Cryptography

Handbook of Applied Cryptography & http://cseweb.ucsd.edu/users/mihir/cse207/

Brief History:

- Proliferation of computers and communication systems in 1960s brought with it a demand to protect *digital information*
  - Feistel's research at IBM in early 1970 lead to *Data Encryption Standard* (DES)
  - Diffie and Hellman published "New Directions in Cryptography" in 1976, and described the revolutionary concept of *public-key cryptography*
  - Rivest, Shamir, and Adleman in 1978 described first practical public-key encryption and signature scheme known as *RSA*
  - ElGamal in 1985 described a second algorithm based on the *discrete logarithm problem*

- More recently:
  - Joan Daemen and Vincent Rijmen algorithm called *Advanced Encryption Standard* (AES) became a NIST standard in 2001
  - Guido Bertoni, Joan Daemen, Michaël Peeters, and Gilles Van Assche algorithm called *Keccak* adopted as SHA-3 standard by NIST in 2015

Cryptography

Cryptography is the study of **mathematical techniques** related to aspects of information security including *confidentiality*, *data integrity*, *authentication*, and *non-repudiation*

Tenets of Information Security:

- **Confidentiality (Privacy)**: Techniques designed to keep information secret
- **Data integrity**: Methods for ensuring information has not been altered
- **Authentication**: Methods and protocols for establishing the source of a message
- **Non-repudiation**: Techniques that undeniably bind messages to entities

Others:

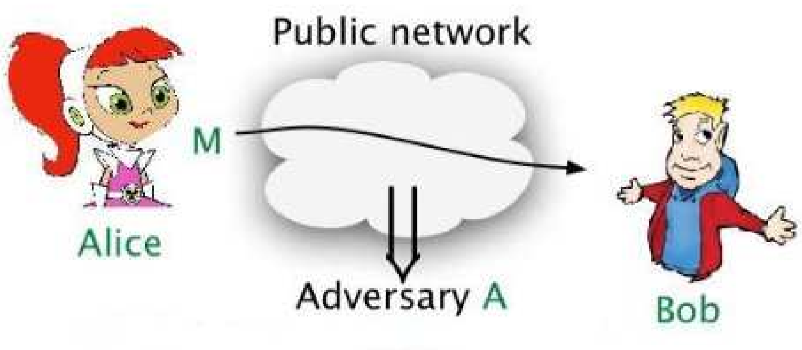
- Anonymity/Privacy: Concealing the identity of an entity involved in some process
- Authorization: Conveyance, to another entity, of official sanction to do something
- Validation: A means to provide timeliness of authorization to use information
- Access control: Restricting access to resources to privileged entities
- Certification: Endorsement of information by a trusted entity
- Witnessing: Verifying the creation or existence of information by an entity other than the creator

Cryptography

Crypto-systems all around us

- ATM machines
- Remote logins using SSH
- Web browsers (https invokes Transport Layer Security (TLS))

Cryptography ensures security of communication across an insecure medium



Adversary is clever person with a powerful computer

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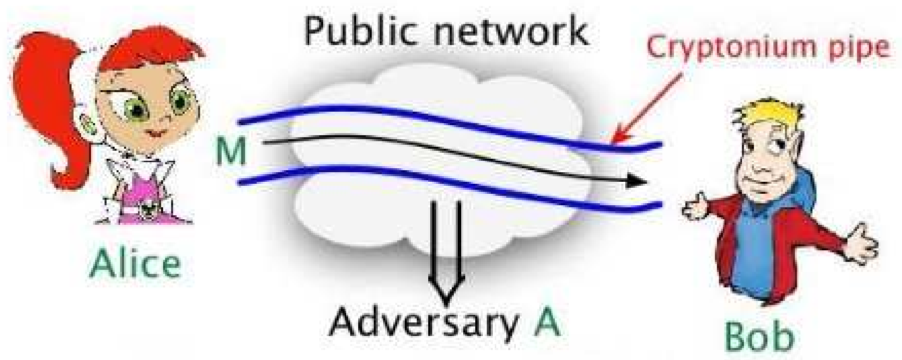
Cryptography

Ideal channel doesn't exist



Cryptonium pipe: Cannot see inside or alter content.

