

Performance of Decentralized Detection in a Resource-constrained Sensor Network with Non-orthogonal Communications

Kossai A. Al Tarzai, Sudharman K. Jayaweera and Visvakumar Aravinthan
 Department of Electrical and Computer Engineering
 Wichita State University, Wichita, KS 67260, USA.
 Email: sudharman.jayaweera@wichita.edu

Abstract—The performance of decentralized detection in a resource-limited wireless sensor network is investigated. The system is assumed to be subjected to a total power constraint and all sensors are assumed to perform analog relay-amplifier local processing. Under these assumptions, we show that, we can monotonically improve the fusion performance by using non-orthogonal sensor-to-fusion center communication, as opposed to orthogonal communications. In order to quantify the performance, we employ circulant matrix theory to derive closed-form asymptotic expressions. These asymptotic results allow us to observe the effect of each parameter on the total system performance. Numerical results show that these asymptotics provide a very good approximation to the exact performance, even for a small number of sensors.

Index Terms—Circulant matrix theory, decentralized detection, data fusion, hypotheses testing, sensor networks, toeplitz matrix.

I. INTRODUCTION

Wireless sensor networks is an emerging technology that has several applications in civil as well as in military communications [1]–[4]. Decentralized detection and data fusion arise as basic problems in such networks. In this paper, the performance of decentralized detection in a total power constrained wireless sensor network is investigated assuming analog relay amplifier local processing. In [5], under the same assumptions, an optimal fusion rule and its performance were derived assuming orthogonal sensor-to-fusion center communication. It was observed in [5] that for a fixed total power, there is a performance floor as sensor network size increases without bound. In this paper, we show that with analog relay amplifier processing this, however, is not necessarily the best achievable performance. In fact, we show that with non-orthogonal communication, we can harness the *coherent gain* to our advantage thereby monotonically improving the performance with sensor network size, for a given total power constraint.

In [6], the performance of decentralized detection of a power constrained network in a band-limited channel was analyzed via large matrix theory, assuming independent sensor observations. Our work can be considered to be a generalization of [6] to the case of dependent sensor observations, since non-orthogonal communication implicitly assumes a band-limited channel. Moreover, we derive closed form expressions

for asymptotic fusion performance via circulant matrix theory. Numerical results are used to show that the derived asymptotic expressions closely match that of exact performance even for a relatively small number of sensors.

The remainder of this paper is organized as follows: In

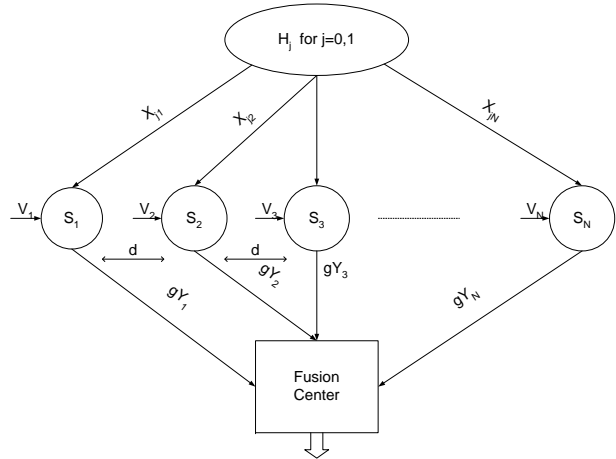


Fig. 1. Distributed Sensor System Model

Section II we present our system model and derive the fusion performance under orthogonal versus general non-orthogonal sensor-to-fusion center communication. Next, in Section III we use circulant matrix theory to obtain a closed form expression for the system asymptotic performance. Finally, in Section IV we provide conclusions of our results.

