Effects of Source and Load Impedance

This slide set shows an alternative means of analyzing transmission lines with src and load impedance (from your text book).

When a signal is introduced on a transmission line (at the src end), only a fraction of the full src voltage propagates down it.

The fraction depends on the frequency and is called $A(\omega)$, the **input acceptance** function.

The value of $A(\omega)$ depends on the src impedance, Z_S, and the transmission line *characteristic impedance*

$$Z_0(\omega) = \sqrt{\frac{R + j\omega L}{j\omega C}} \longrightarrow A(\omega) = \frac{Z_0(\omega)}{Z_S(\omega) + Z_0(\omega)}$$

As the signal propagates, it is attenuated by the propagation function

$$H(\omega) = e^{-l\gamma(\omega)}$$

Effects of Source and Load Impedance

At the far end of the cable a fraction of the attenuated signal amplitude *emerges*.

This fraction is a function of frequency and is called $T(\omega)$, the **output transmission** function

$$T(\omega) = \frac{2Z_L(\omega)}{Z_L(\omega) + Z_0(\omega)} \qquad \text{(ranges from 0 to 2)}$$

A reflected portion of the signal also travels back toward the src.

As it reflects, the signal crosses over the tail of the incoming signal but does not interfere with it.

The fraction of the signal reflected is called $R_2(\omega)$, the **far end reflection** function

$$R_2(\omega) = \frac{Z_L(\omega) - Z_0(\omega)}{Z_L(\omega) + Z_0(\omega)}$$

Effects of Source and Load Impedance

The reflected signal is again attenuated by $H(\omega)$ and reflects off of the src impedance.

The **src end reflection coefficient**

$$R_1(\omega) = \frac{Z_S(\omega) - Z_0(\omega)}{Z_S(\omega) + Z_0(\omega)}$$

This reflection and attenuation continues, and each **emerging signal at the load**, S_i, is described as shown in the following figure, i.e.,

$$S_0(\omega) = A(\omega)H(\omega)T(\omega)$$

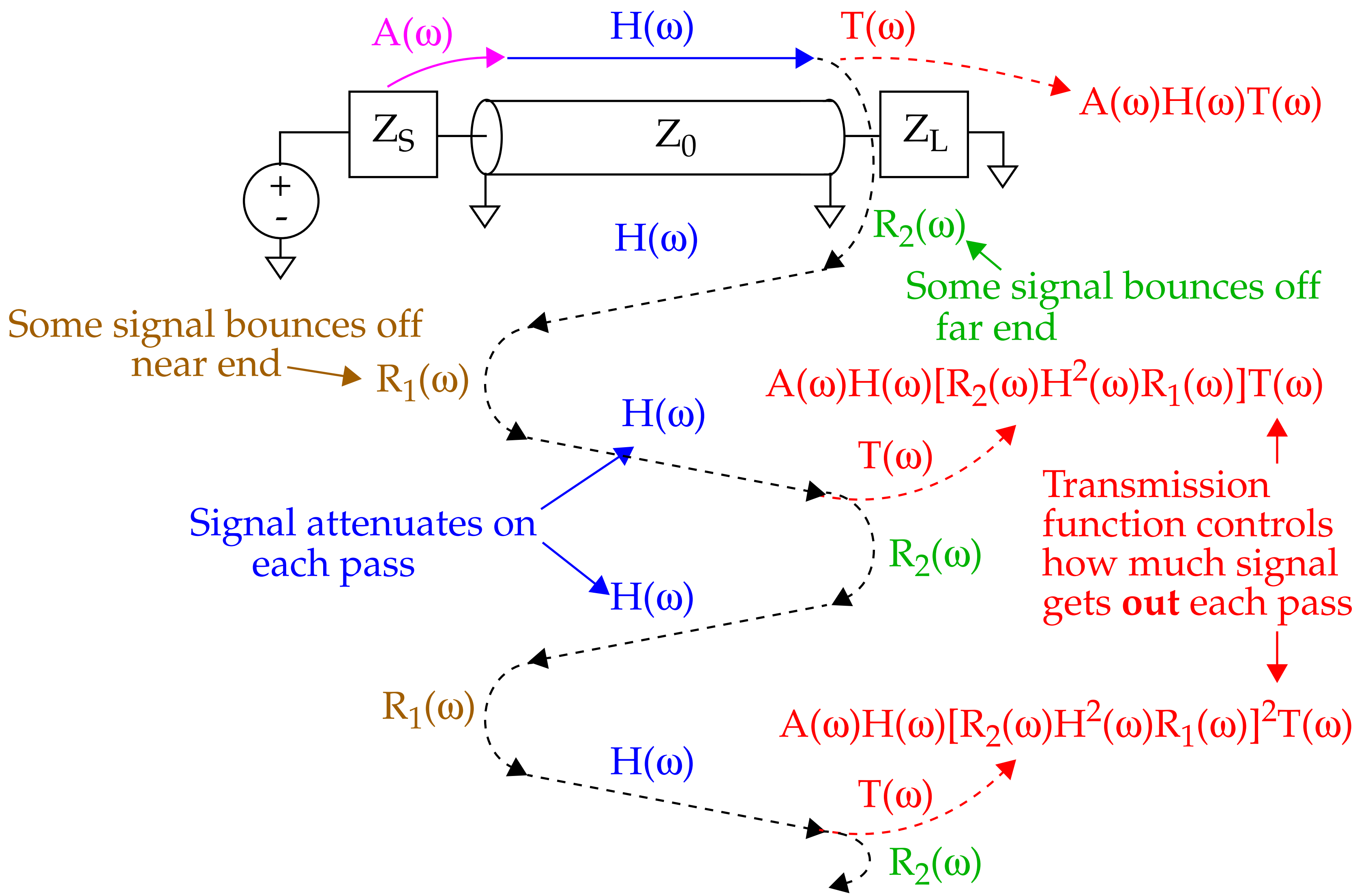
Eventually, all signals N = [0, 1, ..., *inf*] emerge and their sum is given by:

$$S_\infty(\omega) = \sum_{N=0}^{\infty} S_N(\omega)$$

$$S_\infty(\omega) = \frac{A(\omega)H(\omega)T(\omega)}{1 - R_2(\omega)H^2(\omega)R_1(\omega)}$$ (closed form solution)

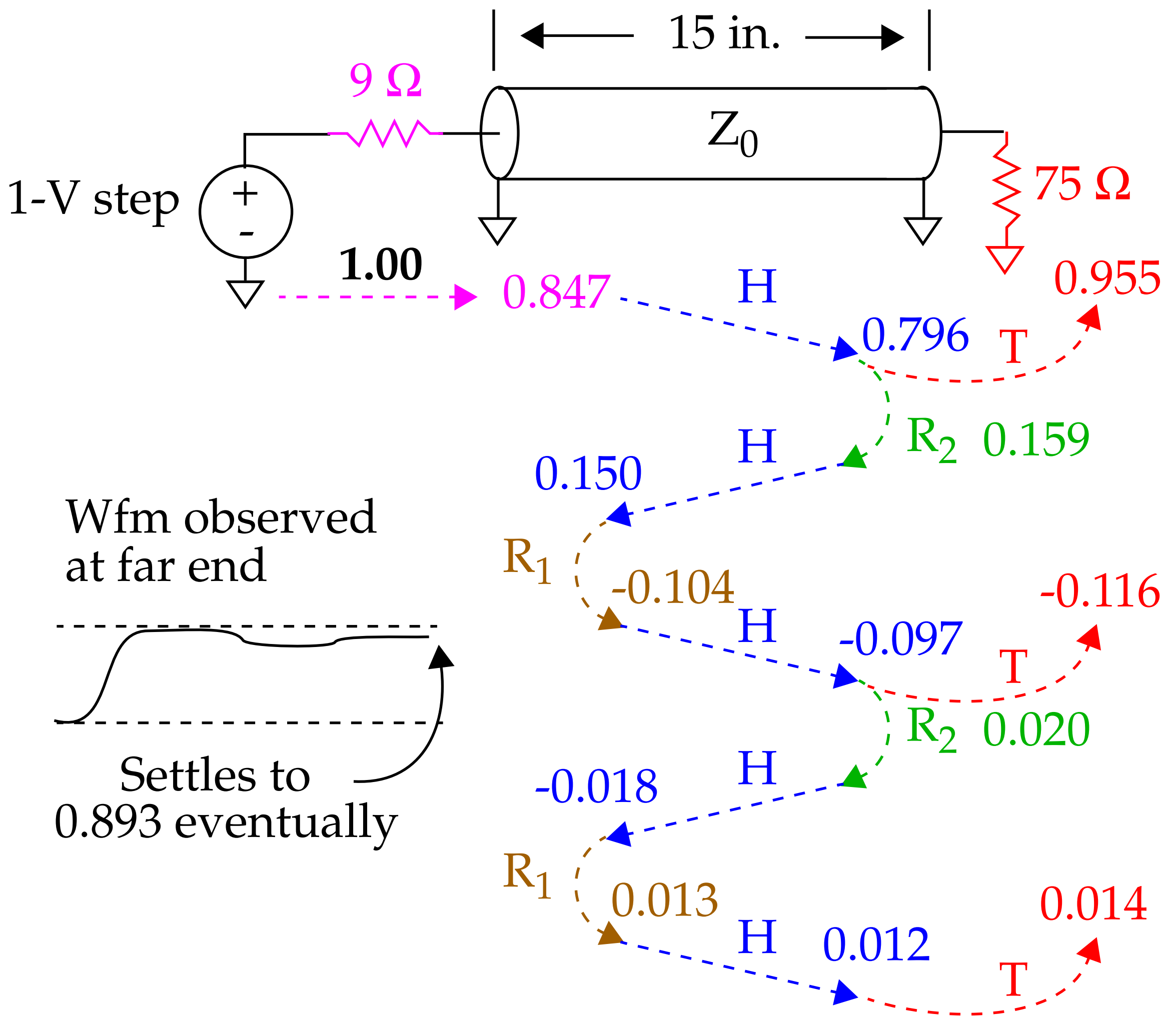
Effects of Source and Load Impedance

Refl ections



Effects of Source and Load Impedance

In this example, the total wire resistance is only 1.2 Ω and therefore we can ignore it and use $Z_0(\omega)$ = 50 Ω (simplifies situation to make refl ections real)



Trace width = 0.006 in.
1-oz. copper
Series R_{DC} = 0.081 Ω/in
Risetime = 1 ns
F_{knee} = 500 MHz
Skin effect R = 0.4 Ω/in.
Total series R = 6 Ω

$A(\omega)$ = 0.847
$H(\omega)$ = 0.94
$T(\omega)$ = 1.200
$R_2(\omega)$ = 0.200
$R_1(\omega)$ = -0.695

50 Ω stripline
ε_r = 4.5
T_p = 180 ps/in.
2700 ps total delay

Effects of Source and Load Impedance

How can we control reflections on a transmission line?

Combining the $T(\omega)$ and $R_2(\omega)$ equations:

$$T(\omega) = \frac{2Z_L(\omega)}{Z_L(\omega) + Z_0(\omega)} \qquad T(\omega) = R_2(\omega) + 1$$

$$R_2(\omega) = \frac{Z_L(\omega) - Z_0(\omega)}{Z_L(\omega) + Z_0(\omega)} \qquad S_\infty(\omega) = \frac{A(\omega)H(\omega)(R_2(\omega) + 1)}{1 - R_2(\omega)H^2(\omega)R_1(\omega)}$$

Assuming that $H(\omega)$ is fixed, we can control two parameters, src and load impedance.

The src impedance controls $A(\omega)$ and $R_1(\omega)$ while the load impedance controls $R_2(\omega)$.