Impenetrable pipe between sender and receiver that no one else can see inside or change what's there

Cryptography cannot achieve all the properties of an ideal pipe

Instead, a few central security goals are targeted, *privacy* and *authenticity/integrity*

Protocols are defined that are designed to achieve these security goals

Cryptography

A protocol is a collection of programs, one for the sender and one for the receiver

Sender program uses a **cryptographic key** to encapsulate while receiver program reverses the process

A **trust model** specifies who has what keys

- Symmetric (shared-key) trust model
- Asymmetric (public-key) trust model

In the **symmetric** model, the sender and receiver *share* a **secret key** that the adversary does not know

Note the secure distribution of the key is not part of the symmetric model (or any model), rather the model specifies how the key is generated and used

Distributing and keeping a key secure is the domain of *computer systems security*

A protocol that is used to provide confidentiality in the symmetric setting is called a **symmetric encryption scheme**

Symmetric Encryption Scheme

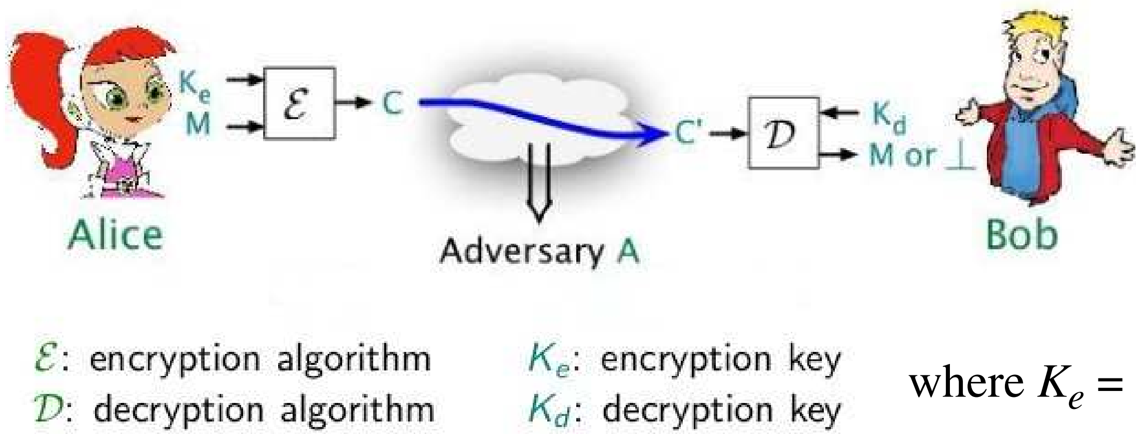
For a symmetric encryption scheme Π, we must specify three algorithms

$$\Pi = (H, E, D)$$

where E represents the encryption algorithm

H represents the algorithm that generates the key K

D represents the decryption algorithm



where $K_e = K_d$

M represent the *message*, which is also called the *plaintext*

Sender encrypts the M by applying algorithm E to K_e and M to obtain *ciphertext*

Decryption may fail and produce "false" or "contradiction"

Symmetric Encryption Scheme

Note that no security scheme can guarantee confidentiality

It can only be evaluated on the grounds that it provides some **probability** of preventing the adversary from breaking it

The *message authentication* problem, in which the receiver can verify the sender, is addressed in the symmetric scheme using a **message authentication code** (MAC)

$\Pi = (H, T, V)$ (Three algorithms)

Sender computes a 'tag' or MAC, σ, by applying T using the shared key K and message M and then transmits the pair (M, σ)

The receiver uses the key K to check if the tag is OK by applying the *verification* algorithm V with K, M and σ.