II. SYSTEM MODEL

We consider a binary hypothesis testing problem, in an N -node distributed sensor system as shown in Fig. 1. The k -th sensor observation, under each of the two hypotheses is given by

$$\begin{aligned} H_0 : y_k &= x_{0,k} + v_k \\ H_1 : y_k &= x_{1,k} + v_k . \end{aligned}$$

In vector notation

$$\mathbf{y} = \mathbf{x} + \mathbf{v} .$$

where \mathbf{v} is a zero mean, Gaussian N -vector of noise samples, with covariance matrix $\Sigma_{\mathbf{v}}$. We assume that sensors are placed on a straight line and are separated by an equal distance d . The noise covariance function is assumed to be of the form of

$$\rho_v(i, j) = \sigma_v^2 \rho^{d|i-j|}.$$

Letting $\rho^d = \rho_d$, $\Sigma_{\mathbf{v}}$ can then be written as

$$\Sigma_{\mathbf{v}} = \sigma_v^2 \begin{bmatrix} 1 & \rho_d & \rho_d^2 & \cdot & \cdot & \cdot & \rho_d^{(N-1)} \\ \rho_d & 1 & \rho_d & \cdot & \cdot & \cdot & \rho_d^{(N-2)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \rho_d^{(N-1)} & \rho_d^{(N-2)} & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}. \quad (1)$$

We consider the fusion of a deterministic signal, so that $x_{0,k} = -m$ under H_0 and $x_{1,k} = m$ under H_1 for $k = 1, \dots, N$. Each sensor node applies analog relay amplifier local processing to its observation by multiplying it by an amplification factor g . Also a signalling waveform (or a code) s_k is assigned for each sensor. The sensors relay these amplified signals to the fusion center after modulating with $s_k(t)$. Assuming a bank of N matched filters at the fusion center, its output is given by

$$\mathbf{u} = \mathbf{R}\mathbf{A}\mathbf{y} + \mathbf{w},$$

where \mathbf{R} is the signalling waveform (code) cross-correlation matrix, $\mathbf{A} = g\mathbf{I}$, and g can be shown to be

$$g = \sqrt{\frac{P}{N(m^2 + \sigma_v^2)}},$$

where P is the total power constraint on the whole sensor system. The channel noise is assumed to be zero mean Gaussian with noise power spectral density σ_w^2 , so that $\mathbf{w} \sim \mathcal{N}(0, \sigma_w^2 \mathbf{R})$. For convenience we assume that the code cross-correlation matrix \mathbf{R} has the form of

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \cdot & \cdot & \cdot & \rho \\ \rho & 1 & \cdot & \cdot & \cdot & \rho \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \rho & \cdot & \cdot & \cdot & \rho & 1 \end{bmatrix}. \quad (2)$$

Under the two hypotheses, it can be shown that

$$H_0: \mathbf{u} \sim \mathcal{N}(\mathbf{R}\mathbf{A}\mathbf{m}_0, g^2 \mathbf{R}\Sigma_{\mathbf{v}}\mathbf{R} + \sigma_w^2 \mathbf{R}),$$

and

$$H_1: \mathbf{u} \sim \mathcal{N}(\mathbf{R}\mathbf{A}\mathbf{m}_1, g^2 \mathbf{R}\Sigma_{\mathbf{v}}\mathbf{R} + \sigma_w^2 \mathbf{R}).$$

Then, the optimal fusion rule is given by the test:

$$\delta(\mathbf{u}) = \begin{cases} 1 & T(\mathbf{u}) \geq \tau \\ 0 & T(\mathbf{u}) < \tau \end{cases},$$

where

$$T(\mathbf{u}) = 2mge^T \mathbf{R}\mathbf{C}^{-1} \mathbf{u},$$

\mathbf{e} is the N -length vector of all ones and we have defined

$$\mathbf{C} = g^2 \mathbf{R}\Sigma_{\mathbf{v}}\mathbf{R} + \sigma_w^2 \mathbf{R}.$$