For a good digital transmission, we want a flat frequency response up to at least the *knee* frequency.

Effects of Source and Load Impedance

There are three accepted methods for ensuring a fl atfrequency response from

$$S_\infty(\omega) = \frac{A(\omega)H_x(\omega)(R_2(\omega)+1)}{1 - R_2(\omega)H_x^2(\omega)R_1(\omega)}$$

These include *end termination*, *series termination* and *short line*.

End Termination:

This method sets $R_2(\omega)$ to zero.

$$S_{\text{end term}} = H(\omega)A(\omega)$$

This eliminates the first refl ection (all energy exits the cable) and any delayed versions of the original signal.

To achieve $R_2(\omega) = 0$, just set $Z_L = Z_0$.

As we indicated, for long cables operating in RC mode, finding a terminating network that matches the Z_0 over a wide frequency range is a challenge.

Effects of Source and Load Impedance

Source Termination: This method sets $R_1(\omega)$ to zero.

$$S_{\text{src term}} = H(\omega)A(\omega)[R_2(\omega) + 1]$$

Physically, this eliminates the second refl ection but not the first.

To achieve $R_1(\omega) = 0$, just set $Z_S = Z_0$.

Note that with $Z_S = Z_0$, the acceptance function becomes 1/2.

This is usually compensated at the far end by leaving the line *unterminated*, $Z_L = inf$.

This, in turn, sets $T(\omega) = 2$ (and $R_2(\omega) = 1$), i.e., the line voltage is **doubled** at its end, compensating for the halving.