If algorithm returns '1', message is accepted as authentic

(https://en.wikipedia.org/wiki/Hash-based_message_authentication_code)

Symmetric Encryption Scheme

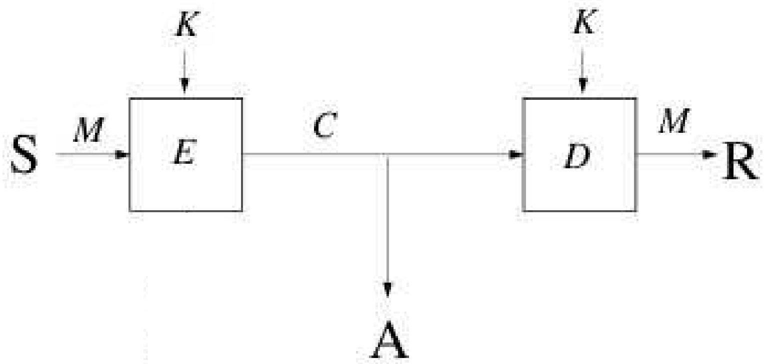


Figure 1.3: Symmetric encryption. The sender and the receiver share a secret key, K. The adversary lacks this key. The message M is the plaintext; the message C is the ciphertext.

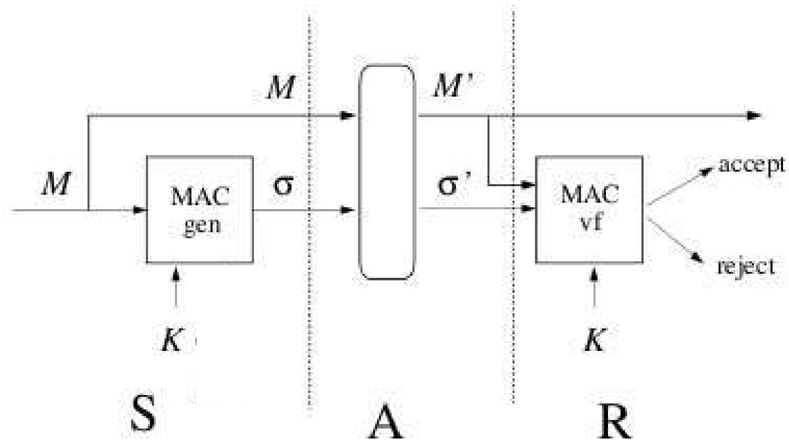


Figure 1.4: A message authentication code. The tag σ accompanies the message M. The receiver R uses it to decide if the message really did originate with the sender S with whom he shares the key K.

Asymmetric Encryption Scheme

In the asymmetric setting, an individual possesses a pair of keys-- a public key, *pk*, and an associated secret key, *sk*

Also called **public-key** setting

The public key is made *publicly known*, e.g., placed in phone book, and bound to its identity

For encryption, the sender is **assumed** to be able to obtain an **authentic copy** pk_R of the receiver's public key (adversary also knows pk_R)

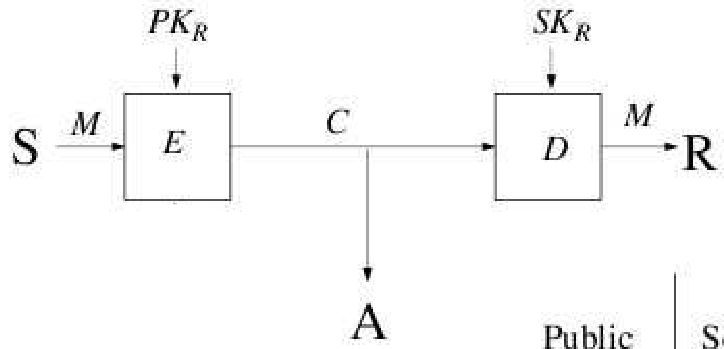
Sender computes ciphertext C <- $E_{pkR}(M)$ and sends C to receiver

Receiver computes M <- $D_{skR}(C)$ using the receiver's secret key sk_R

This is a very useful mechanism

This allows you to look up the receiver's public key and send him/her a message that no one else can read -- even if you've never met the receiver!