The minimum achievable probability of error of this optimum fusion rule is given by (assuming a uniform prior)

$$Pe = \frac{1}{2}(\alpha(\tau) + \beta(\tau)),$$

where $\alpha(\tau)$ and $\beta(\tau)$ are the false-alarm and miss probabilities given by

$$\alpha(\tau) = Q\left(\frac{\tau + m^2 g^2 \mathbf{e}^T \mathbf{R}\mathbf{C}^{-1} \mathbf{R}\mathbf{e}}{\sqrt{2m^2 g^2 \mathbf{e}^T \mathbf{R}\mathbf{C}^{-1} \mathbf{R}\mathbf{e}}}\right), \quad (3)$$

and

$$\beta(\tau) = 1 - Q\left(\frac{\tau - m^2 g^2 \mathbf{e}^T \mathbf{R}\mathbf{C}^{-1} \mathbf{R}\mathbf{e}}{\sqrt{2m^2 g^2 \mathbf{e}^T \mathbf{R}\mathbf{C}^{-1} \mathbf{R}\mathbf{e}}}\right). \quad (4)$$

Since $\tau = 0$ for equi-probable deterministic signal detection, from (3) and (4), we can easily show that

$$Pe = Q\left(mg\sqrt{2\mathbf{e}^T \mathbf{R}\mathbf{C}^{-1} \mathbf{R}\mathbf{e}}\right).$$

In the following we first consider the case of $\mathbf{R} = \mathbf{I}$, which represents orthogonal communication, followed by the general non-orthogonal communication. Since our main objective is to explore the effect of non-orthogonal sensor-to-fusion center communication on the fusion performance, for simplicity we first assume the case of independent observations.

A. Orthogonal codes and independent observation noise at the sensors

Consider the case of $\mathbf{R} = \mathbf{I}$ and $\Sigma_{\mathbf{v}} = \sigma_v^2 \mathbf{I}$. The effective SNR at the fusion center can be shown to be equal to

$$SNR_f = mg\sqrt{\mathbf{e}^T (g^2 \sigma_v^2 \mathbf{I} + \sigma_w^2 \mathbf{I})^{-1} \mathbf{e}}.$$

Substituting $\gamma_0 = \frac{m}{\sigma_v^2}$ and $\gamma_c = \frac{P}{\sigma_w^2}$, which represent the observation and channel SNR respectively, we have that

$$SNR_f = \sqrt{\frac{N\gamma_c\gamma_0}{\gamma_c + N(1 + \gamma_0)}}. \quad (5)$$

Figure 2 shows SNR_f as a function of the number of sensors with orthogonal codes and independent observations. We can see from Fig. 2 that SNR_f has a limit, and does not exceed that limit as we increase the number of sensors. Specifically,

$$\begin{aligned} \lim_{N \rightarrow \infty} SNR_f &= \lim_{N \rightarrow \infty} \sqrt{\frac{N\gamma_c\gamma_0}{\gamma_c + N(1 + \gamma_0)}} \\ &= \sqrt{\frac{\gamma_c}{1 + \frac{1}{\gamma_0}}}. \end{aligned}$$

That is, we cannot improve the fusion performance beyond a certain limit by injecting more sensors into the network when there is a fixed total power constraint on the system. This was also observed in [5].

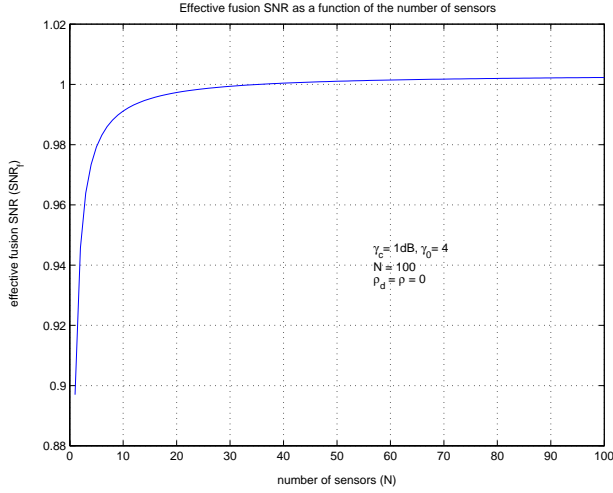


Fig. 2. Dependence of the effective SNR on the number of sensors N .