The drawback is the large signal that refl ects back to the src because $R_2(\omega) = 1$.

Circuits connected between the src and end see a mixed signal, a half sized signal propagating down followed by the other half.

Effects of Source and Load Impedance

Short Line: This method sets *H(ω)* ~= unity (because the line is short).

Under this condition, there is no significant attenuation or phase delay.

$$S_{\infty}(\omega) = \frac{A(\omega)(R_2(\omega)+1)}{1-R_2(\omega)R_1(\omega)}$$

Substituting our previous expressions for these terms and simplifying yields

$$S_{\text{short line}}(\omega) = \frac{Z_L(\omega)}{Z_L(\omega)+Z_S(\omega)}$$

Here, we see the line has no effect and we get a simple *impedance divider.*

However, the line must act as a **lumped circuit** element, i.e. it must be shorter than 1/6 the electrical length of the rising edge.

$$\text{Max Length} < \frac{1}{6}\frac{T_{rise}}{\sqrt{LC}}$$

L = line inductance, H/in.

C = capacitance, F/in.

Special Cases

Unterminated is used to indicate when the src or load impedance does not match the characteristic impedance.

In many cases, *unterminated* load impedances tend to be higher while *unterminated* src can be higher or lower than the characteristic imp.

Assume load impedance is large ($R_2(\omega)$ ~= 1 and $T(\omega)$ ~= 2) in the following analysis that considers the relative value of src impedance.

Low src impedance with unterminated line

Occurs when a resistive, low-impedance output (ECL or high-powered TTL bus driver) drives a transmission line.

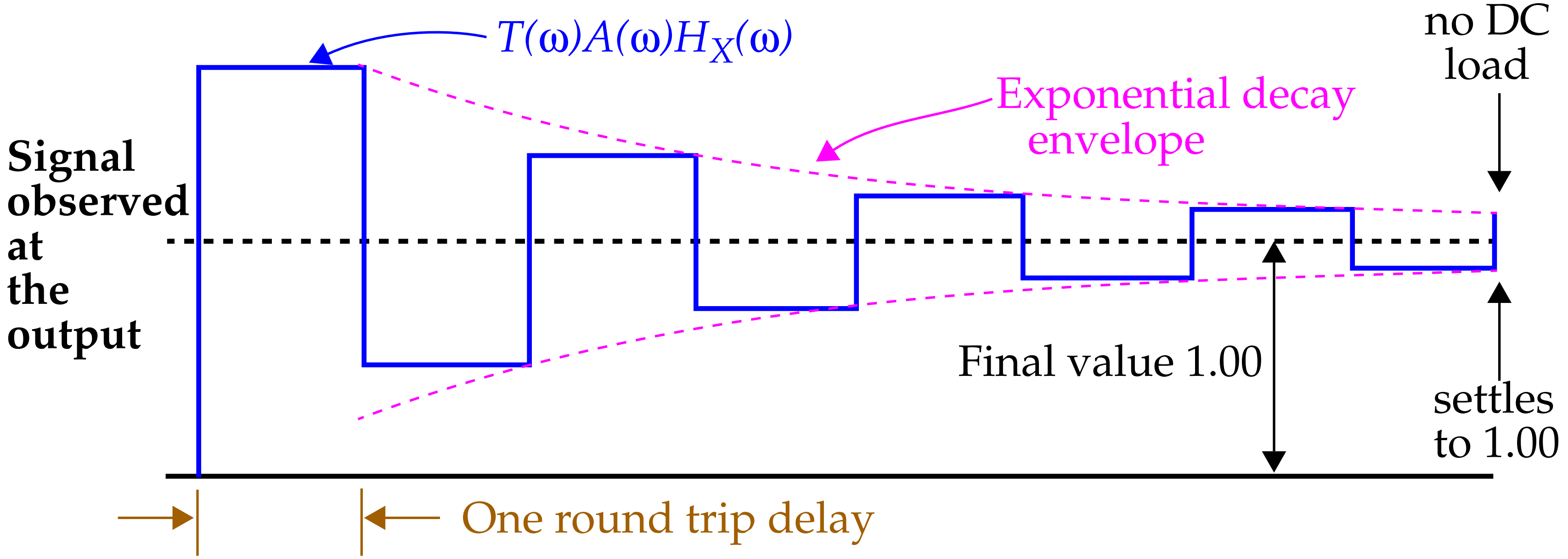
Easy to predict the unit step response here, $A(\omega)$ is near unity and $T(\omega)$ is near +2, the product is then near +2V for a unit step input.