Asymmetric Encryption Scheme



	Public	Secret
$R:$	PK_R	SK_R

Figure 1.5: Asymmetric encryption. The receiver R has a public key, pk_R, which the sender knows belongs to R. The receiver also has a corresponding secret key, sk_R.

The tool used for solving the *message-authentication* problem in the asymmetric setting is a **digital signature**

The sender has a public key pk_S and a corresponding secret key sk_S

The receiver (and adversary) is assumed to know the key pk_S and that it belongs to S

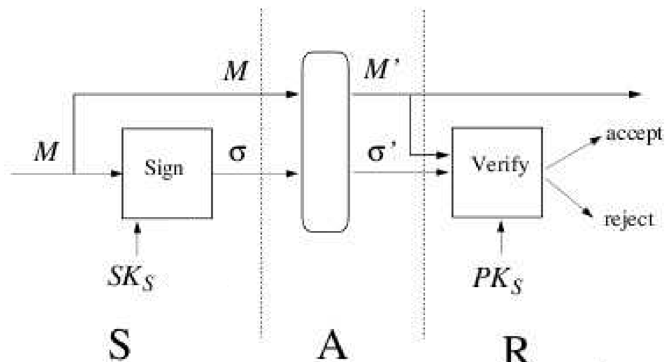
Asymmetric Encryption Scheme

The sender attaches to the message M some extra bits σ (called the *signature*)

The *signature* is computed as a function of M and sk_S using the **Sign** algorithm

The receiver on receipt of M and σ, checks that it is OK using the sender's public key pk_S by applying a **verification** algorithm V - V either accepts or rejects.

$\Pi = (K, \text{Sign}, V)$ (Three algorithms)



Public	Secret
$S: PK_S$	SK_S

(https://en.wikipedia.org/wiki/Diffie%E2%80%93Hellman_key_exchange#Description)

Asymmetric Encryption Scheme

In summary:

	symmetric trust model	**asymmetric trust model**
message privacy	symmetric (a.k.a. private-key) encryption	asymmetric (a.k.a. public-key) encryption
message authenticity	message authentication code (MAC)	digital signature scheme

Figure 1.7: Summary of main goals and trust models.

One difference between MAC and the digital signature concerns the notion of *non-repudiation*

With MAC, anyone who can verify a tagged message can also produce one -- which suggests that authenticity canNOT be proved in a *court of law*

In contrast, a digitally-signed message, the ONLY person who should be able to produce M that verifies under the public key pk_S is the S herself

So S **cannot** claim that the receiver presenting the evidence concocted it

Asymmetric Encryption Scheme

If signature σ authenticates M with pk_S, then it is only S that should have been able to construct σ

S cannot refute that fact

All that sender S can claim is that key sk_S was stolen -- but that may still leave sender S responsible

Other Goals of Cryptography

• Pseudorandom Number Generation

Lots of applications require *random* numbers, including simulation, efficient algorithms and cryptography (key generation and randomized encryption algo)

A pseudorandom number generator is *deterministic* in that it takes a *seed* and produces a sequence of random numbers, that is repeatable for the same seed

The *seed* is a key element -- the task of random number generation is reduced to the task of generating a short random seed

Other Goals of Cryptography

- Pseudorandom Number Generation

  The generation of the seed can be done using a Geiger counter or by computing some function of various system parameters such as time and system load

  The most important element of seed generation is that the process be *completely unpredictable*

- Authenticated Key Exchange

  It is common for an individual to establish a **secure session**

  For example, remote login to a computer or in a web-browsing session

  For situations in which a secret key is shared (symmetric) or public keys are available (asymmetric), a secure session can be established using cryptography

  However, this is not how it is done, rather the parties use their existing keys (*long-lived keys*) to derive a *session key*

  Which is done through a *authenticated key exchange* protocol

  (https://en.wikipedia.org/wiki/Key-agreement_protocol)