B. Non-orthogonal codes with independent noise at the sensors

Now suppose that the code cross-correlation matrix \mathbf{R} has the general form in (2) and that $\Sigma_v = \sigma_v^2 \mathbf{I}$. Then the fusion center effective SNR can be shown to be given by

$$SNR_f = mg \sqrt{\mathbf{e}^T (g^2 \sigma_v^2 \mathbf{I} + \sigma_w^2 \mathbf{R}^{-1})^{-1} \mathbf{e}}. \quad (6)$$

In the special case of $N = 2$, for example, (6) simplifies as

$$SNR_f = \sqrt{\frac{2\gamma_0}{1 + \frac{2(1+\gamma_0)}{\gamma_c(1+\rho)}}}.$$

This clearly shows that increasing the code correlation ρ improves the performance. In other words using the same channel for all sensors is better, than using sperate channels for each sensor, which corresponds to orthogonal communication. To show this in general, let us consider the two extreme cases of (6) as follows:

1. $\rho = 1$:

This means that all sensors use the same code \mathbf{s} . Let us consider the k -th sensor output and denote it as l_k . Then

$$l_k = g\mathbf{s}(m_k + v_k).$$

Denoting the received signal at the fusion center by \mathbf{z} , we have that

$$\begin{aligned} \mathbf{z} &= \sum_{k=1}^N g\mathbf{s}(mb + v_k) + \mathbf{w} \\ &= Ngmb\mathbf{s} + g\mathbf{s}\psi + \mathbf{w}, \end{aligned}$$

where $b = \pm 1$, $\psi = \sum_{k=1}^N v_k$ and the noise term $(g\mathbf{s}\psi + \mathbf{w})$ has the distribution $\mathcal{N}(0, (Ng^2\sigma_v^2 + \sigma_w^2)\mathbf{I})$. The fusion center correlates this signal with the code \mathbf{s} , so that the decision variable is

$$\mathbf{s}^T \mathbf{z} = Ngmb + \eta,$$

where $\eta \sim \mathcal{N}(0, Ng^2\sigma_v^2 + \sigma_w^2)$. Then the effective SNR at the fusion center can be shown to be given by

$$SNR_f = \sqrt{N \frac{\gamma_c \gamma_0}{1 + \gamma_c + \gamma_0}}. \quad (7)$$

It can be shown using the same approach that in the general case of dependent observation noise, with $\rho = 1$, the effective fusion SNR is given by

$$\begin{aligned} SNR_f &= \\ &= \sqrt{\frac{N\gamma_c}{\sigma_v^2(\gamma_0 + 1) + \frac{\gamma_c}{N} \mathbf{e}^T \Sigma_v \mathbf{e}}} \\ &= \sqrt{\frac{N\gamma_c}{\sigma_v^2(\gamma_0 + 1) + \frac{\gamma_c}{N} \left(\frac{2\sigma_v^2}{1-\rho_d} \left(N + 1 - \frac{1-\rho_d^{N+1}}{1-\rho_d} \right) - N \right)}} \end{aligned}$$

where in the last step we have assumed that Σ_v is given by (1).

2. $\rho = 0$:

Following the same method as above, it is easy to show that in this case (6) reduces to (5). Note that in the case of orthogonal communication, the matched filter correlates the channel noise in N orthogonal directions. On the other hand, for the case of $\rho = 1$, it correlates the channel noise only in one direction. As a result we can see from (5) and (7) that SNR_f in the case of $\rho = 1$ is larger than that in the case of $\rho = 0$. In particular SNR_f in (7) is strictly increasing with the number of sensors N . This means that we can improve the performance by increasing the number of nodes and using the same channel for all sensors. Figure 3 shows the improvement of the effective SNR in (6) as a function of code correlation.

Although the above result seems somewhat nonintuitive at the first glance, a closer look at the system reveals that there is in fact nothing surprising here. Essentially, the analog relay-amplifier local processing and non-orthogonal communications, in the case of deterministic signal detection effectively makes the distributed sensor system to act as a cooperative beam-former. In particular, it should be clear that with $\rho = 1$ the system has a perfectly directed beam towards the fusion center that exploits full coherent gain. However, on the other hand when $\rho = 0$, we use separate orthogonal channels to send essentially the same information thereby loosing the cooperative beamforming gain. Clearly, this behavior is a result of the assumption of perfect synchronicity of the whole distributed sensor system and the fusion center. In a practical scenario, however, there is a trade-off between the achievable synchronicity and the performance since resources need to be spent on achieving system synchronization.