Here, $R_1(\omega)$ is nearly -1 (some loss in the line), and so is the product $R_1(\omega)R_2(\omega)$.

Special Cases

The negative sign for $R_1(\omega)R_2(\omega)$ indicates that successive refl ected signals have opposite signs and the response oscillates around the final value.

Four line traversals occur between responses on the output of the same polarity (each line delay is given by the previous equation.)



This overshoot occurs when the *rise time* is shorter than the *round trip delay*.

This causes current to fl ow in the input protection diodes in TTL and CMOS inputs and introduces *ground bounce*.

Special Cases

High src impedance with unterminated line

This occurs when a high-impedance output, e.g. unbuffered CMOS output, drives a transmission line.

$A(\omega)$ in this case is small while $T(\omega)$ is near +2 resulting in a small initial step output.

The reflection coefficient $R_1(\omega)$ is slightly less than +1 (some loss in the line) so the product $R_1(\omega)R_2(\omega)$ is also almost +1

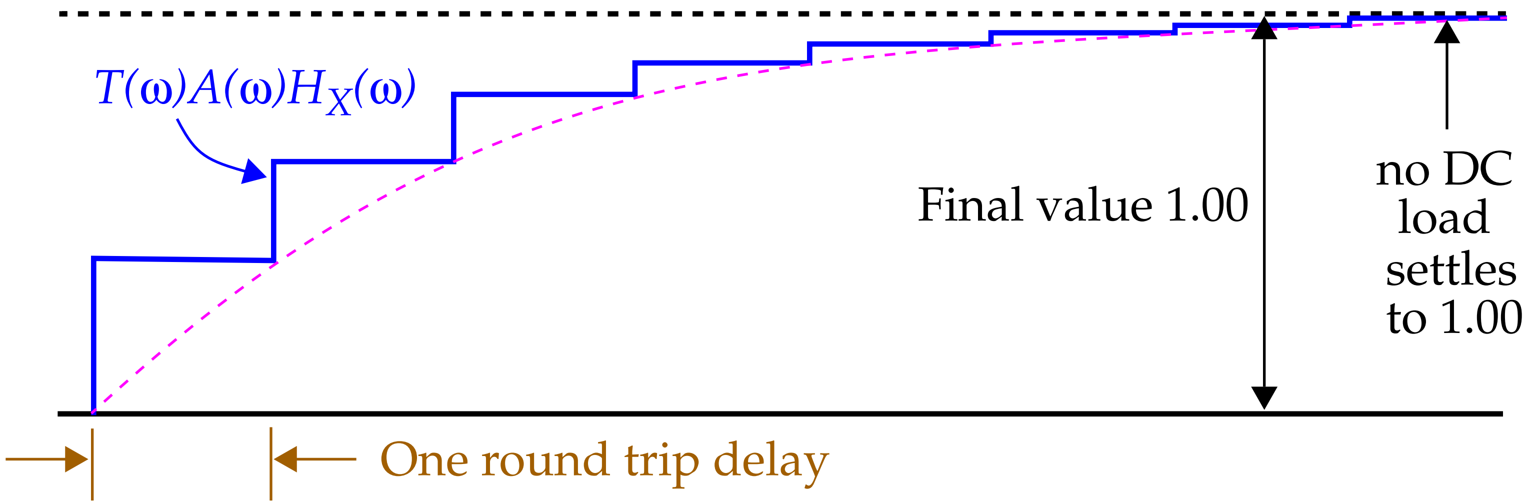
The positive sign for $R_1(\omega)R_2(\omega)$ indicates that successive reflected signals have the same signs and the response "builds" to the final value.

The final response looks like an *RC* filter.