Other Goals of Cryptography

- Coin Flipping

Alice and Bob need to decide who gets the car in a divorce -- Alice proposes that she flip a coin and then have Bob 'call it'

The challenge is how do you implement a system that assures that neither tries to cheat and that each learns the true outcome of the coin toss

Solution: Alice puts a random bit α inside an envelope and sends it to Bob -- Bob announces a random bit β -- Bob opens the envelope and both compute α xor β (the shared bit)

The scheme in which Alice puts a value in an envelope and Bob can't see it is called a *commitment scheme* in cryptography

Other Goals of Cryptography

- Coin Flipping

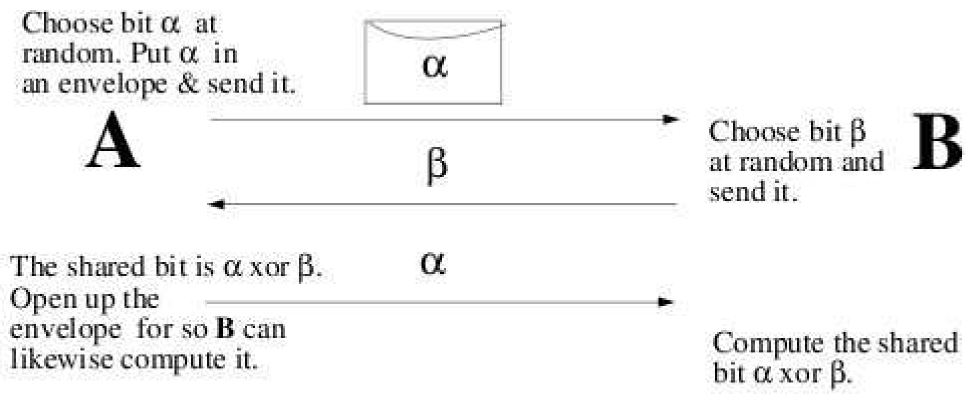


Figure 1.8: Envelope solution to the telephone-coin-flipping 5problem.

A simple scheme to implement the *envelope*: Alice puts a "0" in an envelope by choosing two random 500-bit primes p and q subject to

$p < q$

$p = 1 \pmod 4$

$q = 3 \pmod 4$

The product $N = p*q$ is the commitment to zero, i.e., what Alice would send to commit to "0"

Other Goals of Cryptography

- Coin Flipping

Alice puts a "1" in an envelope by choosing two random 500-bit primes p and q subject to

$p < q$

$p = 3 \pmod 4$

$q = 1 \pmod 4$

The product $N = p*q$ is the commitment to "1"

Poor Bob, seeing N, would like to determine if the smaller of its two prime factors is equal to 1 or to 3 (mod 4)

Unfortunately, there is no way of determining that short of **factoring** N

And we don't know how to factor a 1000 digit number which is the product of 500-digit primes

When Alice wants to *decommit* N (open the envelope), she announces p and q

Bob confirms they are prime (easy), multiply to N, and determines the result

What is Cryptography About?

Cryptography is about constructing and analyzing *protocols*, which are designed to be extremely difficult for adversaries to break

Cryptography problems involve *parties* (humans, computers, gadgets, etc) playing the role as the *good guys*

We design protocols for the parties to use

The protocol tells each party *how to behave* and is typically a (distributed) program

Protocols tell the parties what to do, but not the adversary -- they can do anything

A protocol can be *probabilistic* (can make random choices) and have *state*

$$i \overset{\$}{\leftarrow} \{0, 1, \ldots, n-1\}$$

The former is formalized as 'a random value i from the set on the right is returned (with all values equally likely) for a value $n >= 2$ selected by a party'

What is Cryptography About?

How do we devise and analyze protocols?