III. ASYMPTOTIC FUSION PERFORMANCE

In order to analyze the asymptotic fusion performance of the system we use circulant matrix theory which states that a toplitz matrix goes to a circulant matrix as its dimensions grow [7]. Note that, the observation correlation matrix Σ_v is a toplitz matrix and that the code correlation matrix \mathbf{R}

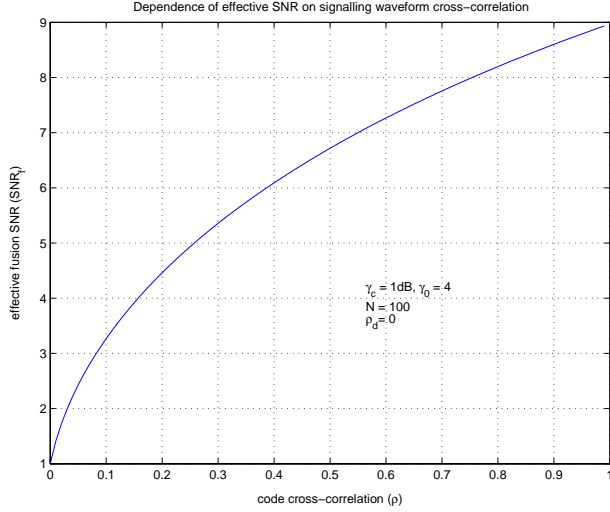


Fig. 3. Dependence of the effective SNR on signalling waveform (code) cross-correlation (Independent sensor observations) ρ .

is already a circulant matrix. Then the distribution of the decision statistic $T(\mathbf{u})$ can shown to be Gaussian with

$$\begin{aligned}\mathbb{E}\{T(\mathbf{u})|H_1\} &= m^2 g^2 \mathbf{e}^T \mathbf{R} \mathbf{C}^{-1} \mathbf{R} \mathbf{e} \\ &= m^2 g^2 \mathbf{e}^T \mathbf{R} (g^2 \mathbf{R} \Sigma_v \mathbf{R} + \sigma_w^2 \mathbf{R})^{-1} \mathbf{R} \mathbf{e}.\end{aligned}$$

After some simple algebra, we get

$$\mathbb{E}\{T(\mathbf{u})/H_1\} = \frac{m^2 g^2}{\sigma_w^2} \left(\mathbf{e}^T \mathbf{R} \mathbf{e} - \frac{\mathbf{e}^T}{\sigma_w^2} \left(\frac{1}{g^2} (\mathbf{R} \Sigma_v \mathbf{R})^{-1} + \frac{\mathbf{R}^{-1}}{\sigma_w^2} \right)^{-1} \mathbf{e} \right). \quad (8)$$

It is easy to show that the inner matrix in the term

$$\frac{\mathbf{e}^T}{\sigma_w^2} \left(\frac{1}{g^2} (\mathbf{R} \Sigma_v \mathbf{R})^{-1} + \frac{\mathbf{R}^{-1}}{\sigma_w^2} \right)^{-1} \mathbf{e} \quad (9)$$

goes to a circulant matrix. Then since the inverse of a circulant matrix is also circulant and using the fact that all circulant matrices have the same eigen-vectors, for asymptotically large dimensions, the term in (9) can be shown to be equal to

$$N \frac{\sigma_v^2 g^2 \lambda_M^2 \mu_M}{\sigma_w^2 + \sigma_v^2 g^2 \lambda_M \mu_M},$$

where λ_M and μ_M are the largest eigenvalues of the matrices \mathbf{R} and Σ_v , respectively. Thus substituting these in (8) and noting that

$$\mathbf{e}^T \mathbf{R} \mathbf{e} = N(1 + \rho(N - 1)),$$

we get

$$\begin{aligned}\mathbb{E}\{T(\mathbf{u})|H_1\} &= \frac{\gamma_c}{1 + \frac{1}{\gamma_0}} \\ &\times \left[1 + \rho(N - 1) - \frac{2\gamma_c(1 + \rho(N - 1))^2}{N(1 + \gamma_0)(1 - \rho_d) + 2\gamma_c(1 + \rho(N - 1))} \right]\end{aligned}$$

