega} = \frac{2\pi}{\sqrt{a^2-b^2}}$, and introduce the definitions of K_0 and σ_1^2 as specified. These substitutions and some manipulations results in the Neyman-Pearson error exponent given in proposition 3. The rest of the proof follows by taking the limit.

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