The first step is to understand the **threats** and **goals** of our particular problem

Once understood, we then attempt to find a protocol solution

The **adversary** is the agent that embodies the *source* of the threat -- their goal is to defeat our protocol's goals

So it is a game -- who is more clever, the protocol designer or the adversary

In formal analysis, the parties may *vanish* (absorbed in the formalization) but the adversary NEVER vanishes and remains at center stage

Cryptography is largely about thinking about the adversary -- what can she do? what is she trying to accomplish?

These questions need answers before we can make progress

Cryptography and Computer Security

Good protocols are the key element of making secure computing systems

Good protocol design is also **hard** and easy to underestimate

Secure systems involve a combination of different factors and define **system security**

An adversary should **not** be able to exploit bugs, break into your system and use your account, buy off your system administrator, steal backup tapes, etc.

The security of a system is only as strong as its weakest link -- for secure systems, all aspects must be addressed

- How do we secure our machines against intruders?
- How do we administer machines to maintain security?
- How do we design good protocols?

A cryptographic protocol is just one piece of the puzzle

There are rules

- We can only overcome the adversary by means of protocols (can't poison her coffee)
- Only *keys* are secret -- protocols should be made *public*

Modern Cryptography

The *systematic* approach to cryptography (where proofs and definitions play a visible role) began with the work of Claude Shannon (father of information theory)

He developed the notion of **perfect security**

Unfortunately, perfect security is not practical because it requires the number of bits in the *key* to be at least as large as the number in the *message*

Modern cryptography introduces a new dimension: the amount of *computing power* available to the adversary

Focus on schemes that are breakable *in principle* but NOT in practice

This takes cryptography from the realm of information theory to computer science

In particular, to *complexity theory* where we study how hard problems are to solve as a function of computational resources invested

The following (probabilistic) principle is the basis:

Assuming the adversary uses **no more** than t computing cycles, her probability of breaking the scheme is at most $t/2^{200}$

Modern Cryptography

There is no notion of *how* the adversary operates, what algorithms or techniques are employed -- which is good because we don't know them

To understand how we get protocols with such properties, let's first look at **atomic primitives**

Higher level protocols are built on top of atomic primitives

Protocols address a cryptographic problem by specifying *how* we encrypt, *how* we distribute a key

Atomic primitives, on the other hand, are generally simpler protocols that have some sort of *hardness* or *security* properties, but by themselves are not useful

Atomic primitives are drawn from two sources:

- Engineered constructs, e.g., **blockciphers** such as AES algorithm
- Mathematical problems, e.g., ECC

Modern Cryptography

Based on the above, modern cryptography is based on computationally *hard* problems

Originally, *NP-complete* problems were looked at for cryptography but failed because they are hard to solve in the worst case, and easy on average

A more suitable example is a **one-way function** f: D->R mapping some domain D to range R, with the following properties:

- f is easy to compute, there is an efficient algorithm given x in D outputs $y = f(x)$ in R
- f is hard to **invert**: an adversary I given a random y in R has a hard time figuring out a point x such that $f(x) = y$ (as long as her computing time is limited)

As noted above, the latter property will be expressed in terms of *probability*

One-way functions are ubiquitous in the real world, e.g., smashing a mirror is easy

Number theory provides a very simple one that is based on *multiplication*

The function f takes as input 2 numbers a and b, and multiplies them to produce $N = a*b$

Modern Cryptography

Interestingly, there is no known algorithm that given a random $N = a*b$, always and quickly recovers a pair of numbers (excluding 1 or N) that are factors of N

The backwards direction is called the *factoring* problem -- a problem that has remained unsolved for hundreds of years

A second example: Let p be prime and consider the function $f(x) = g^x \bmod p$

This is called the *discrete exponentiation* function and its inverse the *discrete logarithm* function: $\log_g(y)$ is the value x such that $y = g^x$

It turns out that there is **no known** fast algorithm that computes discrete logarithms

Given a large enough p, e.g., 1000 bits, the task is *infeasible* given the state of computing power today

Note, like **P** vs **NP**, there is no proof that these are hard functions to invert

We *assume* they are, but if someone comes up with a fast algorithm, a lot of protocols will fail!