Similarly, we can show that

$$\mathbb{E}\{T(\mathbf{u})|H_0\} = -\mathbb{E}\{T(\mathbf{u})|H_1\}.$$

The variance of $T(\mathbf{u})$ is the same under the both hypotheses and is given by

$$\begin{aligned}\text{VAR}\{T(\mathbf{u})\} &= \frac{2\gamma_c}{1 + \frac{1}{\gamma_0}} \\ &\times \left[1 + \rho(N - 1) - \frac{2\gamma_c(1 + \rho(N - 1))^2}{N(1 + \gamma_0)(1 - \rho_d) + 2\gamma_c(1 + \rho(N - 1))} \right].\end{aligned}$$

Using these results the minimum probability of error is asymptotically given by

$$P_e = Q \left[\sqrt{\frac{\frac{\gamma_c}{1 + \frac{1}{\gamma_0}} \left[1 + \rho(N - 1) - \frac{2\gamma_c(1 + \rho(N - 1))^2}{N(1 + \gamma_0)(1 - \rho_d) + 2\gamma_c(1 + \rho(N - 1))} \right]}{2}} \right].$$

In Figs. (4), (5) and (6) we have shown the exact and asymptotic fusion performance. As can be seen from these plots, the derived asymptotic expressions provide a reasonably good approximation even for small number of sensors. Furthermore, it allows us to gain some insight into the effect of each parameter on the system performance. For example, from Fig. (4) we can see the detrimental effect of the observation correlation ρ_d on system performance. This can be understood by noting that as the observation correlation increases the new information added by each additional sensor decreases thus degrading the fusion performance. However, it also shows the improvement that we can get from using non-orthogonal communication. Figure (5) shows clearly that the fusion center probability of error tends to have a performance floor as we decrease the code correlation value ρ . Finally, from Fig. (6) we can see that using a large channel correlation ρ results in having an acceptable performance even for a relatively large observation correlation ρ_d . This means that the system has some kind of robustness against the large values of observation correlation when using non-orthogonal communication. As mentioned earlier, this in fact can be understood either as a cooperative beamforming or a coherent gain.

IV. CONCLUSIONS

In this paper we investigated the decentralized detection and fusion performance of a total power constrained sensor network in a noisy channel under the assumption of analog relay-amplifier local processing. We showed that the performance can be monotonically improved using non-orthogonal sensor-to-fusion center communication. This somewhat nonintuitive result, in fact, is due to the fact that the analog relay-amplifier local processing and non-orthogonal communications, in the case of deterministic signal detection, effectively makes the distributed sensor system to act as a cooperative beam-former. In order to obtain asymptotic performance we made use of the circulant matrix theory. The derived asymptotic expressions show a good approximation for the exact results even for a small number of sensors. These asymptotic expressions also allowed us to gain some insight into the effect of each system parameter on the final fusion performance.

V. ACKNOWLEDGMENT

This research was supported by Kansas National Science Foundation (NSF) EPSCOR program under the grants KUCR # NSF32223/KAN32224 and KUCR # NSF32241.