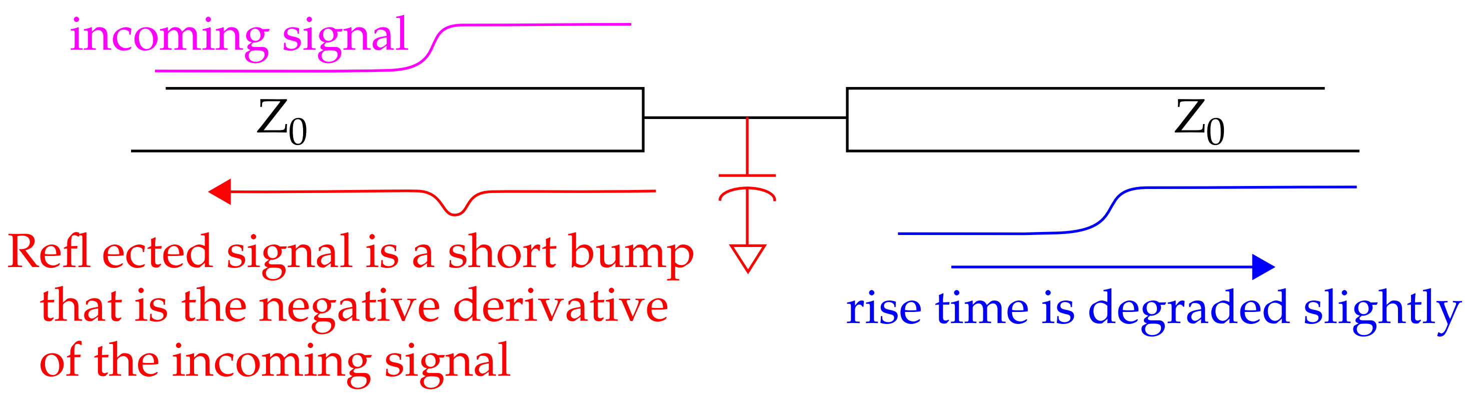
The *time constant* is approximated by the src impedance times the total (lumped) line capacitance.

Special Cases

High src impedance with unterminated line



Capacitive loads connected in the middle of the line



Special Cases

The incoming signal in the circuit shown above is split into a *reflected* portion and a portion that *propagates through*.

The refl ectioncoefficient is a function of frequency -- to compute it, let's start with:

$$R_2(\omega) = \frac{Z_L(\omega) - Z_0(\omega)}{Z_L(\omega) + Z_0(\omega)}$$

Here, we are required to specify the line, Z_0, and terminating impedances, Z_L.

The left section of the transmission line terminates at the capacitor.

So the total terminating load is equal to the reactance of the cap in parallel with the input impedance of the right section.

Without knowing the right hand section's termination conditions, we can't compute the input impedance.

Let's assume *end termination* and *low-loss* (not *RC*) operation region.

Special Cases

Under these conditions, the input impedance is frequency independent:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Therefore, refl ections do not occur (or occur too late to make a diference).

We can substitute the **parallel** combination of C and Z_0 for Z_L as:

$$R_2(\omega) = \frac{Z_L(\omega) - Z_0(\omega)}{Z_L(\omega) + Z_0(\omega)} \longrightarrow R_C(\omega) = \frac{-j\omega C Z_0}{2 + j\omega C Z_0}$$

Refl ection is almost*total* for frequencies above:

$$f_{max} = \frac{1}{C Z_0 \pi}$$

For frequencies below f_{max}, the refl ection coeficient returns a pulse equal to the derivative of the input step.

$$R_C(\omega) = \frac{-j\omega C Z_0}{2 + j\omega C Z_0} \approx \frac{-j\omega C Z_0}{2} \longleftarrow$$

Looks like a negative differentiator

Special Cases

The peak amplitude, P, can be estimated (assuming f_{knee} is below f_{max}):

$$P = C\frac{Z_0}{2}\frac{(-\Delta V)}{T_{rise}}$$

ΔV is incoming voltage step size

Z_0 is the high frequency line imp. $(L/C)^{1/2}$

For the signal transmitted through the right section to the capacitive load, again assume $Z_0 = (L/C)^{1/2}$.

The transmission coefficient is then given by:

$$T(\omega) = R_2(\omega) + 1 \longrightarrow T_c(\omega) = 1 + R_C(\omega) = \frac{1}{1 + j\omega C(Z_0/2)}$$

which is the equation for a low-pass filter with time constant $C(Z_0/2)$.

The 10-90% rise time of this step response will be 2.2. times the τ:

$$T_{\text{10-90 (step response)}} = 2.2C\frac{Z_0}{2}$$

Special Cases

Here, we see the capacitive load deteriorates the rise time of signals propagating past it.

The rise time of the propagating signal can be computed from:

$$T_{\text{rise composite}} = \sqrt{(T_1^2 + T_2^2 + \ldots + T_N^2)}$$

This mixes the rise time of the incoming signal with the capacitor rise time.

These approximation hold if:

- The transmission line is terminated in both directions or
- The transmission line is longer (in both directions) than the rising edge.