REFERENCES

- [1] A. J. Goldsmith and S. B. Wicker, "Design challenges for energy-constrained ad-hoc wireless networks," *IEEE Wireless Commun.*, pp. 8–27, Aug. 2002.
- [2] P. Withington, H. Flusher, and S. Nag, "Enhancing homeland security with advanced uwb sensors," *IEEE Microwave Mag.*, Sep. 2003.
- [3] "Special report: Sensor nation," *IEEE Spectrum*, July 2004.
- [4] A. Mainwaring, J. Polastre, R. Szewczyk, D. Culler, and J. Anderson, "Wireless sensor networks for habit monitoring." Atlanta, GA: in 1st ACM Int. Workshop on Wireless Sensor Networks and Applications, Sep. 2002, pp. 88–97.
- [5] J. F. Chamberland and V. V. Veeravalli, "Decentralized detection in wireless sensor systems with dependent observations," in *International Conference on Computing, Communications and Control Technologies*, Austin, TX, Aug. 2004.
- [6] S. K. Jayaweera, "Large system decentralized detection performance under communication constraints," *IEEE Comm Letters*, vol. 9, pp. 769 – 771, Sep 2005.
- [7] R. M. Gray, "Toeplitz and circulant matrices: A review," Aug. 2002, <http://ee.stanford.edu/~gray/toeplitz.pdf>.

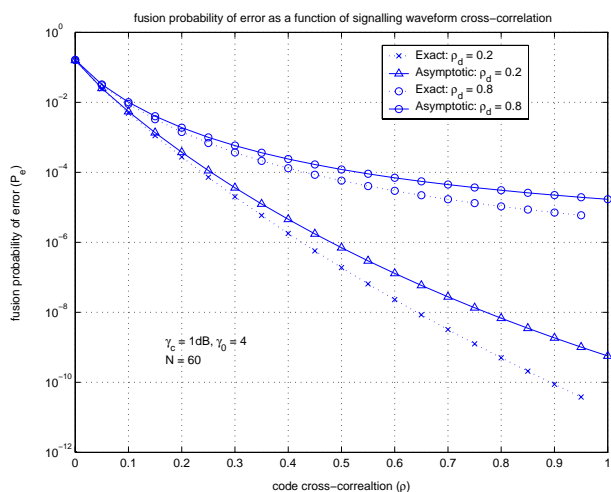


Fig. 4. Dependence of the fusion probability of error on signalling waveform (code) cross-correlation ρ .

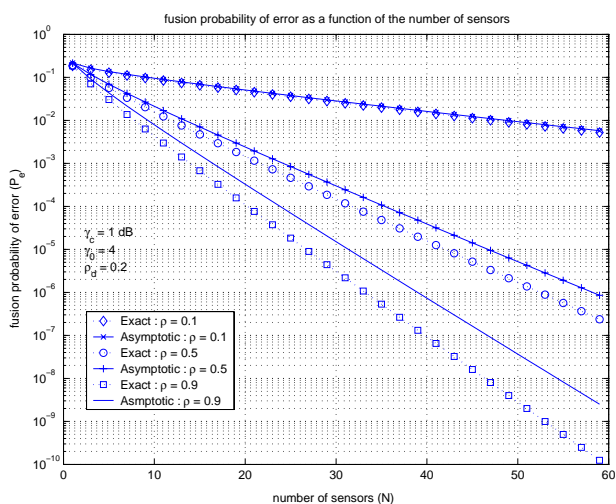


Fig. 5. Dependence of the fusion probability of error on the sensor network size N .

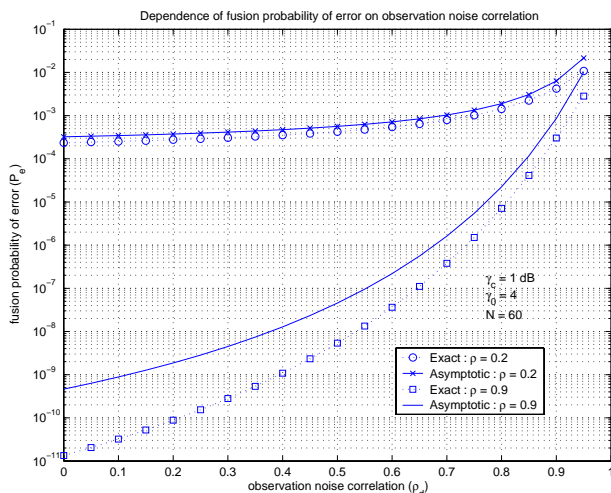


Fig. 6. Dependence of the fusion probability of error on local observation cross-correlation parameter ρ_d .